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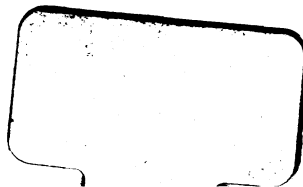
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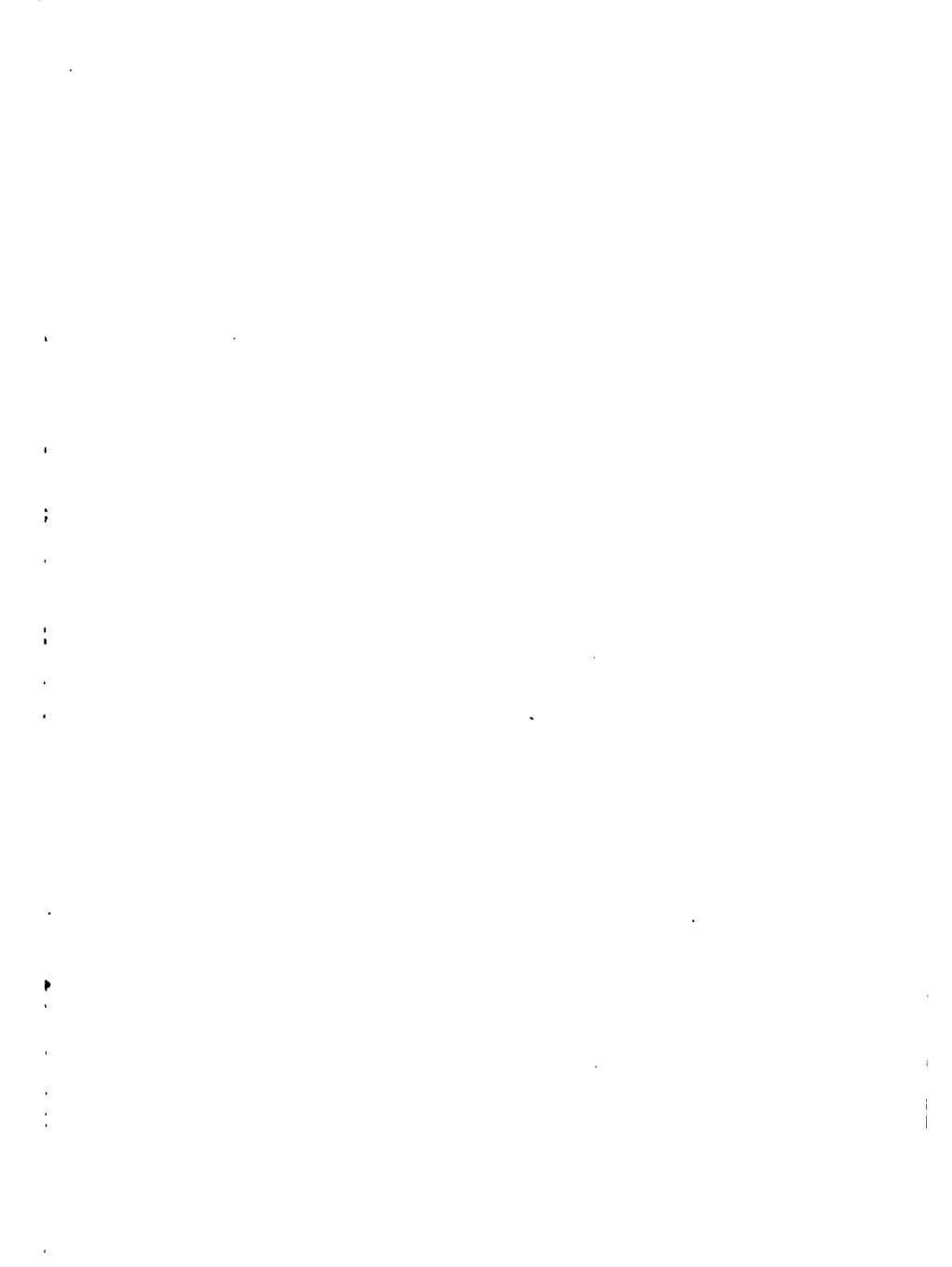
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MILNE'S NEW YORK STATE ARITHMETIC

FIRST BOOK

BY

WILLIAM J. MILNE, Ph.D., LL.D.

PRESIDENT OF NEW YORK STATE COLLEGE FOR TEACHERS

ALBANY, N. Y.



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N. Y. STATE ARITHMETIC, FIRST BOOK.

E. P. 8

PREFACE

THIS series of arithmetics has been written to supply the demand for a sound and modern interpretation of present-day educational ideas concerning the teaching of arithmetic. It is intended especially to meet the need for a definite and complete interpretation of the Course of Study in Arithmetic prescribed by the Education Department of the State of New York.

The series consists of two books. The "First Book" for years three, four, and five, is divided into half-year units, but within each half year the treatment is topical. The "Second Book" is a complete topical arithmetic, covering, in order, the Syllabus requirements for years six, seven, and eight.

The following features of the "First Book" are worthy of attention:

1. The conservative modern treatment that exploits no hobbies.
2. The brief, clear, and definite instruction that carries the work by easy steps, at every stage, to some complete result.
3. The solutions that illustrate the more difficult rather than the easier instances in each case, thus giving help where it is most needed.
4. The exceptionally large number of well-graded abstract drills, both oral and written. The numbers in

these drills are not so large as to cause discouragement nor too small to be useful as a test of power. •

5. The large number of short, easy, practical, and up-to-date problems, concisely and clearly stated, and adapted to the experience of the child.
6. The training for efficiency afforded by the numerous tests and exercises for time work.
7. The systematic reviews, both topical and miscellaneous.

WILLIAM J. MILNE.

CONTENTS

THIRD YEAR

FIRST HALF		SECOND HALF	
	PAGE		PAGE
Reading and Writing Numbers — units to millions	7	Review — topical	63
Addition	13	Division — multiplication and division tables, 10's, 11's, and 12's; short division, divisors of two figures; long divi- sion, divisors of two figures; dividends and divisors end- ing in zeros; divisors of three figures	80
Subtraction	19	Divisors and Factors — tests of divisibility; factoring	96
Multiplication — tables through the 9's; multipliers of one figure, of two figures, of three or more figures	29	Measures — liquid; length; area; squares and rectangles .	99
Division — tables through the 9's; divisors of one figure . .	44	Review — miscellaneous . . .	104
Halves and Fourths	54		
Measures — length; drawing to scale; area	57		

FOURTH YEAR

FIRST HALF		SECOND HALF	
Reading and Writing Numbers — Arabic numerals (review); Roman numerals	111	Divisors and Multiples — great- est common divisor; least common multiple	171
Fundamental Operations (Re- view)	116	Fractions — reduction to higher or lower terms (applying g.c.d.), to least common de- nominator (applying l.c.m.); addition and subtraction of fractions, of mixed numbers; multiplication of fractions, three cases; division of frac- tions, three cases; fractional relations	174
United States Money — coins; making change; reading and writing United States money; addition; subtraction; mul- tiplication; division; prob- lem making	126	Analysis — unitary; by com- parison; problem interpreta- tion; problem making	206
Bills and Accounts	144	Volume Measures	212
Cancellation	147	Review — miscellaneous . . .	216
Dry Measures	152		
Fractions — halves to twelfths; reading and writing frac- tions; principles; reduction; addition and subtraction . .	154		

FIFTH YEAR

FIRST HALF	PAGE	SECOND HALF	PAGE
Decimal Fractions — notation and numeration; reduction; addition; subtraction; multiplication; division; review exercises	223	Review — topical	287
Aliquot Parts	256	Denominate Numbers — liquid measures; dry measures; weight measures; review of length, area, and volume measures; time measures; circular measures; value measures; counting table; stationers' measures; addition and subtraction; multiplication; division . . .	297
Measures and Measurements — length; scale drawings; area; rectangles; parallelograms; triangles; lathing and plastering; painting and calcimining; carpeting; volume; rectangular solids; wood; capacity equivalents; capacity of bins, cisterns, etc.; excavation and masonry	263	Business Problems — bills and statements of account; receipts; simple accounts . .	326
		Review — miscellaneous . . .	334
		Tables — denominate number; multiplication; division . .	343
Glossary			347
Index			351

THIRD YEAR — FIRST HALF

READING AND WRITING NUMBERS

UNITS, TENS, AND HUNDREDS

1. Count by ones, or *units*, to 10 and write the numbers as you count; by *tens* to 100.



Read each number :

2. 30 5. 12 8. 93

3. 40 6. 17 9. 28 11. 42 13. 74 15. 95

4. 70 7. 14 10. 36 12. 57 14. 63 16. 49

Write with figures :

- | | | |
|-------------|----------------|-------------------|
| 17. Forty. | 21. Fifteen. | 25. Ninety-two. |
| 18. Sixty. | 22. Fifty-six. | 26. Forty-seven. |
| 19. Eighty. | 23. Nineteen. | 27. Twenty-four. |
| 20. Eleven. | 24. Sixty-one. | 28. Seventy-five. |

29. Count by *hundreds* and write, filling the blanks :

- | | | |
|-----------------|------------------|------------------|
| 1 hundred, 100. | 4 hundred, ____. | 7 hundred, ____. |
| 2 hundred, 200. | 5 hundred, ____. | 8 hundred, ____. |
| 3 hundred, 300. | 6 hundred, ____. | 9 hundred, ____. |

30. Count by *tens* from 100 to 200 and write :

- | | |
|------------------------|------------------------|
| 1 hundred ten, 110. | 1 hundred forty, ____. |
| 1 hundred twenty, 120. | 1 hundred fifty, ____. |
| 1 hundred thirty, 130. | 1 hundred sixty, ____. |

31. Read: 600 230 720 450 890 780

32. Count by units from 110 to 120: thus, "one hundred eleven, one hundred twelve," etc., and write the numbers 111, 112, etc., as you count.

33. Name the numbers in order from 120 to 170, writing them in columns of ten numbers each.

Read:

34. 123 36. 345 38. 567 40. 789 42. 915

35. 324 37. 456 39. 678 41. 524 43. 894

44. Name and write the numbers in order from 100 to 110: thus, "one hundred one," 101; "one hundred two," 102; and so on.

45. 724 means 7 hundreds 2 tens and 4 units.

503 means — hundreds no tens and — units.

Read, and tell what each figure stands for:

46. 492 49. 568 52. 409 55. 532 58. 904

47. 777 50. 860 53. 290 56. 861 59. 999

48. 395 51. 602 54. 456 57. 692 60. 703

Write with figures:

61. Five hundred forty-six. 65. Two hundred thirty.

62. Six hundred sixty-three. 66. One hundred eleven.

63. One hundred ninety-two. 67. Eight hundred eight.

64. Seven hundred eighty-one. 68. Nine hundred ninety.

The *first* figure, counting from right to left, stands for units; the *second* for tens; and the *third* for hundreds.

THOUSANDS

Ten hundred are **one thousand**, written **1000**. The *fourth* figure from the right stands for **thousands**.

1. Count by *thousands* and write, filling the blanks:

1 thousand, 1000. 4 thousand, ——. 7 thousand, ——.

2 thousand, 2000. 5 thousand, ——. 8 thousand, ——.

3 thousand, ——. 6 thousand, ——. 9 thousand, ——.

2. Name and write the numbers in order from 1000 to 1010: thus, "one thousand one," 1001; "one thousand two," 1002; and so on.

3. Count and write ten numbers in order from 1010.

Name and write ten numbers in order beginning with:

4. 2098 5. 3110 6. 4253 7. 6892

8. Read: 9000 900 90 9 9999

What does 9 mean when it stands in thousands' place?
in hundreds' place? in tens' place? in units' place?

Tell what each figure means in these numbers:

9. 160 11. 3261 13. 7000 15. 3509 17. 6450
10. 375 12. 5824 14. 4073 16. 5006 18. 2090

Read:

19. 706 24. 1038 29. 2500 34. 9604 39. 5005
20. 850 25. 6433 30. 9774 35. 5646 40. 6756
21. 634 26. 8888 31. 3230 36. 7002 41. 3022
22. 509 27. 7103 32. 4112 37. 6960 42. 8004
23. 976 28. 4658 33. 9781 38. 3629 43. 7902

Written Exercises

Write these numbers with figures :

1. Eight hundred three.
2. Seven thousand four.
3. One thousand ninety.
4. Nine hundred twenty-nine.
5. Five thousand two hundred.
6. Three thousand seventy-three.
7. Nine thousand six hundred two.
8. Two thousand one hundred fifteen.
9. Six thousand nine hundred fifty-six.
10. Four thousand four hundred forty-four.
11. Eight thousand seven hundred sixty-six.
12. Seven thousand three hundred ninety-one.

TEN-THOUSANDS AND HUNDRED-THOUSANDS

1. Count (and write the numbers) by thousands from 1 thousand to 10 thousand, **10,000**.

2. Count (and write the numbers) by 10's to 100.

3. Count by 10 *thousands* and write, filling the blanks :

10 thousand, 10,000. 60 thousand, ____.

20 thousand, 20,000. 70 thousand, ____.

30 thousand, _____. 80 thousand, ____.

40 thousand, _____. 90 thousand, ____.

50 thousand, _____. 100 thousand, **100,000**.

In writing numbers as large as these, we put a comma between hundreds' place and thousands' place.

4. Read: 13,000 45,000 89,000 76,000 91,000

Write in figures:

5. Sixteen thousand. 8. Ninety-five thousand.

6. Fifty-one thousand. 9. Forty-eight thousand.

7. Sixty-six thousand. 10. Eighty-four thousand.

11. Count and write the thousands in order from 98 thousand (98,000) to 108 thousand (108,000).

12. Read: 125,000 570,000 80,000 999,000 278,000

Write in figures:

13. Four hundred thousand; one hundred fifty thousand.

14. Nine hundred thousand; six hundred seven thousand.

15. Five hundred thousand; eight hundred ten thousand.

16. Two hundred thousand; three hundred twelve thousand.

Count and write the numbers in order from:

17. 40 thousand 5 (40,005) to 40 thousand 15.

18. 75 thousand 96 (75,096) to 75 thousand 106.

19. 200 thousand 994 (200,994) to 201 thousand 4.

20. 409 thousand 997 (409,997) to 410 thousand 7.

Read, then write in words:

21. 24,635 23. 85,306 25. 423,532 27. 240,605

22. 50,028 24. 66,004 26. 708,029 28. 999,999

Write in figures:

29. Twenty-one thousand, five hundred fourteen.

30. Forty-six thousand, seven hundred eighty-three.

31. Six hundred seventy-two thousand, one hundred.

MILLIONS

The next number of thousands after 999 thousand is 1000 thousand, called **one million, 1,000,000**.

We write any number of millions up to 999 million just as we write any number of thousands up to 999 thousand or any number of units up to 999 units, the millions' group being written at the left of the thousands' group and separated from it by a comma, thus:

9 9 9,	9 9 9,	9 9 9
MILLIONS	THOUSANDS	UNITS

Written Exercises

Read, and then write in words:

- | | | |
|--------------|---------------|-----------------|
| 1. 5,000,000 | 5. 46,000,000 | 9. 625,000,000 |
| 2. 4,250,000 | 6. 72,894,000 | 10. 333,591,000 |
| 3. 2,463,941 | 7. 88,125,290 | 11. 897,279,505 |
| 4. 9,207,008 | 8. 65,437,044 | 12. 400,030,648 |

Write in figures:

- | | |
|--------------------|----------------------------|
| 13. Nine million. | 16. Eighty-three million. |
| 14. Four million. | 17. Five hundred million. |
| 15. Sixty million. | 18. Seven hundred million. |
19. 1 million, 478 thousand, 677.
 20. 30 million, 783 thousand, 834.
 21. 984 million, 71 thousand, 250.
 22. Two hundred thirty-four million, seven thousand.
 23. Nine hundred one million, forty-two thousand, fifty.
 24. Six hundred six million, six hundred thousand, six.

ADDITION

In addition,

The numbers added are **addends**.

The result found by adding is the **sum**.

The sign $+$ is the **sign of addition**. It is read **and** or **plus**.

The sign $=$ is the **sign of equality**. It is read **are** or **equal** (sometimes **is** or **equals**).

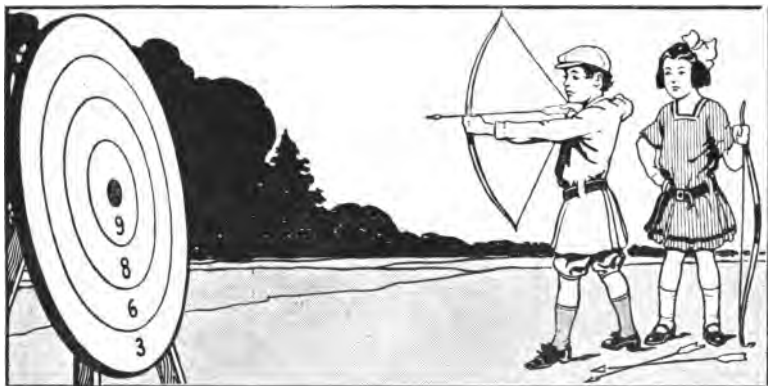
Oral Exercises

Following are the *forty-five* addition combinations. Tell sums instantly by rows; then by columns:

	6.	7.	8.	9.	10.	11.	12.	13.	14.
1.	5	4	7	5	6	2	5	8	3
	<u>1</u>	<u>3</u>	<u>2</u>	<u>5</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>
2.	3	5	1	4	6	7	2	6	4
	<u>2</u>	<u>4</u>	<u>1</u>	<u>2</u>	<u>4</u>	<u>5</u>	<u>1</u>	<u>3</u>	<u>4</u>
3.	7	6	9	9	8	3	8	4	9
	<u>7</u>	<u>2</u>	<u>3</u>	<u>8</u>	<u>4</u>	<u>3</u>	<u>5</u>	<u>1</u>	<u>2</u>
4.	9	7	9	6	8	5	7	9	8
	<u>6</u>	<u>4</u>	<u>1</u>	<u>6</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>9</u>	<u>6</u>
5.	7	9	8	7	9	8	6	8	9
	<u>6</u>	<u>5</u>	<u>8</u>	<u>3</u>	<u>7</u>	<u>1</u>	<u>5</u>	<u>7</u>	<u>4</u>

Oral Exercises

These children are shooting at the target. Each child in turn has two shots, the score being the sum of the numbers given in the rings hit by the arrow.



Find each score :

JOHN'S		RUTH'S		JOHN'S		RUTH'S		JOHN'S		RUTH'S	
1.	6	2.	9	3.	6	4.	3	5.	9	6.	8
	3		6		8		9		8		3
	—		—		—		—		—		—

Tell what number to add to

- | | | |
|----------------------------|------------------------|-----------------|
| 7. 2 to make 5 | 10. 3 to make 6 | 13. 5 to make 7 |
| 8. 6 to make 9 | 11. 4 to make 7 | 14. 3 to make 9 |
| 9. 4 to make 8 | 12. 2 to make 9 | 15. 6 to make 8 |
| 16. 2 and how many are 10? | 4 and how many are 11? | |
| 17. 9 and how many are 13? | 6 and how many are 14? | |
| 18. 7 and how many are 10? | 9 and how many are 17? | |
| 19. 8 and how many are 15? | 7 and how many are 13? | |

Oral Exercises

1. Count by 2's from 0 to 50; 1 to 51.
2. Count by 3's from 0 to 51; 1 to 52; 2 to 53.
3. Count by 4's from 0 to 52; 1 to 53; 2 to 54; 3 to 55.

Count by 5's from :

- | | | |
|-------------|-------------|--------------|
| 4. 0 to 100 | 6. 2 to 102 | 8. 4 to 49 |
| 5. 1 to 101 | 7. 3 to 103 | 9. 49 to 104 |

Count by 6's from :

- | | | |
|--------------|--------------|--------------|
| 10. 0 to 102 | 12. 2 to 104 | 14. 4 to 100 |
| 11. 1 to 103 | 13. 3 to 105 | 15. 5 to 101 |

Add 1, 2, 3, 4, 5, 6, 7, 8, 9 each to the numbers in the table, giving sums quickly by rows; then by columns :

	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.
16.	1	12	24	31	46	58	63	75	86	97
17.	3	11	26	32	42	56	65	78	81	93
18.	4	15	25	34	41	55	67	72	84	96
19.	2	13	21	33	45	57	62	77	89	91
20.	5	14	29	35	43	59	64	71	83	95
21.	7	16	23	37	44	60	66	74	85	98
22.	6	20	22	36	49	54	61	79	82	94
23.	9	18	30	39	47	52	69	73	90	99
24.	8	19	27	38	50	53	70	76	88	92
25.	10	17	28	40	48	51	68	80	87	100

Turn to this table frequently for drill.

Written Exercises

1. Find the sum of 7, 9, and 6.

$$\begin{array}{r}
 7 \\
 9 \\
 6 \\
 \hline
 22
 \end{array}$$

Adding from the bottom toward the top we have 6 and 9 are 15 and 7 are 22, but we add as quickly as we can think, "6, 15, 22."

Test the result by adding downward: "7, 16, 22."

Add quickly and test each result:

2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
5	4	8	6	9	2	9	7	4	1
1	2	7	5	2	6	5	7	9	3
<u>3</u>	<u>6</u>	<u>9</u>	<u>2</u>	<u>1</u>	<u>7</u>	<u>8</u>	<u>3</u>	<u>5</u>	<u>7</u>
12.	13.	14.	15.	16.	17.	18.	19.	20.	21.
3	5	8	2	7	9	8	2	9	3
9	3	4	0	8	3	2	5	7	5
<u>7</u>	<u>4</u>	<u>6</u>	<u>9</u>	<u>5</u>	<u>4</u>	<u>1</u>	<u>3</u>	<u>4</u>	<u>9</u>
22.	23.	24.	25.	26.	27.	28.	29.	30.	31.
4	8	9	5	8	2	6	5	1	9
3	2	9	4	7	4	7	9	2	9
6	1	6	3	9	5	2	0	3	9
<u>5</u>	<u>7</u>	<u>3</u>	<u>6</u>	<u>4</u>	<u>8</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>9</u>

32. 43 Add the units and tens separately, thus:
 25 For the units, we think "5, 8," and write 8.
 68 For the tens, we think "2, 6," and write 6.

33.	34.	35.	36.	37.	38.	39.	40.
23	25	37	63	41	56	75	89
<u>24</u>	<u>64</u>	<u>52</u>	<u>35</u>	<u>38</u>	<u>23</u>	<u>13</u>	<u>10</u>

Written Exercises

1. Find the sum of 267, 593, and 486.

We write the addends so that the units stand in one column, the tens in another, and so on. Adding each column from the bottom up, we think :

267	For the units, "6, 9, 16," and write 6.
593	For the tens, "1 (carried), 9, 18, 24," and write 4.
486	For the hundreds, "2 (carried), 6, 11, 13," and write 13.
<u>1346</u>	The sum is 1346. Read the sum.
	Test by adding in the opposite direction.

Add and test :

2.	3.	4.	5.	6.	7.	8.	9.
12	51	29	44	71	864	728	531
48	94	83	52	84	973	619	427
<u>63</u>	<u>75</u>	<u>67</u>	<u>13</u>	<u>97</u>	<u>251</u>	<u>304</u>	<u>298</u>
10.	11.	12.	13.	14.	15.	16.	17.
34	43	98	57	89	789	637	841
95	66	12	46	23	456	513	937
<u>87</u>	<u>29</u>	<u>36</u>	<u>39</u>	<u>56</u>	<u>734</u>	<u>598</u>	<u>724</u>
18.	19.	20.	21.	22.	23.	24.	25.
46	57	85	75	44	628	479	731
78	81	29	96	78	572	722	695
<u>69</u>	<u>94</u>	<u>56</u>	<u>67</u>	<u>82</u>	<u>965</u>	<u>816</u>	<u>824</u>

26. How many cents are 38 cents and 45 cents ?
27. Find the sum of 185 letters and 219 letters.
28. How many nuts are 26 nuts, 17 nuts, and 25 nuts ?
29. Find the sum of 84 pens, 156 pens, and 228 pens.

Written Exercises

* Add and test, timing yourself for exercises 22-28:

1.	2.	3.	4.	5.	6.	7.
98	43	189	631	5173	7498	5097
76	32	273	809	428	9176	7643
<u>65</u>	<u>21</u>	<u>364</u>	<u>372</u>	<u>6905</u>	<u>532</u>	<u>3528</u>
8.	9.	10.	11.	12.	13.	14.
12	27	398	493	5678	6152	1597
55	94	23	578	102	5189	603
87	85	760	12	456	4376	716
<u>49</u>	<u>61</u>	<u>851</u>	<u>767</u>	<u>7892</u>	<u>9218</u>	<u>4829</u>
15.	16.	17.	18.	19.	20.	21.
95	46	318	568	7429	3928	9873
82	39	967	437	687	4761	549
73	97	42	615	52	5679	8692
64	13	586	729	6349	9543	2054
<u>51</u>	<u>52</u>	<u>245</u>	<u>48</u>	<u>9827</u>	<u>8397</u>	<u>7436</u>
22.	23.	24.	25.	26.	27.	28.
13	27	674	767	9621	5687	8195
97	43	983	493	4539	4321	4870
84	59	749	398	2180	9074	5264
65	61	321	851	8765	6298	9572
<u>27</u>	<u>88</u>	<u>576</u>	<u>634</u>	<u>6928</u>	<u>7937</u>	<u>3439</u>

* In all such time exercises throughout the book, it is intended that the teacher shall keep the record of each pupil, and a few days later have the same exercises worked again with the idea of having the pupils beat their previous records.

SUBTRACTION

In subtraction,

The number from which we subtract is the **minuend**.

The number subtracted is the **subtrahend**.

The result found by subtracting is the **difference**, or **remainder**.

The sign $-$ is the **sign of subtraction**. It is read **less** or **minus**.

Oral Exercises

Give differences quickly by rows; then by columns:

	6.	7.	8.	9.	10.	11.	12.	13.	14.
1.	8 <u>3</u>	11 <u>4</u>	13 <u>9</u>	7 <u>5</u>	14 <u>6</u>	10 <u>7</u>	11 <u>2</u>	9 <u>6</u>	13 <u>7</u>
2.	10 <u>5</u>	9 <u>3</u>	11 <u>5</u>	6 <u>2</u>	8 <u>4</u>	12 <u>3</u>	15 <u>7</u>	17 <u>8</u>	12 <u>4</u>
3.	8 <u>6</u>	10 <u>4</u>	8 <u>2</u>	16 <u>9</u>	11 <u>8</u>	9 <u>7</u>	6 <u>5</u>	10 <u>2</u>	16 <u>8</u>
4.	18 <u>9</u>	7 <u>6</u>	12 <u>5</u>	7 <u>2</u>	14 <u>9</u>	8 <u>5</u>	7 <u>4</u>	15 <u>6</u>	13 <u>5</u>
5.	14 <u>7</u>	12 <u>8</u>	13 <u>6</u>	9 <u>8</u>	12 <u>6</u>	11 <u>3</u>	15 <u>8</u>	9 <u>4</u>	6 <u>3</u>

Give results instantly:

15. $10 - 8$ 17. $11 - 9$ 19. $10 - 3$ 21. $14 - 5$ 23. $15 - 9$
 16. $13 - 4$ 18. $14 - 8$ 20. $13 - 8$ 22. $11 - 7$ 24. $16 - 7$

Oral Exercises

1. How many must be added to 5 to make 7? 9? 8?
14? 13? to 8 to make 10? 15? 17? 12?

2. What number must be added to 7 to make 11? 9?
16? 14? 12? to 6 to make 14? 10? 12? 11? 13?

How many more than

- | | | |
|-------------|---------------|---------------|
| 3. 2 are 6? | 7. 4 are 12? | 11. 6 are 13? |
| 4. 4 are 9? | 8. 9 are 16? | 12. 7 are 14? |
| 5. 3 are 8? | 9. 2 are 10? | 13. 8 are 15? |
| 6. 5 are 7? | 10. 5 are 11? | 14. 9 are 18? |

Read, filling the blanks:

15. 4 and — are 11; 11 less — are 7.
16. 9 and — are 13; 13 less — are 4.
17. 8 and — are 15; 15 less — are 7.
18. 7 and — are 13; 13 less — are 6.
19. 8 and — are 16; 16 less — are 8.
20. $8 + ? = 17$; $17 - ? = 9$. 24. $5 + ? = 14$; $14 - ? = 9$.
21. $4 + ? = 10$; $10 - ? = 6$. 25. $8 + ? = 13$; $13 - ? = 5$.
22. $7 + ? = 14$; $14 - ? = 7$. 26. $7 + ? = 16$; $16 - ? = 9$.
23. $6 + ? = 15$; $15 - ? = 9$. 27. $9 + ? = 15$; $15 - ? = 6$.
28. How many eggs are 9 eggs less 4 eggs?
29. How many hens are 11 hens less 5 hens?
30. How many more cars are 12 cars than 8 cars?
31. How many more cows are 17 cows than 9 cows?
32. How many oranges are 14 oranges less 6 oranges?

Oral Exercises

Read, supplying the number that added to the lower number will give the upper one; thus in exercise 1, "4 and 5 are 9:"

1.	2.	3.	4.	5.	6.	7.	8.	9.
9	7	8	11	12	14	13	10	16
<u>4</u>	<u>2</u>	<u>4</u>	<u>3</u>	<u>5</u>	<u>8</u>	<u>5</u>	<u>6</u>	<u>8</u>
10.	11.	12.	13.	14.	15.	16.	17.	18.
6	17	13	11	15	14	12	18	13
<u>3</u>	<u>9</u>	<u>4</u>	<u>7</u>	<u>9</u>	<u>5</u>	<u>7</u>	<u>9</u>	<u>6</u>

Subtract quickly 1, 2, 3, 4, 5, 6, 7, 8, 9 each from the numbers in the table, first by rows; then by columns:

	29.	30.	31.	32.	33.	34.	35.	36.	37.
19.	17	26	39	40	54	69	71	88	95
20.	15	20	36	42	51	66	73	85	92
21.	19	22	30	44	53	60	78	83	96
22.	16	24	32	41	58	62	77	80	99
23.	10	21	34	43	57	64	75	87	93
24.	12	23	31	48	55	67	79	81	97
25.	14	28	33	47	59	63	76	84	91
26.	11	27	38	45	56	68	70	82	98
27.	13	25	37	49	50	61	72	86	90
28.	18	29	35	46	52	65	74	89	94

Turn to this table frequently for drill.

Written Exercises

1. From 86 subtract 32.

FIRST METHOD *

86 **minuend**
 32 **subtrahend**
54 **remainder**

By the *Austrian* method, the question is to find what number *added* to the subtrahend will give the minuend, thus:

For the units, 2 and 4 are 6; write 4.

For the tens, 3 and 5 are 8; write 5.

The remainder is 54.

SECOND METHOD *

86 **minuend**
 32 **subtrahend**
54 **remainder**

By the "taking-away" method, the question is to find the number that is left when the subtrahend is taken from the minuend, thus:

For the units, 2 from 6 leaves 4; write 4.

For the tens, 3 from 8 leaves 5; write 5.

The remainder is 54.

Test. — The sum of the remainder and the subtrahend should equal the minuend; thus for exercise 1, $54 + 32 = 86$.

Subtract and test:

2. 97 46 <u> </u>	3. 84 30 <u> </u>	4. 39 27 <u> </u>	5. 65 14 <u> </u>	6. 43 21 <u> </u>	7. 29 16 <u> </u>
8. 52 12 <u> </u>	9. 36 25 <u> </u>	10. 99 33 <u> </u>	11. 83 62 <u> </u>	12. 42 11 <u> </u>	13. 78 66 <u> </u>

* NOTE TO THE TEACHER. — Because some schools use the *Austrian*, or **addition**, method of subtraction while others retain the so-called "taking-away" method, this text gives instruction in both, calling them respectively "First Method" and "Second Method." Under no circumstances, however, should both methods be presented to the same class.

14. From 75 subtract 26.

FIRST METHOD

Because 5 is less than 6, no arithmetical number added to 6 will give 5, so we add enough to 6 to give $10 + 5$, or 15. We thus increase the minuend by 10; but this will change the remainder

unless we also increase the subtrahend by the same number, which we do by carrying 1 ten to the 2 tens, as in addition, making it 3 tens. Thus, we have:

75

26

49

For the units, 6 and 9 are 15; write 9.

For the tens, 3 and 4 are 7; write 4.

The remainder is 49.

SECOND METHOD

We cannot take 6 units from 5 units so we take 1 ten (= 10 units) from the 7 tens to unite with the 5 units, making 15 units.

Then, 6 units from 15 units are 9 units; write 9.

75

26

49

Having taken 1 ten from the 7 tens we have 6 tens left in the minuend.

Then, 2 tens from 6 tens are 4 tens; write 4.

The following arrangement of the numbers may serve to make the work plainer:

Minuend = $75 = 7 \text{ tens} + 5 \text{ units} = 6 \text{ tens} + 15 \text{ units}$

Subtrahend = $26 = 2 \text{ tens} + 6 \text{ units} = 2 \text{ tens} + 6 \text{ units}$

Remainder = $4 \text{ tens} + 9 \text{ units} = 49$

In practice we simply think:

For the units, "6 from 15, 9"; write 9.

For the tens, "2 from 6, 4"; write 4.

The remainder is 49.

Subtract and test:

15.	85	16.	61	17.	50	18.	93	19.	74	20.	62
	<u>46</u>		<u>29</u>		<u>27</u>		<u>38</u>		<u>25</u>		<u>49</u>

Subtract and test :

21. 25 <u>12</u>	22. 83 <u>35</u>	23. 52 <u>34</u>	24. 54 <u>32</u>	25. 60 <u>41</u>	26. 41 <u>30</u>
27. 91 <u>56</u>	28. 43 <u>27</u>	29. 77 <u>43</u>	30. 81 <u>25</u>	31. 98 <u>51</u>	32. 72 <u>59</u>
33. 68 <u>42</u>	34. 75 <u>59</u>	35. 59 <u>13</u>	36. 58 <u>29</u>	37. 47 <u>19</u>	38. 75 <u>23</u>
39. 35 <u>16</u>	40. 66 <u>24</u>	41. 80 <u>53</u>	42. 87 <u>32</u>	43. 41 <u>25</u>	44. 79 <u>53</u>

45. How many roses are 42 roses less 20 roses ?
 46. How many cards are 59 cards less 12 cards ?
 47. How many acorns are 31 acorns less 18 acorns ?
 48. How many more deer are 45 deer than 27 deer ?

Subtract and test quickly, timing yourself :

49. 57 <u>34</u>	50. 55 <u>31</u>	51. 83 <u>46</u>	52. 63 <u>39</u>	53. 58 <u>45</u>	54. 47 <u>13</u>
55. 72 <u>36</u>	56. 91 <u>41</u>	57. 36 <u>17</u>	58. 89 <u>25</u>	59. 90 <u>57</u>	60. 45 <u>28</u>
61. 46 <u>19</u>	62. 70 <u>22</u>	63. 77 <u>49</u>	64. 65 <u>25</u>	65. 63 <u>38</u>	66. 96 <u>68</u>
67. 75 <u>27</u>	68. 70 <u>53</u>	69. 42 <u>26</u>	70. 57 <u>39</u>	71. 74 <u>46</u>	72. 51 <u>17</u>

Written Exercises

1. From 675 subtract 492.

FIRST METHOD

675	For the units,	2 and 3 are 5; write 3.
492	For the tens,	9 and 8 are 17; write 8.
<u>183</u>	For the hundreds,	5 and 1 are 6; write 1.
	The remainder is 183.	

SECOND METHOD

675	For the units,	2 from 5, 3; write 3.
492	For the tens,	9 from 17, 8; write 8.
<u>183</u>	For the hundreds,	4 from 5, 1; write 1.
	The remainder is 183.	

Subtract:

2. 452	3. 580	4. 865	5. 934	6. 344	7. 802
239	257	507	256	69	538
<u>213</u>	<u>323</u>	<u>358</u>	<u>678</u>	<u>275</u>	<u>264</u>

Subtract and test:

8. 749	9. 818	10. 556	11. 927	12. 453	13. 646
<u>385</u>	<u>542</u>	<u>283</u>	<u>462</u>	<u>428</u>	<u>339</u>
14. 860	15. 790	16. 473	17. 925	18. 587	19. 432
<u>528</u>	<u>288</u>	<u>206</u>	<u>609</u>	<u>395</u>	<u>154</u>
20. 578	21. 861	22. 342	23. 675	24. 777	25. 234
<u>299</u>	<u>476</u>	<u>81</u>	<u>96</u>	<u>88</u>	<u>75</u>
26. 307	27. 806	28. 504	29. 902	30. 718	31. 564
<u>165</u>	<u>248</u>	<u>346</u>	<u>705</u>	<u>640</u>	<u>365</u>

Subtract and test:

32. 859 <u>325</u>	33. 918 <u>542</u>	34. 239 <u>72</u>	35. 749 <u>385</u>	36. 426 <u>348</u>	37. 838 <u>549</u>
38. 381 <u>193</u>	39. 888 <u>79</u>	40. 586 <u>297</u>	41. 735 <u>486</u>	42. 626 <u>27</u>	43. 504 <u>226</u>
44. 702 <u>538</u>	45. 934 <u>256</u>	46. 863 <u>475</u>	47. 636 <u>84</u>	48. 801 <u>426</u>	49. 208 <u>79</u>
50. 758 <u>324</u>	51. 596 <u>206</u>	52. 145 <u>75</u>	53. 581 <u>257</u>	54. 465 <u>98</u>	55. 239 <u>174</u>

56. How many balls are 124 balls less 109 balls?
 57. How many trees are 429 trees less 184 trees?
 58. How many cookies are 325 cookies less 178 cookies?
 59. How many dollars are 521 dollars less 285 dollars?

Subtract and test quickly, timing yourself:

60. 648 <u>70</u>	61. 803 <u>466</u>	62. 947 <u>684</u>	63. 243 <u>91</u>	64. 405 <u>83</u>	65. 708 <u>640</u>
66. 314 <u>128</u>	67. 365 <u>84</u>	68. 876 <u>345</u>	69. 707 <u>278</u>	70. 105 <u>47</u>	71. 666 <u>99</u>
72. 644 <u>67</u>	73. 263 <u>249</u>	74. 208 <u>79</u>	75. 765 <u>208</u>	76. 375 <u>243</u>	77. 808 <u>709</u>
78. 505 <u>268</u>	79. 633 <u>38</u>	80. 363 <u>184</u>	81. 403 <u>272</u>	82. 807 <u>629</u>	83. 740 <u>309</u>

Written Exercises

1. From 8000 subtract 2379.

FIRST METHOD

8000	For the units,	9 and 1 are 10; write 1.
2379	For the tens,	8 and 2 are 10; write 2.
<u>5621</u>	For the hundreds,	4 and 6 are 10; write 6.
	For the thousands,	3 and 5 are 8; write 5.
	The remainder is 5621.	

SECOND METHOD

8000	For the units,	9 from 10, 1; write 1.
2379	For the tens,	7 from 9, 2; write 2.
<u>5621</u>	For the hundreds,	3 from 9, 6; write 6.
	For the thousands,	2 from 7, 5; write 5.
	The remainder is 5621.	

Subtract :

2. 5623	3. 7368	4. 9300	5. 4214	6. 1608
2346	5832	3936	1569	843
<u>3277</u>	<u>1536</u>	<u>5364</u>	<u>2645</u>	<u>765</u>

Find answers and test :

7. 400 - 288	14. 1736 - 848	21. 3000 - 1846
8. 500 - 372	15. 1406 - 563	22. 5438 - 4259
9. 613 - 408	16. 2000 - 745	23. 8000 - 2068
10. 800 - 493	17. 5004 - 656	24. 4399 - 1933
11. 304 - 125	18. 1672 - 948	25. 3002 - 2499
12. 600 - 579	19. 7005 - 680	26. 6000 - 3734
13. 891 - 483	20. 9000 - 547	27. 7050 - 4182

Miscellaneous Exercises

Find results rapidly :

1. $8 + 7 + 4 + 8 + 5$

4. $8 + 6 + 9 + 0 + 5 + 7$

2. $6 + 8 + 0 + 9 + 8$

5. $9 + 7 - 5 - 4 + 6 - 9$

3. $7 + 9 - 5 + 8 - 7$

6. $7 + 6 + 4 + 9 + 8 + 5$

Add at sight :

7.	8.	9	10.	11.	12.	13.
38	27	85	56	39	72	61
<u>40</u>	<u>60</u>	<u>20</u>	<u>22</u>	<u>11</u>	<u>22</u>	<u>31</u>
14.	15.	16.	17.	18.	19.	20.
45	20	74	66	55	48	87
<u>15</u>	<u>57</u>	<u>26</u>	<u>33</u>	<u>40</u>	<u>30</u>	<u>82</u>

Subtract and test :

21. 84	22. 63	23. 475	24. 894	25. 3296	26. 9041
<u>16</u>	<u>25</u>	<u>182</u>	<u>538</u>	<u>2466</u>	<u>4218</u>
27. 52	28. 80	29. 721	30. 914	31. 6148	32. 7230
<u>36</u>	<u>57</u>	<u>406</u>	<u>117</u>	<u>2267</u>	<u>4235</u>

Add and test :

33. 16	34. 34	35. 215	36. 589	37. 2130	38. 4261
47	87	504	638	4625	7848
30	51	438	750	6078	8935
68	90	723	196	3914	3674
72	66	376	807	8352	9057
<u>59</u>	<u>79</u>	<u>601</u>	<u>475</u>	<u>5643</u>	<u>5706</u>

MULTIPLICATION

In multiplication,

The number multiplied is the **multiplicand**.

The number by which we multiply is the **multiplier**.

The result found by multiplying is the **product**.

Review the multiplication tables through 9×9 , reading the sign \times , "**times**." Then complete the tables through 12×9 , and thoroughly commit them to memory.

MULTIPLICATION TABLES

Complete, write in full, and memorize the table of :

Twos	Threes	Fours	Fives
$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$
$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$
etc.	etc.	etc.	etc.
$10 \times 2 = 20$	$10 \times 3 = 30$	$10 \times 4 = 40$	$10 \times 5 = 50$
$11 \times 2 = 22$	$11 \times 3 = 33$	$11 \times 4 = 44$	$11 \times 5 = 55$
$12 \times 2 = 24$	$12 \times 3 = 36$	$12 \times 4 = 48$	$12 \times 5 = 60$
Sixes	Sevens	Eights	Nines
$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
etc.	etc.	etc.	etc.
$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$	$10 \times 9 = 90$
$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$	$11 \times 9 = 99$
$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$	$12 \times 9 = 108$

For complete tables through 12×12 see page 345.

Oral Exercises

Give results quickly in each group first by rows; then by columns:

5.	6.	7.	8.	9.
1. 1×2	2×3	5×2	2×2	10×2
2. 3×2	1×7	2×4	4×3	11×3
3. 2×5	3×3	1×8	4×2	12×2
4. 3×4	1×9	3×5	2×6	10×3
14.	15.	16.	17.	18.
10. 6×2	2×7	9×8	3×6	11×2
11. 2×8	5×3	2×9	4×5	10×4
12. 3×7	4×4	3×8	7×2	12×3
13. 6×3	3×9	5×4	4×6	11×4
24.	25.	26.	27.	28.
19. 5×5	7×3	8×2	4×7	10×5
20. 6×4	4×8	5×6	9×2	12×4
21. 4×9	6×5	8×3	7×4	10×6
22. 7×5	5×8	6×6	5×9	11×5
23. 9×3	8×4	5×7	6×8	10×7
34.	35.	36.	37.	38.
29. 8×5	9×4	6×7	10×8	12×6
30. 6×9	8×6	7×9	11×6	11×8
31. 8×7	9×5	8×8	12×5	12×7
32. 9×6	8×9	7×8	10×9	11×9
33. 7×7	9×7	9×9	11×7	12×8

Turn to this page frequently for drill.

Oral Exercises

1. At 2 cents each, how much will 3 lemons cost? 6 lemons? 10 lemons? 8 lemons? 12 lemons? 7 lemons?

2. At 3 cents each, what is the cost of 5 bags of popcorn? of 8 bags? of 6 bags? of 10 bags? of 12 bags?

3. At 4 cents each, how much will 3 oranges cost? 6 oranges? 10 oranges? 7 oranges? 12 oranges? 5 oranges? 8 oranges?

4. How much do 3 roses cost at 5 cents each? 5 roses? 8 roses? 6 roses? 9 roses? 12 roses?

5. At 6 cents a quart, what is the cost of 2 quarts of milk? of 5 quarts? of 9 quarts? of 11 quarts?

6. When you can buy 7 cards for a cent, how many can you buy for 5 cents? for 8 cents? for 10 cents?

7. A room has 7 rows of 8 seats each. How many seats are there in the room? in 3 rows? in 6 rows?

8. Find the cost of 3 pads at 9 cents each; of 8 such pads; of 5 pads; of 7 pads; of 11 pads; of 9 pads.

9. How many snowballs did Elmer make, if he had 6 piles with 6 snowballs in each pile?

10. Our country's flag has 6 rows of stars and 8 stars in each row. How many stars are there in the flag?

11. Agnes fed 9 nuts to each of 5 squirrels. How many nuts did she feed to the squirrels?



MULTIPLIERS OF ONE FIGURE

Oral Exercises

In each exercise, add in the first column, multiply in the second, and compare results :

1.	2.	3.	4.
2 2	3 3	4 4	6 6
<u>2</u> <u>2</u>	<u>3</u> <u>2</u>	<u>4</u> <u>2</u>	<u>6</u> <u>2</u>
5.	6.	7.	8.
7 7	5 5	9 9	8 8
<u>7</u> <u>2</u>	<u>5</u> <u>2</u>	<u>9</u> <u>2</u>	<u>8</u> <u>2</u>
9.	10.	11.	12.
3	6	8	5
3 3	6 6	8 8	5 5
<u>3</u> <u>3</u>	<u>6</u> <u>3</u>	<u>8</u> <u>3</u>	<u>5</u> <u>3</u>

Written Exercises

1. How many are three 42's, or 3 times 42 ?

42 The sum of three 42's is 126.

42 Three 42's may be added like this :

42 Three 2's are 6, the number of units; three 4's
126 are 12, the number of tens.

sum The sum is 126.

Here is a shorter way of finding 3 times 42.

42 multiplicand
 3 multiplier
126 product

Write 3 under the units' figure of 42.

For the units, 3 times 2 are 6; write 6.

For the tens, 3 times 4 are 12; write 12.

You have multiplied 42 by 3.

The product is 126.

Multiplication is a short way of adding equal numbers.

There are two short ways of writing $42 + 42 + 42$.

First way: 3×42 , read "3 times 42."

Second way: 42×3 , read "42 multiplied by 3."

We see, then, that the sign \times is read "*times*" when it is *before* the number to be multiplied, and "*multiplied by*" when it *follows* that number.

2. Find the value of $33 + 33 + 33$ first by addition; then by multiplication. Which way is the shorter?

3. Find, in the shortest way, the value of $21 + 21 + 21$.

In each exercise, add in the first column, multiply in the second, and compare results:

4.	5.	6.	7.
14 14	33 33	54 54	40 40
<u>14</u> <u>2</u>	<u>33</u> <u>2</u>	<u>54</u> <u>2</u>	<u>40</u> <u>2</u>
8.	9.	10.	11.
13	22	61	30
13 13	22 22	61 61	30 30
<u>13</u> <u>3</u>	<u>22</u> <u>3</u>	<u>61</u> <u>3</u>	<u>30</u> <u>3</u>

Multiply:

12. 23	13. 34	14. 80	15. 44	16. 71	17. 63
<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>
18. 52	19. 43	20. 31	21. 50	22. 62	23. 33
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>
24. 22	25. 31	26. 40	27. 21	28. 80	29. 91
<u>4</u>	<u>6</u>	<u>8</u>	<u>5</u>	<u>7</u>	<u>9</u>

30. Multiply 47 by 5.

$$\begin{array}{r} 47 \\ 5 \\ \hline 235 \end{array}$$

For the units, $5 \times 7 = 35$; write 5; carry 3.
 For the tens, $5 \times 4 = 20$;
 $20 + 3$ (carried) = 23; write 23.
 The product is 235.

Test. — Find the result by addition.

Multiply, testing each result by addition :

31. $\begin{array}{r} 66 \\ 2 \\ \hline \end{array}$	32. $\begin{array}{r} 48 \\ 2 \\ \hline \end{array}$	33. $\begin{array}{r} 54 \\ 3 \\ \hline \end{array}$	34. $\begin{array}{r} 66 \\ 3 \\ \hline \end{array}$	35. $\begin{array}{r} 75 \\ 4 \\ \hline \end{array}$	36. $\begin{array}{r} 96 \\ 4 \\ \hline \end{array}$
37. $\begin{array}{r} 34 \\ 5 \\ \hline \end{array}$	38. $\begin{array}{r} 86 \\ 5 \\ \hline \end{array}$	39. $\begin{array}{r} 37 \\ 4 \\ \hline \end{array}$	40. $\begin{array}{r} 63 \\ 4 \\ \hline \end{array}$	41. $\begin{array}{r} 64 \\ 3 \\ \hline \end{array}$	42. $\begin{array}{r} 86 \\ 2 \\ \hline \end{array}$

Multiply :

- | | | | |
|-------------|-------------|-------------|-------------|
| 43. 22 by 5 | 47. 53 by 6 | 51. 76 by 5 | 55. 43 by 9 |
| 44. 29 by 7 | 48. 49 by 8 | 52. 86 by 8 | 56. 77 by 6 |
| 45. 77 by 4 | 49. 99 by 5 | 53. 99 by 4 | 57. 85 by 8 |
| 46. 36 by 9 | 50. 88 by 9 | 54. 78 by 7 | 58. 54 by 9 |

59. How many tons are 6 times 32 tons?

60. How many weeks are 4 times 52 weeks?

61. How many pounds are 5 times 38 pounds?

Multiplying the larger number by the smaller, find :

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| 62. 47×2 | 67. 9×65 | 72. 86×4 | 77. 6×72 |
| 63. 38×3 | 68. 5×52 | 73. 38×5 | 78. 9×47 |
| 64. 82×5 | 69. 7×97 | 74. 95×9 | 79. 8×65 |
| 65. 89×4 | 70. 4×85 | 75. 87×6 | 80. 5×84 |
| 66. 96×3 | 71. 8×57 | 76. 99×8 | 81. 7×58 |

Written Exercises

1. Multiply 756 by 5.

756 For the units, $5 \times 6 = 30$; write 0; carry 3.
 5 For the tens, $5 \times 5 = 25$; $25 + 3 = 28$; write 8; carry 2.
 3780 For the hundreds, $5 \times 7 = 35$; $35 + 2 = 37$; write 37.
 The product is 3780.
 Test your answer by adding five 756's.

Multiply and test:

2. 627 2 —	3. 762 3 —	4. 250 4 —	5. 481 3 —	6. 288 2 —
7. 851 2 —	8. 296 4 —	9. 807 5 —	10. 285 7 —	11. 344 3 —

12. How many pens are 6 times 144 pens?
 13. How many yards are 4 times 384 yards?
 14. How many miles are 5 times 408 miles?
 15. How many minutes are 8 times 476 minutes?

Multiply:

- | | | | |
|--------------|--------------|--------------|--------------|
| 16. 213 by 3 | 25. 443 by 2 | 34. 536 by 4 | 43. 288 by 5 |
| 17. 105 by 2 | 26. 625 by 3 | 35. 612 by 6 | 44. 316 by 8 |
| 18. 452 by 4 | 27. 219 by 5 | 36. 409 by 8 | 45. 537 by 7 |
| 19. 341 by 6 | 28. 508 by 4 | 37. 812 by 7 | 46. 698 by 3 |
| 20. 502 by 5 | 29. 362 by 6 | 38. 358 by 5 | 47. 735 by 6 |
| 21. 273 by 2 | 30. 772 by 2 | 39. 143 by 9 | 48. 947 by 4 |
| 22. 614 by 4 | 31. 207 by 7 | 40. 857 by 3 | 49. 215 by 9 |
| 23. 308 by 3 | 32. 632 by 5 | 41. 528 by 6 | 50. 834 by 5 |
| 24. 216 by 5 | 33. 391 by 3 | 42. 745 by 4 | 51. 525 by 8 |

MULTIPLIERS OF TWO FIGURES

Oral Exercises

Multiply at sight:

- | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1. 5 | 2. 7 | 3. 10 | 4. 20 | 5. 30 | 6. 50 | 7. 500 |
| <u>10</u> | <u>10</u> | <u>10</u> | <u>10</u> | <u>10</u> | <u>10</u> | <u>10</u> |

8. What figure written after 5 will change it from 5 units to 5 tens, or to 10 times 5?

9. How can you multiply 15 by 10? 22 by 10? 18 by 10? any number by 10?

Multiply by 10:

- | | | | | | |
|-------|--------|--------|--------|---------|---------|
| 10. 4 | 13. 11 | 16. 19 | 19. 52 | 22. 750 | 25. 900 |
| 11. 9 | 14. 15 | 17. 24 | 20. 60 | 23. 816 | 26. 999 |
| 12. 8 | 15. 12 | 18. 38 | 21. 75 | 24. 605 | 27. 830 |

Written Exercises

When you multiply by 11 or by 12, you should obtain the product just as you do with a one-figure multiplier.

$$\begin{array}{r} 1. \quad 465 \\ \quad \quad 6 \\ \hline 2790 \end{array}$$

$$\begin{array}{r} 2. \quad 523 \\ \quad \quad 11 \\ \hline 5753 \end{array}$$

$$\begin{array}{r} 3. \quad 745 \\ \quad \quad 12 \\ \hline 8940 \end{array}$$

Multiply by 11:

- | | | |
|-------|--------|---------|
| 4. 13 | 9. 36 | 14. 150 |
| 5. 20 | 10. 55 | 15. 309 |
| 6. 41 | 11. 62 | 16. 223 |
| 7. 32 | 12. 83 | 17. 415 |
| 8. 28 | 13. 74 | 18. 531 |

Multiply by 12:

- | | | |
|--------|--------|---------|
| 19. 15 | 24. 27 | 29. 105 |
| 20. 31 | 25. 53 | 30. 230 |
| 21. 22 | 26. 44 | 31. 512 |
| 22. 45 | 27. 62 | 32. 365 |
| 23. 36 | 28. 75 | 33. 453 |

Written Exercises

1. Multiply 32 by 13.

EXPLANATION		PRACTICAL PROCESS
32	multiplicand	32
13	multiplier	13
3 times 32 = 96	partial product	96
10 times 32 = 320	partial product	32
13 times 32 = 416	product	416

The explanation shows that to multiply by 13, we first multiply by 3, then by 10, and add the partial products to obtain 10 + 3, or 13, times 32; but in practice we put down only what is shown on the right, thus:

We multiply 32 by 3 (units) and write the partial product 96 as *units* by placing 6 in units' column.

Then, we multiply 32 by 1 (ten) and write the partial product 32 as *tens* by placing 2 in tens' column.

The figure 0 of the explanation does not appear in the practical process.

The right-hand figure of each partial product is written under the figure by which we are multiplying.

2. Multiply 23 by 14; 19 by 18; 86 by 15.

23	19	86
14	18	15
92	152	430
23	19	86
322	342	1290

Multiply:

3. 41	4. 33	5. 63	6. 56	7. 81	8. 94
<u>13</u>	<u>15</u>	<u>14</u>	<u>16</u>	<u>18</u>	<u>17</u>

Multiply:

9. 35 by 13 12. 63 by 14 15. 22 by 18 18. 21 by 19
 10. 27 by 14 13. 18 by 17 16. 43 by 16 19. 45 by 18
 11. 72 by 15 14. 54 by 15 17. 37 by 14 20. 84 by 16
21. Multiply 62 by 23 and test the result.

62	23
23	62
<u>186</u>	<u>46</u>
124	138
<u>1426</u>	<u>1426</u>

Test. — The correctness of the result obtained by multiplying 62 by 23 may be tested by multiplying 23 by 62 as shown on the right.

Find each product and test:

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 22. 36×21 | 29. 73×42 | 36. 58×61 | 43. 63×74 |
| 23. 45×24 | 30. 54×45 | 37. 64×63 | 44. 87×89 |
| 24. 75×25 | 31. 58×48 | 38. 94×69 | 45. 75×84 |
| 25. 61×32 | 32. 94×29 | 39. 77×71 | 46. 93×66 |
| 26. 92×31 | 33. 51×49 | 40. 82×75 | 47. 57×94 |
| 27. 66×39 | 34. 68×55 | 41. 93×79 | 48. 99×88 |
| 28. 80×37 | 35. 72×57 | 42. 85×81 | 49. 95×96 |

50. How many pints are 18 times 21 pints?
 51. How many quarts are 15 times 32 quarts?
 52. How many bushels are 25 times 56 bushels?
 53. If a train goes at the rate of 48 miles per hour, how far will it go in 24 hours?

Written Exercises

1. Multiply 462 by 14; also 3265 by 46.

$$\begin{array}{r}
 462 \\
 14 \\
 \hline
 1848 \\
 462 \\
 \hline
 6468
 \end{array}$$

$$\begin{array}{r}
 3265 \\
 46 \\
 \hline
 19590 \\
 13060 \\
 \hline
 150190
 \end{array}$$

Multiply :

- | | | |
|---------------|---------------|----------------|
| 2. 585 by 15 | 15. 332 by 47 | 28. 2468 by 13 |
| 3. 473 by 14 | 16. 541 by 58 | 29. 2537 by 27 |
| 4. 821 by 22 | 17. 736 by 36 | 30. 3409 by 35 |
| 5. 232 by 19 | 18. 804 by 45 | 31. 5254 by 49 |
| 6. 394 by 21 | 19. 960 by 88 | 32. 4721 by 53 |
| 7. 465 by 18 | 20. 653 by 74 | 33. 3970 by 66 |
| 8. 283 by 16 | 21. 298 by 63 | 34. 6345 by 73 |
| 9. 602 by 24 | 22. 767 by 56 | 35. 7128 by 39 |
| 10. 735 by 31 | 23. 509 by 48 | 36. 9209 by 85 |
| 11. 564 by 23 | 24. 639 by 75 | 37. 5674 by 78 |
| 12. 407 by 17 | 25. 827 by 95 | 38. 3491 by 99 |
| 13. 264 by 25 | 26. 498 by 59 | 39. 8065 by 72 |
| 14. 375 by 34 | 27. 736 by 86 | 40. 4927 by 87 |

Multiply quickly, timing yourself :

- | | | |
|---------------|---------------|----------------|
| 41. 681 by 26 | 44. 980 by 79 | 47. 7396 by 65 |
| 42. 243 by 32 | 45. 866 by 92 | 48. 6835 by 54 |
| 43. 505 by 29 | 46. 689 by 67 | 49. 8763 by 98 |

MULTIPLIERS OF THREE OR MORE FIGURES**Oral Exercises**

1. How many are 10 times 4? 10×12 ? 10×25 ?
How may any number be multiplied by 10?

2. How many are 100 times 5? 100×7 ? 100×11 ?
How many zeros annexed to 5 will change 5 units to 5 hundreds? How may any number be multiplied by 100?

3. How many are 1000 times 3? 1000×16 ?

How many zeros annexed to 3 will change 3 units to 3 thousands? How may any number be multiplied by 1000?

Multiply :

- | | | |
|--------------|--------------|----------------|
| 4. 256 by 10 | 7. 25 by 100 | 10. 75 by 1000 |
| 5. 481 by 10 | 8. 40 by 100 | 11. 64 by 1000 |
| 6. 980 by 10 | 9. 63 by 100 | 12. 80 by 1000 |

Written Exercises

1. Multiply 43 by 2000.

43 Since 2000 equals 2×1000 , we multiply by 2 and
 $\begin{array}{r} 2000 \\ \times 43 \\ \hline 86000 \end{array}$ then by 1000; that is, we multiply by 2 and annex
three 0's.

Multiply :

- | | | |
|--------------|----------------|----------------|
| 2. 560 by 30 | 7. 166 by 200 | 12. 15 by 7000 |
| 3. 409 by 50 | 8. 225 by 600 | 13. 84 by 2000 |
| 4. 175 by 70 | 9. 605 by 400 | 14. 78 by 6000 |
| 5. 280 by 60 | 10. 430 by 800 | 15. 67 by 5000 |
| 6. 800 by 90 | 11. 975 by 700 | 16. 46 by 9000 |

Written Exercises

1. Multiply 346 by 278.

$\begin{array}{r} 346 \\ 278 \\ \hline 2768 \\ 2422 \\ \hline 96188 \end{array}$	<p>The <i>first</i> partial product is 2768; the <i>second</i> partial product is 2422 (tens), or 24,220; the <i>third</i> partial product is 692 (hundreds), or 69,200.</p> <p>The <i>entire</i> product is the sum of all the partial products, or 96,188.</p> <p>Test the answer by multiplying 278 by 346.</p>
--	--

Multiply and test :

2. 135	4. 166	6. 396	8. 415	10. 578	12. 289
<u>247</u>	<u>585</u>	<u>295</u>	<u>175</u>	<u>967</u>	<u>788</u>
3. 214	5. 317	7. 168	9. 742	11. 536	13. 658
<u>128</u>	<u>236</u>	<u>421</u>	<u>284</u>	<u>478</u>	<u>926</u>

Find products, timing yourself on the first column :

14. 151×214	26. 914×452	38. 467×476
15. 216×152	27. 675×343	39. 872×654
16. 342×235	28. 587×462	40. 584×919
17. 185×163	29. 398×647	41. 947×326
18. 456×237	30. 929×145	42. 738×429
19. 726×123	31. 637×296	43. 685×299
20. 281×218	32. 481×764	44. 748×814
21. 267×197	33. 777×398	45. 986×549
22. 519×468	34. 796×425	46. 837×558
23. 845×316	35. 845×386	47. 694×649
24. 916×712	36. 594×653	48. 989×374
25. 638×254	37. 857×786	49. 679×897

50. Multiply 347 by 206.

$\begin{array}{r} 347 \\ 206 \\ \hline 2082 \\ 000 \\ \hline 694 \\ \hline 71482 \end{array}$	<p>The partial product by 0 (tens) may be omitted, as in the right-hand process.</p>	$\begin{array}{r} 347 \\ 206 \\ \hline 2082 \\ 694 \\ \hline 71482 \end{array}$
---	--	---

Multiply :

- | | | |
|----------------|----------------|----------------|
| 51. 235 by 102 | 56. 583 by 308 | 61. 473 by 703 |
| 52. 416 by 304 | 57. 715 by 209 | 62. 633 by 608 |
| 53. 372 by 203 | 58. 636 by 506 | 63. 579 by 801 |
| 54. 624 by 405 | 59. 485 by 604 | 64. 824 by 705 |
| 55. 431 by 502 | 60. 547 by 408 | 65. 715 by 904 |

Multiply and test, timing yourself on exercises 82-89 :

- | | | |
|----------------|----------------|----------------|
| 66. 456 by 307 | 74. 859 by 407 | 82. 763 by 802 |
| 67. 563 by 208 | 75. 567 by 504 | 83. 549 by 708 |
| 68. 398 by 104 | 76. 925 by 602 | 84. 476 by 609 |
| 69. 639 by 205 | 77. 739 by 707 | 85. 887 by 905 |
| 70. 486 by 305 | 78. 678 by 806 | 86. 659 by 805 |
| 71. 589 by 206 | 79. 499 by 507 | 87. 987 by 706 |
| 72. 684 by 403 | 80. 847 by 605 | 88. 999 by 908 |
| 73. 746 by 309 | 81. 932 by 409 | 89. 676 by 809 |

90. How many rods are 115 times 320 rods ?

91. How many flags are 104 times 112 flags ?

92. How many gallons are 125 times 410 gallons ?

Miscellaneous Exercises

Tell results instantly by rows; then by columns:

10.	11.	12.	13.	14.
1. $8+7$	6×8	9×8	12×5	$18-10$
2. $9-6$	$8+9$	$16-9$	$10+7$	12×8
3. 4×5	7×8	$14+7$	$27+9$	$24+12$
4. $9+4$	$9-2$	6×9	11×5	$12+11$
5. 7×3	8×4	$17-8$	10×7	$40-10$
6. $8-5$	9×7	$12+9$	$11+6$	11×8
7. 4×9	$8-2$	9×9	12×7	12×9
8. $9+9$	6×7	$19-7$	11×6	11×9
9. 9×3	$9-4$	$16-7$	12×6	$24-12$

15. How many doors are 24 doors and 18 doors?

16. How many windows are 12 times 20 windows?

Perform the operations indicated:

17. 87×32	27. 640×29	37. 936×409	47. 754×506
18. 59×25	28. $178-89$	38. $578+695$	48. $349+837$
19. $32-16$	29. $879+74$	39. $824-576$	49. $917-608$
20. 64×47	30. 689×57	40. $603-259$	50. $893+257$
21. $81-36$	31. 756×37	41. $769+588$	51. 706×708
22. $89+67$	32. $896+97$	42. 857×680	52. 508×690
23. $93+49$	33. $437-69$	43. 908×704	53. $962-578$
24. $75-38$	34. $680-85$	44. $465-198$	54. $407+897$
25. 69×87	35. $392+99$	45. 376×589	55. 968×406
26. $96+99$	36. 907×83	46. $999+867$	56. 876×792

DIVISION

In division,

The number divided is the **dividend**.

The number by which we divide is the **divisor**.

The result found by dividing is the **quotient**.

The sign \div is read "**divided by**."

The pupil has studied the division tables through $81 \div 9$ in connection with the corresponding multiplication tables.

He should now review the tables, complete them through $108 \div 9$, and thoroughly commit them to memory.

DIVISION TABLES

Complete, write in full, and memorize the table of:

Twos	Threes	Fours	Fives
$2 \div 2 = 1$	$3 \div 3 = 1$	$4 \div 4 = 1$	$5 \div 5 = 1$
$4 \div 2 = 2$	$6 \div 3 = 2$	$8 \div 4 = 2$	$10 \div 5 = 2$
etc.	etc.	etc.	etc.
$20 \div 2 = 10$	$30 \div 3 = 10$	$40 \div 4 = 10$	$50 \div 5 = 10$
$22 \div 2 = 11$	$33 \div 3 = 11$	$44 \div 4 = 11$	$55 \div 5 = 11$
$24 \div 2 = 12$	$36 \div 3 = 12$	$48 \div 4 = 12$	$60 \div 5 = 12$
Sixes	Sevens	Eights	Nines
$6 \div 6 = 1$	$7 \div 7 = 1$	$8 \div 8 = 1$	$9 \div 9 = 1$
$12 \div 6 = 2$	$14 \div 7 = 2$	$16 \div 8 = 2$	$18 \div 9 = 2$
etc.	etc.	etc.	etc.
$60 \div 6 = 10$	$70 \div 7 = 10$	$80 \div 8 = 10$	$90 \div 9 = 10$
$66 \div 6 = 11$	$77 \div 7 = 11$	$88 \div 8 = 11$	$99 \div 9 = 11$
$72 \div 6 = 12$	$84 \div 7 = 12$	$96 \div 8 = 12$	$108 \div 9 = 12$

The division tables through $144 \div 12$ are given on page 346.

Oral Exercises

Give results quickly in each group by rows; then by columns:

	5.	6.	7.	8.	9.
1.	$3 \div 3$	$6 \div 2$	$9 \div 3$	$10 \div 2$	$12 \div 4$
2.	$4 \div 2$	$8 \div 8$	$6 \div 3$	$12 \div 6$	$14 \div 7$
3.	$6 \div 6$	$9 \div 3$	$4 \div 4$	$10 \div 5$	$12 \div 2$
4.	$8 \div 4$	$7 \div 7$	$8 \div 2$	$12 \div 3$	$15 \div 5$
	14.	15.	16.	17.	18.
10.	$16 \div 8$	$18 \div 6$	$20 \div 5$	$18 \div 3$	$21 \div 7$
11.	$15 \div 3$	$16 \div 4$	$18 \div 9$	$25 \div 5$	$16 \div 2$
12.	$14 \div 2$	$24 \div 8$	$30 \div 5$	$24 \div 4$	$21 \div 3$
13.	$20 \div 4$	$18 \div 2$	$20 \div 2$	$27 \div 9$	$24 \div 6$
	24.	25.	26.	27.	28.
19.	$22 \div 2$	$28 \div 7$	$30 \div 6$	$32 \div 8$	$35 \div 7$
20.	$40 \div 8$	$36 \div 9$	$28 \div 4$	$24 \div 3$	$32 \div 4$
21.	$27 \div 3$	$35 \div 5$	$45 \div 9$	$36 \div 6$	$42 \div 7$
22.	$24 \div 2$	$42 \div 6$	$40 \div 5$	$30 \div 3$	$48 \div 8$
23.	$36 \div 4$	$54 \div 9$	$49 \div 7$	$45 \div 5$	$33 \div 3$
	35.	36.	37.	38.	39.
29.	$48 \div 6$	$56 \div 8$	$63 \div 9$	$56 \div 7$	$40 \div 4$
30.	$50 \div 5$	$54 \div 6$	$64 \div 8$	$60 \div 6$	$55 \div 5$
31.	$63 \div 7$	$48 \div 4$	$60 \div 5$	$72 \div 9$	$36 \div 3$
32.	$81 \div 9$	$80 \div 8$	$90 \div 9$	$77 \div 7$	$72 \div 8$
33.	$72 \div 6$	$99 \div 9$	$66 \div 6$	$96 \div 8$	$84 \div 7$
34.	$70 \div 7$	$88 \div 8$	$44 \div 4$	$48 \div 8$	$45 \div 9$

Turn to this page frequently for drill.

Oral Exercises

1. How many 2-cent postage stamps can you buy for 4 cents? for 10 cents? for 14 cents? for 24 cents?
2. At 3 dollars each, how many barrels of apples can be bought for 12 dollars? for 9 dollars? for 24 dollars?
3. If a pound of flour costs 4 cents, how many pounds can be bought for 16 cents? for 24 cents? for 36 cents?
4. How many 5-cent pencils can be bought for 15 cents? for 25 cents? for 40 cents? for 60 cents?
5. If it takes 6 yards of cloth for a dress, how many dresses can you make from 36 yards? from 54 yards?
6. If James earns 7 cents a day, how long will it take him to earn 21 cents? 35 cents? 56 cents? 63 cents?
7. A horse eats 8 quarts of oats a day. How long do 24 quarts last him? 48 quarts? 64 quarts? 96 quarts?
8. At 9 cents a yard, how much ribbon can be bought for 18 cents? for 36 cents? for 27 cents? for 63 cents?
9. Helen, Arthur, and I have 12 cookies in our lunch basket. How many cookies are there for each of us?
10. There are 7 days in a week. How many weeks are there in 42 days? in 84 days?
11. A man wishes to plant 45 trees in 5 equal rows. How many trees should he plant in each row?
12. There are 9 players in a baseball team. How many baseball teams can be formed from 81 players?
13. If Max's father saves 8 dollars a week, how long will it take him to save 56 dollars? 72 dollars?

DIVISORS OF ONE FIGURE

Oral Exercises

1. How many times does 6 contain 2? $6 \div 2 = 3$. ○ ○ ○ ○ ○ ○

2. When 6 is separated into 2 equal parts, how many are there in each part? $6 \div 2 = 3$. ○ ○ ○ ○ ○ ○

3. $6 \div 2$, read "6 divided by 2," means either "Find how many times 6 contains 2" or "Find how many there are in each part when 6 is separated into 2 equal parts."

Some other ways to write $6 \div 2$ are $2 \overline{)6}$ and $\frac{6}{2}$.

4. What two meanings may $8 \div 4$ have? $3 \overline{)9}$? $\frac{10}{5}$?

Tell results quickly:

5. $12 \div 3$ $3 \overline{)12}$ $\frac{12}{3}$ $5 \overline{)15}$ $4 \overline{)16}$
 6. $2 \overline{)8}$ $2 \overline{)8}$ tens $2 \overline{)80}$ $2 \overline{)80 + 6}$ $2 \overline{)86}$
 7. $2 \overline{)4}$ $2 \overline{)4}$ hundreds $2 \overline{)400}$ $2 \overline{)400 + 80 + 6}$ $2 \overline{)486}$

Written Exercises

1. How many times is 2 contained in 86?

$$\begin{array}{r} \text{divisor} \quad 2 \overline{)86} \quad \text{dividend} \\ \quad \quad \quad 43 \quad \quad \text{quotient} \end{array}$$

2 is contained in 8 (tens), 4 (tens) times; write 4 under the tens.

2 is contained in 6, 3 times; write 3 under the units.

You have divided 86 by 2. The quotient is 43.

Test. — $43 + 43$, or 43 multiplied by 2, is equal to 86.

2. Divide 486 by 2.

Think :

$$\begin{array}{r} 2 \overline{)486} \\ 243 \end{array}$$

For the hundreds, "2 in 4, 2 times"; write 2.

For the tens, "2 in 8, 4 times"; write 4.

For the units, "2 in 6, 3 times"; write 3.

The quotient is 243.

Test. — 243 multiplied by 2 gives 486, the number divided.

Copy, divide, and test :

$$\begin{array}{r} 3. \\ 2 \overline{)48} \end{array}$$

$$\begin{array}{r} 7. \\ 4 \overline{)88} \end{array}$$

$$\begin{array}{r} 11. \\ 2 \overline{)264} \end{array}$$

$$\begin{array}{r} 15. \\ 7 \overline{)770} \end{array}$$

$$\begin{array}{r} 19. \\ 6 \overline{)6060} \end{array}$$

$$\begin{array}{r} 4. \\ 3 \overline{)39} \end{array}$$

$$\begin{array}{r} 8. \\ 2 \overline{)68} \end{array}$$

$$\begin{array}{r} 12. \\ 3 \overline{)630} \end{array}$$

$$\begin{array}{r} 16. \\ 5 \overline{)555} \end{array}$$

$$\begin{array}{r} 20. \\ 2 \overline{)4628} \end{array}$$

$$\begin{array}{r} 5. \\ 2 \overline{)28} \end{array}$$

$$\begin{array}{r} 9. \\ 3 \overline{)69} \end{array}$$

$$\begin{array}{r} 13. \\ 9 \overline{)909} \end{array}$$

$$\begin{array}{r} 17. \\ 4 \overline{)484} \end{array}$$

$$\begin{array}{r} 21. \\ 5 \overline{)5500} \end{array}$$

$$\begin{array}{r} 6. \\ 3 \overline{)63} \end{array}$$

$$\begin{array}{r} 10. \\ 2 \overline{)82} \end{array}$$

$$\begin{array}{r} 14. \\ 6 \overline{)666} \end{array}$$

$$\begin{array}{r} 18. \\ 3 \overline{)963} \end{array}$$

$$\begin{array}{r} 22. \\ 3 \overline{)6939} \end{array}$$

23. Find how many times 84 plums contain 4 plums.

24. How many times can 3 pears be taken from 63 pears ?

25. How many times are 2 apples contained in 86 apples ? 4 peaches in 88 peaches ?

Find the value of each of the following :

26. $84 \div 4$

30. $505 \div 5$

34. $\frac{60}{3}$

38. $\frac{648}{2}$

42. $\frac{8024}{2}$

27. $96 \div 3$

31. $488 \div 4$

35. $\frac{84}{2}$

39. $\frac{369}{3}$

43. $\frac{4084}{4}$

28. $64 \div 2$

32. $669 \div 3$

36. $\frac{99}{3}$

40. $\frac{707}{7}$

44. $\frac{9396}{3}$

29. $66 \div 3$

33. $862 \div 2$

37. $\frac{62}{2}$

41. $\frac{888}{8}$

45. $\frac{2648}{2}$

Oral Exercises

Read, filling the blanks :

1. 2 is contained in 5 — times and — over.
2. 3 is contained in 8 — times and — over.
3. 2 is contained in 1 no times and 1 over.
4. 9 is contained in 6 — times and — over.
5. 6 is contained in 50 — times and — over.

6-25. Divide each number in the first *two* columns of this table by 2, giving each quotient and the remainder when there is one.

1	11	21	31	41	51	61	71	81
2	12	22	32	42	52	62	72	82
3	13	23	33	43	53	63	73	83
4	14	24	34	44	54	64	74	84
5	15	25	35	45	55	65	75	85
6	16	26	36	46	56	66	76	86
7	17	27	37	47	57	67	77	87
8	18	28	38	48	58	68	78	88
9	19	29	39	49	59	69	79	89
10	20	30	40	50	60	70	80	90

26-55. Divide the numbers in the first *three* columns by 3, giving quotients and remainders as in exercises 6-25.

In a similar manner, divide the numbers in the first
56-95. *Four* columns by 4. **206-275.** *Seven* columns by 7
96-145. *Five* columns by 5. **276-355.** *Eight* columns by 8
146-205. *Six* columns by 6. **356-445.** *Nine* columns by 9.

Written Exercises

1. Divide 25 by 2; 637 by 3; 807 by 4.

$$\begin{array}{r} 2 \overline{)25} \\ 12, 1 \text{ remainder} \end{array} \qquad \begin{array}{r} 3 \overline{)637} \\ 212, 1 \text{ rem.} \end{array} \qquad \begin{array}{r} 4 \overline{)807} \\ 201, 3 \text{ rem.} \end{array}$$

Tests. — $2 \times 12, +1 = 25$. $3 \times 212, +1 = 637$. $4 \times 201, +3 = 807$.

To the product of the quotient and the divisor add the remainder.
The result should equal the dividend.

Divide and test:

- | | | | |
|------------|--------------|--------------|--------------|
| 2. 38 by 3 | 9. 395 by 3 | 16. 889 by 8 | 23. 394 by 3 |
| 3. 47 by 2 | 10. 486 by 4 | 17. 708 by 7 | 24. 669 by 6 |
| 4. 89 by 4 | 11. 845 by 2 | 18. 483 by 2 | 25. 806 by 4 |
| 5. 67 by 2 | 12. 507 by 5 | 19. 558 by 5 | 26. 865 by 2 |
| 6. 79 by 7 | 13. 608 by 6 | 20. 637 by 3 | 27. 603 by 6 |
| 7. 95 by 3 | 14. 938 by 3 | 21. 847 by 4 | 28. 805 by 8 |
| 8. 69 by 6 | 15. 685 by 2 | 22. 779 by 7 | 29. 967 by 3 |

30. Divide 978 by 3.

3 is contained in 9 (hundreds), 3 (hundreds) times;
 $\begin{array}{r} 3 \overline{)978} \\ 326 \end{array}$ write 3 under the hundreds.

326 3 is contained in 7 (tens), 2 (tens) times with a remainder of 1 (ten), or 10; write 2 under the tens.

3 is contained in $10 + 8$, or in 18, 6 times; write 6 under the units. The quotient is 326.

In practice, think:

For the hundreds, "3 in 9, 3"; write 3.

For the tens, "3 in 7, 2 with 1 rem."; write 2.

For the units, "3 in 18, 6"; write 6.

Test. — $3 \times 326 = 978$.

Divide and test :

31. <u>2)456</u>	32. <u>3)675</u>	33. <u>5)575</u>	34. <u>4)876</u>	35. <u>7)791</u>	36. <u>6)684</u>
---------------------	---------------------	---------------------	---------------------	---------------------	---------------------

37. <u>8)896</u>	38. <u>7)784</u>	39. <u>2)698</u>	40. <u>3)987</u>	41. <u>5)590</u>	42. <u>4)468</u>
---------------------	---------------------	---------------------	---------------------	---------------------	---------------------

- | | | | |
|--------------|--------------|--------------|--------------|
| 43. 858 by 2 | 48. 892 by 4 | 53. 696 by 6 | 58. 560 by 5 |
| 44. 492 by 4 | 49. 296 by 2 | 54. 975 by 3 | 59. 694 by 2 |
| 45. 378 by 3 | 50. 681 by 3 | 55. 476 by 4 | 60. 672 by 6 |
| 46. 565 by 5 | 51. 452 by 4 | 56. 872 by 2 | 61. 948 by 3 |
| 47. 690 by 6 | 52. 570 by 5 | 57. 657 by 3 | 62. 494 by 2 |

Divide :

63. <u>2)520</u> 260	64. <u>2)528</u> 264	65. <u>3)759</u> 253	66. <u>7)350</u> 50	67. <u>7)357</u> 51	68. <u>3)186</u> 62
----------------------------	----------------------------	----------------------------	---------------------------	---------------------------	---------------------------

- | | | | |
|--------------|--------------|--------------|--------------|
| 69. 879 by 3 | 76. 546 by 3 | 83. 742 by 2 | 90. 944 by 2 |
| 70. 546 by 2 | 77. 917 by 7 | 84. 968 by 8 | 91. 168 by 8 |
| 71. 728 by 4 | 78. 324 by 4 | 85. 459 by 9 | 92. 369 by 9 |
| 72. 246 by 6 | 79. 144 by 2 | 86. 655 by 5 | 93. 768 by 4 |
| 73. 605 by 5 | 80. 924 by 4 | 87. 608 by 4 | 94. 489 by 3 |
| 74. 847 by 7 | 81. 726 by 6 | 88. 987 by 7 | 95. 855 by 5 |
| 75. 480 by 8 | 82. 255 by 5 | 89. 846 by 3 | 96. 966 by 6 |

97. Find how many times 186 pints contain 2 pints.

98. Find how many times 126 inches contain 6 inches.

99. How many times can 3 feet be taken from 216 feet?

100. How many times are 4 yards contained in 564 yards? 8 rods in 320 rods?

Written Exercises

1. Divide 834 by 6.

$$\begin{array}{r} 6 \overline{)834} \\ 139 \end{array}$$

For the hundreds, 6 in 8, 1 with 2 rem.; write 1.

For the tens, 6 in 23, 3 with 5 rem.; write 3.

For the units, 6 in 54, 9; write 9.

The quotient is 139.

Test. — $6 \times 139 = 834$.

Divide and test:

2.	3.	4.	5.	6.	7.
2) <u>572</u>	4) <u>664</u>	3) <u>534</u>	7) <u>875</u>	6) <u>738</u>	8) <u>904</u>

- | | | | |
|--------------|--------------|--------------|--------------|
| 8. 342 by 6 | 13. 925 by 5 | 18. 456 by 8 | 23. 774 by 6 |
| 9. 936 by 8 | 14. 178 by 2 | 19. 576 by 4 | 24. 598 by 2 |
| 10. 810 by 5 | 15. 954 by 6 | 20. 745 by 5 | 25. 952 by 4 |
| 11. 282 by 3 | 16. 396 by 2 | 21. 462 by 7 | 26. 177 by 3 |
| 12. 754 by 2 | 17. 861 by 3 | 22. 592 by 8 | 27. 651 by 7 |

28.	29.	30.
9) <u>947</u>	5) <u>804</u>	4) <u>9759</u>
105, 2 rem.	160, 4 rem.	2439, 3 rem.

Divide by 2; by 5; by 4; by 6; by 8; by 3; by 7; by 9:

- | | | | | |
|---------|---------|---------|----------|----------|
| 31. 465 | 37. 923 | 43. 754 | 49. 7638 | 55. 6281 |
| 32. 720 | 38. 701 | 44. 808 | 50. 5673 | 56. 4967 |
| 33. 463 | 39. 862 | 45. 647 | 51. 8679 | 57. 7420 |
| 34. 512 | 40. 823 | 46. 725 | 52. 9560 | 58. 8562 |
| 35. 713 | 41. 753 | 47. 845 | 53. 8275 | 59. 6205 |
| 36. 802 | 42. 869 | 48. 938 | 54. 9753 | 60. 9854 |

Miscellaneous Exercises

Give results at sight:

- | | | | |
|-----------------|-------------------|-------------------|-------------------|
| 1. 9×6 | 6. $36 \div 4$ | 11. $11 + 9$ | 16. $96 \div 8$ |
| 2. $8 + 8$ | 7. $17 - 8$ | 12. $63 \div 7$ | 17. $16 + 8$ |
| 3. $9 - 3$ | 8. 12×4 | 13. 12×8 | 18. 12×9 |
| 4. 7×9 | 9. $42 \div 7$ | 14. $15 - 7$ | 19. $54 \div 6$ |
| 5. $9 + 8$ | 10. 12×6 | 15. $56 \div 8$ | 20. $16 - 9$ |

Perform the operations indicated, and test:

- | | | |
|-----------------|------------------|-------------------|
| 21. $432 + 688$ | 24. $5209 - 675$ | 27. $4567 - 3879$ |
| 22. $576 - 249$ | 25. $6167 + 824$ | 28. $5328 - 1564$ |
| 23. $389 + 716$ | 26. $5614 - 925$ | 29. $6000 - 3217$ |

Add and test:

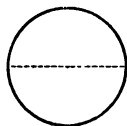
- | | | | | | |
|-----------|-----------|------------|------------|-------------|-------------|
| 30. 88 | 31. 62 | 32. 296 | 33. 439 | 34. 2354 | 35. 4183 |
| 23 | 83 | 69 | 199 | 6132 | 453 |
| 37 | 29 | 386 | 78 | 1483 | 6375 |
| 40 | 37 | 178 | 68 | 2569 | 2846 |
| 92 | 62 | 99 | 345 | 1342 | 794 |
| <u>61</u> | <u>89</u> | <u>167</u> | <u>276</u> | <u>7645</u> | <u>5328</u> |

Find answers and test:

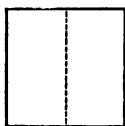
- | | | | |
|--------------------|------------------|---------------------|-------------------|
| 36. 690×4 | 42. $738 \div 3$ | 48. 295×62 | 54. $3227 \div 7$ |
| 37. 508×7 | 43. $549 \div 9$ | 49. 408×73 | 55. $5904 \div 9$ |
| 38. 728×6 | 44. $848 \div 4$ | 50. 587×58 | 56. $8649 \div 8$ |
| 39. 807×9 | 45. $696 \div 6$ | 51. 792×64 | 57. $4695 \div 7$ |
| 40. 568×8 | 46. $987 \div 7$ | 52. 698×95 | 58. $7968 \div 9$ |
| 41. 869×7 | 47. $865 \div 5$ | 53. 854×82 | 59. $6984 \div 8$ |

HALVES AND FOURTHS

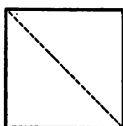
1. Into how many parts is the circle divided? each square? the oblong?



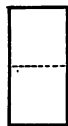
CIRCLE



SQUARE



SQUARE



OBLONG

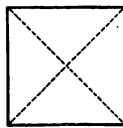
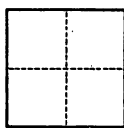
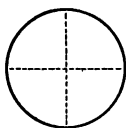
2. How do the parts of the circle compare in size? the parts of each square? of the oblong?

3. One of the *two equal* parts of anything is **one half** of it.

4. Draw a line and divide it into halves. What do you do to anything to get one half of it?

5. One half is written $\frac{1}{2}$; two halves, $\frac{2}{2}$.

6. Divide each half of the circle into two equal parts. Do the same with each square; with the oblong.



7. Into how many equal parts has each object been divided?

8. One of the *four equal* parts of anything is **one fourth**, or **one quarter**, of it.

9. One fourth is written $\frac{1}{4}$. Write 2 fourths; 3 fourths; fourths.

Oral Exercises

1. If six pears are divided into *two equal* groups, how many pears are there in each group?

2. What part of the pears is in each group? How many pears are $\frac{1}{2}$ of 6 pears?



3. How many blocks are there in this row of blocks?



4. Into how many equal groups are the blocks divided? What part of the blocks is in each group?

How many blocks are $\frac{1}{4}$ of 8 blocks?

5. How many of the blocks are black? white?

How many blocks are $\frac{1}{2}$ of 8 blocks?

6. Compare " $\frac{1}{2}$ of 6 pears" with " $6 \text{ pears} \div 2$ "; " $\frac{1}{4}$ of 8 blocks" with " $8 \text{ blocks} \div 4$ "; " $\frac{1}{2}$ of 8 blocks" with " $8 \text{ blocks} \div 2$."

7. Observe that " $\frac{1}{2}$ of 6" means the same as " $6 \div 2$."

What does $\frac{1}{4}$ of 8 mean? $\frac{1}{2}$ of 8?

8. What do you do to anything, or a group of things, or a number, to find $\frac{1}{2}$ of it? $\frac{1}{4}$ of it?

Give answers quickly:

- | | | | |
|------------------------|-------------------------|-------------------------|-------------------------|
| 9. $4 \div 2$ | 13. $\frac{1}{2}$ of 10 | 17. $\frac{1}{2}$ of 20 | 21. $\frac{1}{4}$ of 40 |
| 10. $\frac{1}{2}$ of 4 | 14. $\frac{1}{4}$ of 12 | 18. $\frac{1}{4}$ of 24 | 22. $\frac{1}{4}$ of 48 |
| 11. $4 \div 4$ | 15. $\frac{1}{2}$ of 12 | 19. $\frac{1}{2}$ of 18 | 23. $\frac{1}{2}$ of 24 |
| 12. $\frac{1}{4}$ of 4 | 16. $\frac{1}{4}$ of 16 | 20. $\frac{1}{4}$ of 28 | 24. $\frac{1}{4}$ of 36 |

Written Exercises

1. Find
- $\frac{1}{2}$
- of 144; also
- $\frac{1}{4}$
- of 376.

$$\begin{array}{r} 2 \overline{)144} \\ 72 \end{array}$$

$$\begin{array}{r} 4 \overline{)376} \\ 94 \end{array}$$

To find $\frac{1}{2}$ of 144, divide 144
by 2; $\frac{1}{2}$ of 144 = 72.

To find $\frac{1}{4}$ of 376, divide 376
by 4; $\frac{1}{4}$ of 376 = 94.

Find :

- | | | | |
|------------------------|-------------------------|--------------------------|--------------------------|
| 2. $\frac{1}{2}$ of 46 | 10. $\frac{1}{4}$ of 84 | 18. $\frac{1}{2}$ of 204 | 26. $\frac{1}{4}$ of 328 |
| 3. $\frac{1}{2}$ of 36 | 11. $\frac{1}{4}$ of 52 | 19. $\frac{1}{4}$ of 484 | 27. $\frac{1}{4}$ of 356 |
| 4. $\frac{1}{2}$ of 58 | 12. $\frac{1}{4}$ of 88 | 20. $\frac{1}{2}$ of 186 | 28. $\frac{1}{2}$ of 568 |
| 5. $\frac{1}{2}$ of 86 | 13. $\frac{1}{4}$ of 56 | 21. $\frac{1}{2}$ of 172 | 29. $\frac{1}{4}$ of 592 |
| 6. $\frac{1}{2}$ of 54 | 14. $\frac{1}{4}$ of 68 | 22. $\frac{1}{4}$ of 168 | 30. $\frac{1}{2}$ of 652 |
| 7. $\frac{1}{2}$ of 72 | 15. $\frac{1}{4}$ of 72 | 23. $\frac{1}{4}$ of 176 | 31. $\frac{1}{2}$ of 776 |
| 8. $\frac{1}{2}$ of 96 | 16. $\frac{1}{4}$ of 64 | 24. $\frac{1}{2}$ of 274 | 32. $\frac{1}{4}$ of 628 |
| 9. $\frac{1}{2}$ of 78 | 17. $\frac{1}{4}$ of 96 | 25. $\frac{1}{4}$ of 288 | 33. $\frac{1}{2}$ of 994 |

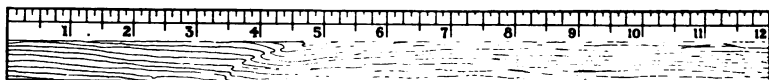
Find $\frac{1}{2}$ of:

- | | | | | |
|---------|---------|----------|----------|----------|
| 34. 164 | 40. 592 | 46. 1864 | 52. 6456 | 58. 1756 |
| 35. 192 | 41. 536 | 47. 2756 | 53. 5732 | 59. 2572 |
| 36. 260 | 42. 672 | 48. 2608 | 54. 6996 | 60. 4968 |
| 37. 276 | 43. 804 | 49. 3764 | 55. 7932 | 61. 6748 |
| 38. 392 | 44. 736 | 50. 3940 | 56. 8976 | 62. 7480 |
| 39. 384 | 45. 968 | 51. 4684 | 57. 9900 | 63. 9876 |

64-93. Find $\frac{1}{4}$ of each of the numbers in exercises 34-63.

MEASURES

LENGTH MEASURES



1. This picture represents a foot rule drawn one third of its length. Examine a foot rule and notice the long marks that are numbered. Observe the distance from one of these marks to the next one. It is the same as the length of this line: _____. This length is **one inch**.

2. Draw a line twelve inches long on the blackboard. This length is **one foot**.

3. Count the number of inches in the length of a foot rule. How many inches are there in one foot?

Twelve inches equal one foot.

12 in. = 1 ft.

4. Name some things that are measured by the inch; by the foot.

Oral Exercises

Read, filling the blanks :

1. $\frac{1}{2}$ of 12 in. = ____ in.

2. $\frac{1}{2}$ of 1 ft. = ____ in.

3. $\frac{1}{4}$ of 12 in. = ____ in.

4. $\frac{1}{4}$ of 1 ft. = ____ in.

5. What part of a foot are 6 in.? 3 in.?

6. My sheet of paper is 1 ft. long. How many inches wide is it, if its width is $\frac{1}{2}$ of its length? $\frac{1}{4}$ of its length?

7. How many inches are there in 3 ft.? in 2 ft. 6 in.?

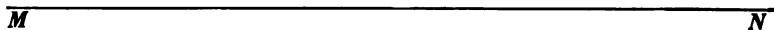
Measuring Exercises




1. Measure the length of your desk ; the height.
2. Without measuring, draw a line on the blackboard as nearly 1 ft. long as you can. Measure it with a rule.
3. In the same way draw a line $\frac{1}{2}$ ft. long and test your estimate ; $\frac{1}{4}$ ft. long ; 3 in. long ; 2 in. long.
4. Estimate in inches the length, width, and height of a chalk box. Test your estimate by measuring.
5. Estimate and measure the width of a window.
6. Estimate and measure the length and width of a pane of glass in the window of your schoolroom.
7. Estimate how much wider your copy book is than this book. Test by measuring.
8. Is your pad more or less than $\frac{1}{2}$ ft. wide ? $\frac{1}{4}$ ft. ? how many inches more or less in each case ?
9. How many feet long do you think the schoolroom is ? how wide ? Measure to see.
10. Estimate, in feet, the length of each blackboard in the room. Test your estimate by measuring.

DRAWING TO SCALE


Measure the line MN . How long is it?



If we wish to represent this line in a shorter space we may do so by drawing a line AB ,
which is $\frac{1}{2}$ as long as MN . 

MN is then said to be drawn to the scale $\frac{1}{2}$, or 1 to 2, each $\frac{1}{2}$ in. of AB representing 1 in. of the length of MN .

If we desire to represent MN by a line still shorter, we may take any other scale as $\frac{1}{4}$, or 1 to 4.

Drawn to this scale MN is represented 
by CD , in which each $\frac{1}{4}$ in. represents 1 in. of the length of MN .

Oral Exercises

1. Notice how each inch on your foot rule is divided. Point to $\frac{1}{2}$ in. on your rule; to $\frac{1}{4}$ in.

2. When a line 1 in. long represents 4 in., what is the scale? when it represents 8 in.?

3. When a line 1 in. long represents 1 ft., the scale is 1 in. to 1 ft.

4. Measure this line and tell what length it represents when the scale is 1 to 2; 1 to 4; 1 to 6; _____
1 to 8; 1 in. to 1 ft.; 1 in. to 2 ft.

5. The picture of a foot rule shown on page 57 is 4 in. long. What length does it represent?

Then, how many inches does each inch of the length of the picture represent? To what scale is the picture drawn?

Written Exercises

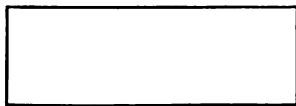
Represent each length by a line drawn to the scale given:

1. 6 in., $\frac{1}{2}$.
2. 8 in., $\frac{1}{4}$.
3. 12 in., $\frac{1}{4}$.
4. 16 in., 1 to 8.
5. 24 in., 1 to 6.
6. 36 in., 1 to 9.
7. 2 ft., 1 in. to 1 ft.
8. 6 ft., 1 in. to 2 ft.
9. 8 ft., 1 in. to 4 ft.

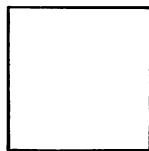
Each of these figures is drawn to the scale $\frac{1}{4}$.



TRIANGLE



RECTANGLE



SQUARE

10. What is the length of each side of the triangle that is represented here? How far is it around the sides?

The distance around the triangle is its **perimeter**.

11. Find the perimeter of the rectangle; of the square.
12. Draw to the scale $\frac{1}{2}$, a rectangle 8 in. long and 4 in. wide, or "8 in. by 4 in." Find its perimeter.
13. The sides of a square are equal. Draw to the scale $\frac{1}{4}$, a square 4 in. by 4 in., or a "4-inch square."

Find the perimeter of, and represent by a drawing:

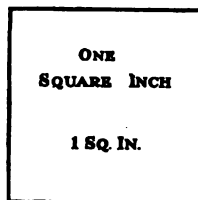
14. A floor, 24 ft. by 16 ft., scale 1 in. to 4 ft.
15. A hall floor, 28 ft. by 7 ft., scale 1 in. to 7 ft.
16. A garden plot, 40 ft. by 32 ft., scale 1 in. to 8 ft.
17. A basketball court, 70 ft. by 50 ft., scale 1 in. to 10 ft.

AREA MEASURES

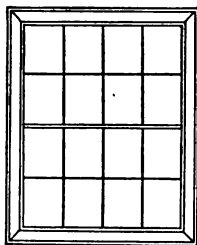
1. A square whose sides are each 1 inch long is a **square inch**.

2. What name is given to a square each of whose sides is 1 foot long?

3. The panes of glass are arranged in **vertical** (up and down) rows; also in



rows across the window, **horizontal** rows.



How many panes of glass are there in each horizontal row? how many such rows?

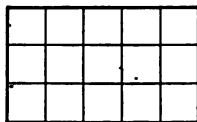
Then, how many times 4 panes of glass are there in the window? how many panes?

4×4 panes are — panes.

4. This picture represents a rectangle 5 in. by 3 in. divided into square inches.

How many square inches are there in 1 horizontal row? how many such rows?

Then, how many times 5 square inches are there in the rectangle? how many square inches?



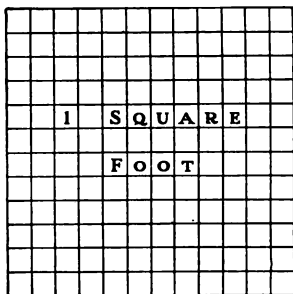
3×5 sq. in. are — sq. in.

This is the **area** of the rectangle.

In the same way, find the area of:

- | | |
|---------------------------|---------------------------------|
| 5. A 5-inch square. | 9. A mirror, 6 ft. by 3 ft. |
| 6. A rug, 5 ft. by 3 ft. | 10. A picture, 8 in. by 4 in. |
| 7. A door, 7 ft. by 3 ft. | 11. A fur robe, 6 ft. by 5 ft. |
| 8. A sign, 9 ft. by 2 ft. | 12. A tablecloth, 9 ft. square. |

1. How long is each side of an inch square? of a foot square? Can a foot square be drawn on this page?



2. On the blackboard draw a square foot divided into square inches, *full size*.

In this picture of a square foot, divided into square inches, the scale is 1 to 8.

3. Since there are 12 inches in a foot, how many square inches are there in each row? in 2 rows? in 12 rows?
4. How many square inches are there in a square foot?

144 square inches equal 1 square foot.

144 sq. in. = 1 sq. ft.

Written Exercises

1. Find the number of square inches in 2 sq. ft.; in $\frac{1}{2}$ sq. ft.; in 5 sq. ft.; in $\frac{1}{4}$ sq. ft.

Represent by a drawing and find the area of:

2. A rectangle, 8 in. by 10 in., scale $\frac{1}{2}$.
3. A square, 8 ft. by 8 ft., scale 1 in. to 4 ft.
4. A platform, 12 ft. by 9 ft., scale 1 in. to 3 ft.
5. A roof, 40 ft. by 30 ft., scale 1 in. to 10 ft.
6. A flower bed, 25 ft. by 15 ft., scale 1 in. to 5 ft.
7. Measure and find the perimeter and area of doors, windows, blackboards, etc., in your schoolroom.

8-11. Find the area of the *surfaces* in Ex. 14-17, p. 60.

THIRD YEAR—SECOND HALF

REVIEW

ADDITION

Oral Exercises

Add, giving results at sight :

1.	8 <u>7</u>	2.	5 <u>9</u>	3.	6 <u>5</u>	4.	9 <u>8</u>	5.	7 <u>6</u>	6.	9 <u>9</u>
7.	52 <u>6</u>	8.	74 <u>3</u>	9.	5 <u>24</u>	10.	5 <u>75</u>	11.	57 <u>4</u>	12.	49 <u>6</u>
13.	64 <u>10</u>	14.	30 <u>17</u>	15.	48 <u>40</u>	16.	20 <u>52</u>	17.	24 <u>30</u>	18.	13 <u>70</u>
19.	14 <u>11</u>	20.	21 <u>23</u>	21.	55 <u>22</u>	22.	32 <u>21</u>	23.	22 <u>44</u>	24.	66 <u>33</u>

25. This morning I saw 7 crows in a tree and 4 on the ground. How many crows did I see?

26. In a game of ringtoss Carl threw the ring over the peg 9 times and missed it 6 times. How many times did he throw?

27. Herbert found 15 chestnuts and Hugh found 8. How many chestnuts did both boys find?

28. Alfred can throw a ball 67 feet and Edgar can throw it 4 feet farther. How far can Edgar throw the ball?

Written Exercises

Add:

1.	2.	3.	4.	5.	6.
21	54	471	309	1240	2358
45	38	135	562	6735	7963
<u>36</u>	<u>62</u>	<u>268</u>	<u>784</u>	<u>8069</u>	<u>4180</u>
7.	8.	9.	10.	11.	12.
15	67	234	528	3146	4701
33	45	561	174	5720	3586
76	83	409	836	4687	5259
<u>28</u>	<u>94</u>	<u>785</u>	<u>954</u>	<u>8235</u>	<u>9647</u>

Find, for each table, the sum of the numbers in each column; in each row:

	13.	14.	15.	16.	17.		22.	23.	24.	25.	26.
18.	32	74	66	32	14	27.	12	18	36	27	44
19.	49	67	88	96	29	28.	27	45	96	89	72
20.	13	20	94	19	37	29.	19	24	83	46	30
21.	83	76	67	73	61	30.	78	39	47	58	77

Add and test:

In examples like these the dollar sign \$ is written only with the first number and with the answer.

31.	32.	33.	34.	35.	36.
\$18	\$45	\$875	\$236	\$5790	\$5267
36	60	914	543	7423	8452
28	79	289	696	8963	5675
63	23	432	535	2970	8952
<u>97</u>	<u>54</u>	<u>807</u>	<u>637</u>	<u>6432</u>	<u>6345</u>

Written Exercises

1. What was the value of a farmer's crop, if it consisted of corn worth \$ 325, oats \$ 298, and potatoes \$ 663 ?

MODEL SOLUTION

\$ 325,	value of corn
298,	value of oats
<u>663,</u>	value of potatoes
\$1286,	value of crop

2. Ruth paid 52¢ for some nuts and 23¢ for raisins. How much did she pay for both ?

The sign ¢ stands for cent or cents.

3. Two boys bought a football. If one paid 39¢ toward it and the other 59¢, how much did the football cost ?

4. A school garden was worked by 148 boys and 86 girls. How many children worked in the garden ?

5. How many books are there in three bookcases, if one contains 241 books, another 196, and the third 275 ?

6. A fire engine cost \$4625 and a chemical engine \$2075. How much did both cost ?

7. If one of Mr. Fay's horses weighs 1473 pounds and the other 1514 pounds, how much does the team weigh ?

8. A milk dealer's sales for a day were 152 quarts from one wagon, 136 from another, and 148 from another. How many quarts of milk did he sell that day ?

9. A fruit dealer bought four crates of oranges. There were 150 oranges in one, 172 in another, 126 in a third, and 128 in the fourth. How many oranges did he buy ?

Written Exercises

Add and test, timing yourself on exercises 19-24:

1.	2.	3.	4.	5.	6.
97	653	2871	4658	5736	35,791
21	849	5639	397	9258	627
38	720	219	40	1824	20,318
<u>45</u>	<u>135</u>	<u>3748</u>	<u>9532</u>	<u>3692</u>	<u>67,854</u>
7.	8.	9.	10.	11.	12.
25	397	5431	8154	7632	59,687
76	58	7698	729	8054	87,321
83	621	254	6483	72	65,473
91	943	9175	9576	823	3,908
<u>47</u>	<u>75</u>	<u>8368</u>	<u>2312</u>	<u>5745</u>	<u>76,154</u>
13.	14.	15.	16.	17.	18.
48	493	2357	3465	9465	13,579
97	872	648	7281	2379	92,468
64	156	9506	5973	814	75,120
72	647	4764	69	8723	715
<u>58</u>	<u>738</u>	<u>293</u>	<u>8346</u>	<u>958</u>	<u>86,934</u>
19.	20.	21.	22.	23.	24.
57	842	1794	4698	5261	78,465
64	761	8563	9327	6789	19,361
39	935	2419	1861	4512	82,529
81	287	7028	2074	9176	34,712
95	513	3695	4519	8457	95,847
<u>72</u>	<u>976</u>	<u>9871</u>	<u>3256</u>	<u>1904</u>	<u>63,092</u>

SUBTRACTION

Oral Exercises

Subtract, giving results instantly :

1. 10 <u>7</u>	2. 11 <u>2</u>	3. 12 <u>4</u>	4. 14 <u>8</u>	5. 13 <u>7</u>	6. 16 <u>9</u>
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7. 12 <u>5</u>	8. 14 <u>7</u>	9. 15 <u>9</u>	10. 11 <u>6</u>	11. 18 <u>9</u>	12. 15 <u>7</u>
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13. 44 <u>8</u>	14. 30 <u>5</u>	15. 52 <u>4</u>	16. 90 <u>9</u>	17. 25 <u>7</u>	18. 64 <u>8</u>
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19. 36 - 8	23. 33 - 9	27. 85 - 50	31. 26 - 24
------------	------------	-------------	-------------

20. 53 - 7	24. 47 - 8	28. 92 - 20	32. 68 - 62
------------	------------	-------------	-------------

21. 74 - 9	25. 62 - 7	29. 74 - 40	33. 85 - 55
------------	------------	-------------	-------------

22. 81 - 7	26. 93 - 6	30. 89 - 60	34. 93 - 13
------------	------------	-------------	-------------

35. How many words are 10 words less 2 words ?

36. Twelve things equal a dozen. Emma set out a dozen pansy plants. If 3 of them died, how many lived ?

37. Richard now has 8¢. How much more money must he have to buy a toy costing 15¢ ?

38. Max and his father caught 16 trout. If Max caught 7 of them, how many did his father catch ?

39. A farmer took 56 pumpkins to market and sold all but 4 of them. How many pumpkins did he sell ?

40. There were 95 bananas in a bunch. After 40 of them had been sold, how many bananas were left ?

Written Exercises

Subtract and test:

1.	5.	9.	13.	17.	21.
77	981	4573	1924	4642	93,062
<u>58</u>	<u>487</u>	<u>1626</u>	<u>980</u>	<u>2925</u>	<u>57,625</u>

2.	6.	10.	14.	18.	22.
62	725	7326	4319	1000	52,849
<u>27</u>	<u>486</u>	<u>3542</u>	<u>2684</u>	<u>248</u>	<u>24,638</u>

3.	7.	11.	15.	19.	23.
44	505	1372	1534	5406	87,246
<u>16</u>	<u>268</u>	<u>1098</u>	<u>569</u>	<u>2159</u>	<u>9,384</u>

4.	8.	12.	16.	20.	24.
93	700	3642	2790	8000	64,047
<u>75</u>	<u>637</u>	<u>2975</u>	<u>1882</u>	<u>508</u>	<u>40,285</u>

Subtract and test quickly, timing yourself:

25.	28.	31.	34.	37.	40.
\$ 65	\$ 406	\$ 1802	\$ 1791	\$ 3002	\$ 20,000
<u>29</u>	<u>372</u>	<u>971</u>	<u>384</u>	<u>1499</u>	<u>374</u>

26.	29.	32.	35.	38.	41.
\$ 75	\$ 782	\$ 9700	\$ 1906	\$ 4281	\$ 81,006
<u>36</u>	<u>294</u>	<u>1811</u>	<u>1052</u>	<u>3462</u>	<u>56,309</u>

27.	30.	33.	36.	39.	42.
\$ 82	\$ 900	\$ 8261	\$ 8796	\$ 6208	\$ 85,704
<u>45</u>	<u>602</u>	<u>4894</u>	<u>4321</u>	<u>1432</u>	<u>46,070</u>

Written Exercises

1. If Donald earned 42¢ on Saturday and spent 25¢ of it to see a ball game, how much money had he left?

MODEL SOLUTION

42¢, money earned

25, money spent

17¢, money left

2. A doll's sweater and cap together cost 45¢. If the cap cost 15¢, how much did the sweater cost?

3. There were 36 children at a party. If 17 of them were boys, how many girls were there?

4. Mildred is 52 inches tall and May is 44 inches tall. How much taller is Mildred than May?

5. Julia spent 31 days in the country. If all but 12 of them were fair, how many fair days were there?

6. David can jump 52 inches and Ivan can jump 39 inches. How much farther can David jump than Ivan?

7. There are 84 trees in an orchard. If 48 of them are apple trees, how many other trees are there in the orchard?

8. A fast train from Chicago was on time in New York. 278 days out of 365 days. On how many days was it late?

9. While coasting, Guy's sled went 716 feet and Roy's 674 feet. How much farther did Guy's sled go than Roy's?

10. A loaded wagon weighed 3678 pounds. If the wagon weighed 1235 pounds, find the weight of the load.

11. A man who had \$5750 bought a house for \$2425. How much money had he left?

MULTIPLICATION

Oral Exercises

Give answers quickly :

- | | | | |
|-----------------|------------------|-------------------|-------------------|
| 1. 5×4 | 6. 2×9 | 11. 8×11 | 16. 2×40 |
| 2. 6×3 | 7. 8×4 | 12. 7×12 | 17. 4×50 |
| 3. 4×9 | 8. 7×6 | 13. 2×20 | 18. 3×13 |
| 4. 3×7 | 9. 8×9 | 14. 9×12 | 19. 2×15 |
| 5. 7×5 | 10. 9×7 | 15. 3×30 | 20. 4×21 |

Find the cost of :

21. 5 handkerchiefs at 8 cents each.
22. 6 pieces of tape at 9 cents each.
23. 2 dozen eggs at 30 cents a dozen.
24. 9 crochet needles at 4 cents each.
25. 7 yards of lace at 8 cents a yard.
26. 2 boxes of honey at 14 cents a box.
27. 9 yards of ribbon at 7 cents a yard.
28. 3 dozen oranges at 30 cents a dozen.
29. 8 pounds of sugar at 6 cents a pound.
30. 12 spools of thread at 5 cents a spool.
31. 4 pecks of potatoes at 20 cents a peck.
32. 2 yards of gingham at 25 cents a yard.
33. 8 pounds of crackers at 9 cents a pound.
34. 12 bunches of celery at 8 cents a bunch.
35. Twenty things equal a score. How many are 3 score? 5 score?

Written Exercises

Multiply:

1. 28	5. \$ 47	9. 213	13. \$ 521	17. 3265	21. 1320
<u>2</u>	<u>6</u>	<u>4</u>	<u>9</u>	<u>3</u>	<u>11</u>
2. 37	6. \$ 54	10. 156	14. \$ 637	18. 1408	22. 2115
<u>3</u>	<u>5</u>	<u>2</u>	<u>5</u>	<u>6</u>	<u>12</u>
3. 26	7. \$ 87	11. 308	15. \$ 494	19. 4237	23. 4062
<u>5</u>	<u>3</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>11</u>
4. 63	8. \$ 71	12. 435	16. \$ 705	20. 5065	24. 3540
<u>4</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>12</u>

Find the cost of:

25. 2 cows at \$ 65 each ; at \$ 72 each.
26. 3 muffs at \$ 19 each ; at \$ 23 each.
27. 2 combs at 35¢ each ; at 39¢ each.
28. 5 towels at 18¢ each ; at 19¢ each.
29. 3 stoves at \$ 42 each ; at \$ 54 each.
30. 4 wagons at \$ 48 each ; at \$ 65 each.
31. 6 saddles at \$ 18 each ; at \$ 24 each.
32. 2 horses at \$ 175 each ; at \$ 225 each.
33. 3 pianos at \$ 425 each ; at \$ 475 each.
34. 7 rowboats at \$ 28 each ; at \$ 32 each.
35. 8 wall tents at \$ 24 each ; at \$ 36 each.
36. 9 office desks at \$ 35 each ; at \$ 55 each.
37. 12 kitchen cabinets at \$ 19 each ; at \$ 24 each.

Written Exercises

1. A grocer bought 3 boxes of lemons, each containing 360 lemons. How many lemons did he buy?

360 lemons

3

1080 lemons

He bought 3 times 360 lemons, or 1080 lemons.

2. At a fruit store Ruth saw 4 boxes, each containing 150 oranges. How many oranges did all the boxes contain?

3. How many pineapples are there in 3 crates, if each crate contains 36 pineapples?

4. A flour barrel holds 196 pounds of flour. How many pounds of flour will 8 such barrels hold?

5. Find the cost of 24 chairs at \$ 2 each.

\$ 24

2

\$ 48

At \$ 1 each, 24 chairs would cost \$ 24.

At \$ 2 each, 24 chairs cost 2 times \$ 24, or \$ 48.

Find the cost of:

Find the value of:

6. 25 hats at \$ 4 each.

9. 96 ten-dollar bills.

7. 32 pears at 3¢ each.

10. 38 five-dollar bills.

8. 48 books at \$ 2 each.

11. 75 two-dollar bills.

12. If you can solve 3 exercises in a minute, how many can you solve in an hour, or 60 minutes?

13. A woman bought 15 packages of flower seeds at 6 cents a package. How much did they cost?

14. The tailor has a pressing iron that weighs 14 pounds. How much did it cost at 5 cents a pound?

Written Exercises

Multiply and test, timing yourself on the second column :

- | | | |
|--------------|----------------|----------------|
| 1. 26 by 14 | 12. 134 by 25 | 23. 452 by 300 |
| 2. 35 by 40 | 13. 205 by 18 | 24. 536 by 160 |
| 3. 51 by 32 | 14. 463 by 30 | 25. 217 by 235 |
| 4. 43 by 27 | 15. 317 by 42 | 26. 368 by 506 |
| 5. 62 by 55 | 16. 608 by 64 | 27. 642 by 424 |
| 6. 57 by 70 | 17. 826 by 59 | 28. 805 by 337 |
| 7. 48 by 63 | 18. 2109 by 35 | 29. 543 by 690 |
| 8. 95 by 16 | 19. 4731 by 90 | 30. 462 by 538 |
| 9. 74 by 82 | 20. 5065 by 73 | 31. 736 by 805 |
| 10. 92 by 48 | 21. 3827 by 86 | 32. 824 by 713 |
| 11. 88 by 36 | 22. 7249 by 95 | 33. 937 by 824 |

34. How many pins are there in a paper of pins that contains 14 rows with 20 pins in each row ?

35. How many dozen eggs are there in 28 cases, if each case contains 30 dozen eggs ?

36. If a rubber tree yields 18 pounds of rubber a year, how much will 25 such trees yield in that time ?

37. A dealer bought 54 pieces of cloth each containing 48 yards. How many yards of cloth did he buy ?

38. Multiplicand = 86 ; multiplier = 57 ; find the product.

39. How many firecrackers are there in a box containing 40 packages of 64 firecrackers each ?

40. How many pounds of coffee are there in 48 bags, if each bag contains 132 pounds ?

41. A poultry car holds 112 crates, each having room for 60 fowls. How many fowls will the car hold?
42. A train consisted of 120 coal cars, each carrying 55 tons of coal. Find the weight of the train load.
43. How many candles can be packed in 75 cases, each case containing 180 candles?
44. How many crates of strawberries were there on a train of 26 cars, each of which contained 225 crates?
45. There are 320 rods in a mile. How many rods does a man walk on a tramp of 17 miles?
46. If 56 carloads of potatoes, of 525 bushels each, are used in New York daily, how many bushels are used a day?
47. In a day there are 1440 minutes. How many minutes are there in 31 days?
48. How many cans of salmon were there in a shipment of 2500 cases, each containing 48 cans?
49. Find the cost of 14 houses at \$8425 each.
50. There are twelve dozen (144) in a gross. How many brushes are there in 150 gross?
51. There are 5280 feet in a mile. How many feet does an automobile go on a 25-mile trip?
52. Find the total weight of 475 bales of raw silk, each bale weighing 106 pounds.
53. How many pounds of dates are there in 820 packages, if a package contains 108 pounds?
54. What is the weight of 905 bags of rice, if each bag weighs 175 pounds?

DIVISION

Oral Exercises

Tell quotients instantly :

- | | | | |
|----------------|-----------------|-----------------|-----------------|
| 1. $16 \div 8$ | 6. $27 \div 3$ | 11. $54 \div 6$ | 16. $64 \div 8$ |
| 2. $28 \div 7$ | 7. $24 \div 2$ | 12. $48 \div 4$ | 17. $72 \div 9$ |
| 3. $24 \div 4$ | 8. $42 \div 7$ | 13. $56 \div 7$ | 18. $63 \div 7$ |
| 4. $35 \div 5$ | 9. $36 \div 4$ | 14. $72 \div 6$ | 19. $81 \div 9$ |
| 5. $48 \div 6$ | 10. $88 \div 8$ | 15. $84 \div 7$ | 20. $96 \div 8$ |

Give quotients and remainders :

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 21. $2 \overline{)9}$ | 26. $5 \overline{)12}$ | 31. $3 \overline{)28}$ | 36. $6 \overline{)57}$ |
| 22. $3 \overline{)16}$ | 27. $3 \overline{)23}$ | 32. $6 \overline{)65}$ | 37. $8 \overline{)25}$ |
| 23. $4 \overline{)18}$ | 28. $6 \overline{)25}$ | 33. $7 \overline{)16}$ | 38. $5 \overline{)33}$ |
| 24. $2 \overline{)17}$ | 29. $4 \overline{)38}$ | 34. $4 \overline{)27}$ | 39. $9 \overline{)19}$ |
| 25. $4 \overline{)21}$ | 30. $3 \overline{)31}$ | 35. $7 \overline{)25}$ | 40. $8 \overline{)75}$ |

41. Divide 12 nuts equally among 4 children.
42. At 3¢ each, how many pencils can be bought for 15¢ ?
43. Three boys sold lemonade and earned \$9, which they divided equally. How much money did each receive ?
44. Jane has 20 cents. How many 5-cent measures of peanuts can she buy ? how many 4-cent measures ?
45. Arthur carries in 6 sticks of wood at a time. How many trips does he make to carry in 30 sticks ? 48 sticks ?
46. In 9 weeks a man saved \$72. If he saved the same amount each week, how much did he save per week ?

Written Exercises

Divide :

1. 2) <u>56</u>	11. 3) <u>351</u>	21. 5) <u>\$ 540</u>	31. 4) <u>4260</u>	41. 2) <u>\$ 3816</u>
2. 4) <u>68</u>	12. 2) <u>548</u>	22. 3) <u>\$ 705</u>	32. 5) <u>3065</u>	42. 4) <u>\$ 5808</u>
3. 3) <u>72</u>	13. 5) <u>325</u>	23. 2) <u>\$ 618</u>	33. 3) <u>8121</u>	43. 5) <u>\$ 7390</u>
4. 5) <u>65</u>	14. 4) <u>824</u>	24. 6) <u>\$ 432</u>	34. 2) <u>7538</u>	44. 3) <u>\$ 5751</u>
5. 6) <u>84</u>	15. 8) <u>656</u>	25. 7) <u>\$ 826</u>	35. 6) <u>5670</u>	45. 7) <u>\$ 8855</u>
6. 7) <u>91</u>	16. 6) <u>342</u>	26. 4) <u>\$ 272</u>	36. 9) <u>3816</u>	46. 6) <u>\$ 4254</u>
7. 5) <u>80</u>	17. 7) <u>469</u>	27. 8) <u>\$ 456</u>	37. 7) <u>6342</u>	47. 4) <u>\$ 2916</u>
8. 4) <u>76</u>	18. 5) <u>730</u>	28. 6) <u>\$ 768</u>	38. 8) <u>9904</u>	48. 8) <u>\$ 7000</u>
9. 6) <u>90</u>	19. 4) <u>316</u>	29. 9) <u>\$ 954</u>	39. 5) <u>3940</u>	49. 9) <u>\$ 6543</u>
10. 7) <u>98</u>	20. 9) <u>837</u>	30. 8) <u>\$ 976</u>	40. 9) <u>7569</u>	50. 7) <u>\$ 5635</u>

Written Exercises

1. If 63 pens are divided equally among 3 classes, how many pens will each class receive?

$$\begin{array}{r} 3 \overline{)63} \text{ pens} \\ 21 \text{ pens} \end{array}$$

When 63 is divided into 3 equal parts, there are 21 in each part.

Then, each class will receive 21 pens.

2. If 48 picture cards are divided equally between 2 children, how many cards does each child receive?

3. Four boys made a bobsled that cost 84¢. If they shared the expense equally, how much did each boy pay?

4. A log 65 feet long was sawed crosswise into 5 equal parts. How long was each part?

5. If a boy earns \$2 a week, how many weeks will it take him to earn \$46?

$$\begin{array}{r} \$2 \overline{) \$46} \end{array}$$

23 times

23 weeks

The number of *weeks* it will take him is the same as the number of *times* that \$2 is contained in \$46.

\$2 is contained 23 times in \$46.

Then, it will take him 23 weeks to earn \$46.

6. How many 2-cent stamps can be bought for 68¢?

7. If 60 boys march "four abreast," or in 4 columns, how many boys will there be in each column?

8. How many 5-cent stamps can be bought for 70¢?

9. There are 3 feet in a **yard**. A road is 66 feet wide. How many yards wide is it?

10. There are 4 quarts in a **gallon**. How many gallons are there in 92 quarts?

Written Exercises

Divide by 2:

1. 89 2. 576 3. 685 4. 5238 5. 7309 6. 59,639

Divide by 3:

7. 67 8. 708 9. 847 10. 5658 11. 8462 12. 29,637

Divide by 4

13. 79 14. 889 15. 652 16. 7508 17. 2546 18. 68,954

Divide by 5:

19. 83 20. 855 21. 578 22. 2405 23. 7398 24. 38,775

Divide by 6:

25. 92 26. 728 27. 423 28. 7896 29. 4000 30. 48,763

Divide by 7:

31. 75 32. 469 33. 976 34. 4876 35. 9083 36. 59,367

Divide by 8:

37. 99 38. 655 39. 827 40. 8976 41. 6573 42. 86,452

Divide by 9:

43. 96 44. 805 45. 938 46. 5342 47. 7837 48. 38,095

Find results and test, timing yourself:

- | | | | |
|-----------------|------------------|-------------------|---------------------|
| 49. $65 \div 2$ | 54. $648 \div 9$ | 59. $8396 \div 4$ | 64. $50,000 \div 8$ |
| 50. $98 \div 8$ | 55. $356 \div 6$ | 60. $7983 \div 7$ | 65. $83,692 \div 6$ |
| 51. $74 \div 3$ | 56. $845 \div 7$ | 61. $6000 \div 9$ | 66. $23,689 \div 5$ |
| 52. $86 \div 4$ | 57. $954 \div 3$ | 62. $5868 \div 6$ | 67. $58,797 \div 9$ |
| 53. $78 \div 5$ | 58. $836 \div 8$ | 63. $9206 \div 3$ | 68. $60,344 \div 7$ |

Written Exercises

1. How many chairs at \$2 each can be bought for \$120?
2. At \$3 each, how many hats can be bought for \$156?
3. A warship sailed 259 miles in 7 hours. At that rate, how far did it go per hour?
4. How many months will it take to pay for a piano costing \$225, if \$5 is paid each month?
5. A blacksmith has 172 horseshoes. How many horses can he shoe with 4 shoes each?
6. A certain church will hold 420 persons, if 6 persons sit in each pew. How many pews are there?
7. There are 1500 peach trees in 3 equal rows. How many trees are there in each row?
8. There are 8 pints in a gallon. How many gallons are there in 736 pints?
9. The yield from 6 acres of cranberries was 576 bushels. Find the yield per acre.
10. In a fire drill 364 school children marched out 2 abreast. How many rows of 2 children were there?
11. If 2000 soldiers march 8 abreast, how many rows of 8 soldiers will there be?
12. In coaling a battleship 1664 tons of coal were loaded in 4 hours. How many tons were loaded per hour?
13. A furniture dealer paid \$2460 for bedsteads at \$6 each. How many did he buy?
14. A debt of \$7236 was paid in 9 years, an equal amount in each year. How much was paid per year?

DIVISION

TABLES OF TENS

Memorize :

1. Multiplication Table

$1 \times 10 = 10$	$7 \times 10 = 70$
$2 \times 10 = 20$	$8 \times 10 = 80$
$3 \times 10 = 30$	$9 \times 10 = 90$
$4 \times 10 = 40$	$10 \times 10 = 100$
$5 \times 10 = 50$	$11 \times 10 = 110$
$6 \times 10 = 60$	$12 \times 10 = 120$

2. Division Table

$10 \div 10 = 1$	$70 \div 10 = 7$
$20 \div 10 = 2$	$80 \div 10 = 8$
$30 \div 10 = 3$	$90 \div 10 = 9$
$40 \div 10 = 4$	$100 \div 10 = 10$
$50 \div 10 = 5$	$110 \div 10 = 11$
$60 \div 10 = 6$	$120 \div 10 = 12$

3. Give results quickly: 4×10 ; $30 \div 10$; 2×10 ; $50 \div 10$; 8×10 ; $70 \div 10$; 6×10 ; $90 \div 10$; 11×10 ; $120 \div 10$.

TABLES OF ELEVENS

Memorize :

1. Multiplication Table

$1 \times 11 = 11$	$7 \times 11 = 77$
$2 \times 11 = 22$	$8 \times 11 = 88$
$3 \times 11 = 33$	$9 \times 11 = 99$
$4 \times 11 = 44$	$10 \times 11 = 110$
$5 \times 11 = 55$	$11 \times 11 = 121$
$6 \times 11 = 66$	$12 \times 11 = 132$

2. Division Table

$11 \div 11 = 1$	$77 \div 11 = 7$
$22 \div 11 = 2$	$88 \div 11 = 8$
$33 \div 11 = 3$	$99 \div 11 = 9$
$44 \div 11 = 4$	$110 \div 11 = 10$
$55 \div 11 = 5$	$121 \div 11 = 11$
$66 \div 11 = 6$	$132 \div 11 = 12$

3. Give results instantly: $22 \div 11$; 3×11 ; $44 \div 11$; 5×11 ; $66 \div 11$; 7×11 ; $88 \div 11$; 9×11 ; $121 \div 11$; 12×11 .

TABLES OF TWELVES

Memorize :

1. Multiplication Table

$1 \times 12 = 12$	$7 \times 12 = 84$
$2 \times 12 = 24$	$8 \times 12 = 96$
$3 \times 12 = 36$	$9 \times 12 = 108$
$4 \times 12 = 48$	$10 \times 12 = 120$
$5 \times 12 = 60$	$11 \times 12 = 132$
$6 \times 12 = 72$	$12 \times 12 = 144$

2. Division Table

$12 \div 12 = 1$	$84 \div 12 = 7$
$24 \div 12 = 2$	$96 \div 12 = 8$
$36 \div 12 = 3$	$108 \div 12 = 9$
$48 \div 12 = 4$	$120 \div 12 = 10$
$60 \div 12 = 5$	$132 \div 12 = 11$
$72 \div 12 = 6$	$144 \div 12 = 12$

3. Give results quickly: 9×12 ; $36 \div 12$; 4×12 ; $24 \div 12$; 6×12 ; $120 \div 12$; 11×12 ; $60 \div 12$; 7×12 ; $144 \div 12$.

Oral Exercises

Give results quickly :

1. 4×11	5. 10×11	9. $80 \div 10$	13. $110 \div 11$
2. 5×12	6. 11×12	10. $84 \div 12$	14. $108 \div 12$
3. 9×10	7. 11×11	11. $99 \div 11$	15. $132 \div 11$
4. 8×12	8. 12×12	12. $96 \div 12$	16. $144 \div 12$

Divide by 11 :

17. 33	20. 77	23. 121
18. 66	21. 99	24. 110
19. 44	22. 88	25. 132

Divide by 12 :

26. 36	29. 84	32. 132
27. 60	30. 72	33. 108
28. 48	31. 96	34. 144

35. Divide by 10, 11, and 12, naming quotients and remainders: 17, 25, 45, 61, 78, 89, 101, 122.

SHORT DIVISION — DIVISORS OF TWO FIGURES

Oral Exercises

1. How many 10's are there in 30? in 50? in 100? in 120? in 150? in 240? in 360? in 500?

Read these numbers as *tens* :

2. 60 3. 90 4. 140 5. 200 6. 340 7. 480

8. When a number ends in 0, how do you find out how many times it contains 10?

Divide by 10 :

9. 70	12. 160	15. 280	18. \$ 700	21. 3000
10. 40	13. 330	16. 600	19. \$ 850	22. 4200
11. 80	14. 670	17. 410	20. \$ 920	23. 9580

Written Exercises

1. Divide 840 by 20.

$\begin{array}{r} 20 \overline{)840} \\ 42 \end{array}$	<p>840 = 84 tens; 20 = 2 tens. 84 tens contains 2 tens as many times as 84 contains 2. Then, cut off or cancel the last figure of each number and divide 84 by 2.</p>
---	--

Divide :

2. 640 by 20	8. \$ 650 by \$ 50	14. 720 by 60
3. 960 by 30	9. \$ 420 by \$ 60	15. 490 by 70
4. 750 by 30	10. \$ 870 by \$ 30	16. 540 by 90
5. 360 by 20	11. \$ 1200 by \$ 50	17. 5600 by 80
6. 760 by 40	12. \$ 1560 by \$ 40	18. 6580 by 70
7. 350 by 50	13. \$ 3450 by \$ 50	19. 5120 by 80

Divide by 11 or by 12 just as with a one-figure divisor, that is, by **short division**, as shown here :

$$\begin{array}{r} 20. \\ 6 \overline{)2790} \\ \underline{465} \end{array}$$

6 in 27, **4**;
6 in 39, **6**;
6 in 30, **5**.

$$\begin{array}{r} 21. \\ 11 \overline{)5753} \\ \underline{523} \end{array}$$

11 in 57, **5**;
11 in 25, **2**;
11 in 33, **3**.

$$\begin{array}{r} 22. \\ 12 \overline{)4863} \\ \underline{405} \end{array} \text{ 3 rem.}$$

12 in 48, **4**;
12 in 6, **0**;
12 in 63, **5**, **3** rem.

Divide by 11 :

23. 154	26. 781	29. 2464	32. 6721	35. 7964
24. 385	27. 583	30. 4587	33. 5093	36. 6193
25. 462	28. 605	31. 3652	34. 8162	37. 9152

Divide by 12 :

38. 180	41. 384	44. 1404	47. 4248	50. 6528
39. 300	42. 612	45. 3720	48. 8472	51. 8760
40. 492	43. 744	46. 2100	49. 5064	52. 7656

Divide by 11 ; then by 12 :

53. 253	56. 420	59. 1562	62. 6432	65. 7755
54. 372	57. 517	60. 3768	63. 4818	66. 8592
55. 638	58. 864	61. 2057	64. 5088	67. 9960

68. Mrs. Avery bought 156 clothespins. How many dozen clothespins did she buy ?

69. The yield from 11 acres of oats was 528 bushels. Find the yield per acre.

70. One day 720 people attended a school exhibition. How many score of people attended the exhibition ?

LONG DIVISION—DIVISORS OF TWO FIGURES

Written Exercises

1. Divide 483 by 21.

$$\begin{array}{r}
 23 \\
 21 \overline{)483} \\
 \underline{42} \\
 63 \\
 \underline{63} \\
 0
 \end{array}$$

21 is not contained in 4, but it is contained in 48 about as many times as 20 is contained in 40, or about as many times as 2 is contained in 4, that is, about 2 times. Try 2.

Write 2 in the quotient over 8, the last figure of 483 used; multiply 21 by 2, giving 42; subtract 42 from 48, giving 6, which does not contain 21; then, 2 is the right quotient figure.

Bring down 3; 21 is contained in 63, 3 times; write 3 in the quotient; multiply 21 by 3; subtract; there is no remainder.

Then, the quotient is 23.

- The five steps in dividing*
- | | | |
|---|--|---|
| <i>The
five steps
in
dividing</i> | $\left\{ \begin{array}{l} (1) \text{ Divide} \\ (2) \text{ Write quotient figure} \\ (3) \text{ Multiply} \\ (4) \text{ Subtract} \\ (5) \text{ Bring down next figure} \end{array} \right.$ | (1) Divide . . . Thus, in the process, $48 \div 21 = 2$. |
| | | (2) Write quotient figure Write 2. |
| | | (3) Multiply $2 \times 21 = 42$. |
| | | (4) Subtract $48 - 42 = 6$. |
| | | (5) Bring down next figure 6 tens + 3 = 63. |

Point out the steps in the following:

2.
$$\begin{array}{r}
 31 \\
 21 \overline{)651} \\
 \underline{63} \\
 21 \\
 \underline{21} \\
 0
 \end{array}$$

3.
$$\begin{array}{r}
 42 \\
 21 \overline{)882} \\
 \underline{84} \\
 42 \\
 \underline{42} \\
 0
 \end{array}$$

4.
$$\begin{array}{r}
 54 \\
 21 \overline{)1134} \\
 \underline{105} \\
 84 \\
 \underline{84} \\
 0
 \end{array}$$

The processes on this page illustrate long division.

The quotient is usually written *below* the dividend in short division and *above* it in long division.

Find quotients and test:

$$\begin{array}{r} 32 \\ 5. \ 31 \overline{)992} \\ \underline{93} \\ 62 \\ \underline{62} \end{array}$$

$$\begin{array}{r} 24 \\ 6. \ 22 \overline{)528} \\ \underline{44} \\ 88 \\ \underline{88} \end{array}$$

$$\begin{array}{r} 45 \\ 7. \ 31 \overline{)1395} \\ \underline{124} \\ 155 \\ \underline{155} \end{array}$$

Tests. — $31 \times 32 = 992.$

$22 \times 24 = 528.$

$31 \times 45 = 1395.$

Divide:

Divide and test:

Divide:

- | | | |
|---------------|----------------|----------------|
| 8. 672 by 21 | 25. 882 by 21 | 42. 1088 by 32 |
| 9. 561 by 51 | 26. 748 by 22 | 43. 3567 by 41 |
| 10. 682 by 22 | 27. 775 by 31 | 44. 1872 by 52 |
| 11. 483 by 21 | 28. 832 by 32 | 45. 4284 by 51 |
| 12. 651 by 31 | 29. 656 by 41 | 46. 2976 by 62 |
| 13. 992 by 32 | 30. 798 by 42 | 47. 4608 by 72 |
| 14. 525 by 21 | 31. 765 by 51 | 48. 5551 by 61 |
| 15. 793 by 61 | 32. 676 by 52 | 49. 3645 by 81 |
| 16. 496 by 31 | 33. 1071 by 21 | 50. 5084 by 62 |
| 17. 945 by 21 | 34. 1364 by 22 | 51. 6912 by 72 |
| 18. 624 by 52 | 35. 1147 by 31 | 52. 8281 by 91 |
| 19. 744 by 31 | 36. 1224 by 51 | 53. 5964 by 71 |
| 20. 504 by 42 | 37. 1488 by 31 | 54. 3286 by 62 |
| 21. 714 by 51 | 38. 1134 by 42 | 55. 6966 by 81 |
| 22. 448 by 32 | 39. 2601 by 51 | 56. 3094 by 91 |
| 23. 943 by 41 | 40. 1664 by 52 | 57. 4784 by 92 |
| 24. 924 by 42 | 41. 2132 by 41 | 58. 7872 by 82 |

Written Exercises

1. Divide 2856 by 42.

$$\begin{array}{r} 7^* \\ 42 \overline{)2856} \\ \underline{294} \end{array}$$

*Quotient too large

$$\begin{array}{r} 5^* \\ 42 \overline{)2856} \\ \underline{210} \\ 75 \end{array}$$

*Quotient too small

$$\begin{array}{r} 68 \\ 42 \overline{)2856} \\ \underline{252} \\ 336 \\ \underline{336} \end{array}$$

Since 4 is contained in 28 only 7 times, 42 is contained in 285 not more than 7 times, but perhaps only 5 or 6 times.

The first figure of the quotient cannot be 7, for 294 cannot be subtracted from 285. It is not 5, for the remainder 75, being larger than 42, will contain 42 again. It is 6, for 6×42 , or 252, can be subtracted from 285, and the remainder 33 is less than 42.

The solution is completed as shown in the right-hand process.

Divide :

- | | | |
|----------------|----------------|----------------|
| 2. 1672 by 22 | 13. 3608 by 41 | 24. 4898 by 62 |
| 3. 2079 by 21 | 14. 3519 by 51 | 25. 6319 by 71 |
| 4. 2728 by 31 | 15. 2028 by 52 | 26. 5658 by 82 |
| 5. 3040 by 32 | 16. 4209 by 61 | 27. 7268 by 92 |
| 6. 2436 by 42 | 17. 5049 by 51 | 28. 6336 by 72 |
| 7. 2759 by 31 | 18. 3658 by 62 | 29. 7209 by 81 |
| 8. 1536 by 32 | 19. 4576 by 52 | 30. 6478 by 82 |
| 9. 3239 by 41 | 20. 5609 by 71 | 31. 9009 by 91 |
| 10. 3612 by 42 | 21. 4819 by 61 | 32. 6348 by 92 |
| 11. 2176 by 32 | 22. 2808 by 72 | 33. 7216 by 82 |
| 12. 3009 by 51 | 23. 4056 by 52 | 34. 8188 by 92 |

$$\begin{array}{r}
 231 \\
 35. \ 32 \overline{)7392} \\
 \underline{64} \\
 99 \\
 \underline{96} \\
 32 \\
 \underline{32} \\
 0
 \end{array}$$

$$\begin{array}{r}
 125 \\
 36. \ 52 \overline{)6500} \\
 \underline{52} \\
 130 \\
 \underline{104} \\
 260 \\
 \underline{260} \\
 0
 \end{array}$$

$$\begin{array}{r}
 346 \\
 37. \ 72 \overline{)24912} \\
 \underline{216} \\
 331 \\
 \underline{288} \\
 432 \\
 \underline{432} \\
 0
 \end{array}$$

38. 5661 by 51

41. 9086 by 22

44. 24,864 by 42

39. 9102 by 41

42. 8601 by 61

45. 37,882 by 62

40. 9345 by 21

43. 9984 by 32

46. 70,848 by 72

$$\begin{array}{r}
 150 \\
 47. \ 61 \overline{)9150} \\
 \underline{61} \\
 305 \\
 \underline{305} \\
 0
 \end{array}$$

$$\begin{array}{r}
 204 \\
 48. \ 42 \overline{)8568} \\
 \underline{84} \\
 168 \\
 \underline{168} \\
 0
 \end{array}$$

$$\begin{array}{r}
 608 \\
 49. \ 32 \overline{)19456} \\
 \underline{192} \\
 256 \\
 \underline{256} \\
 0
 \end{array}$$

After the second division, which is 0 and write 0 in the quotient.

After bringing down 6, as 42 is not contained in 16, write 0 in the quotient and bring down 8.

After bringing down 5, as 32 is not contained in 25, write 0 in the quotient and bring down 6.

Bring down only one figure at a time and write a figure in the quotient (it may be 0) for each figure brought down.

50. 6820 by 31

54. 6527 by 61

58. 69,120 by 72

51. 8405 by 41

55. 8520 by 71

59. 16,929 by 81

52. 6496 by 32

56. 6448 by 62

60. 26,416 by 52

53. 9020 by 82

57. 9919 by 91

61. 65,044 by 92

Written Exercises

1. Divide 25,272 by 78.

$$\begin{array}{r}
 324 \\
 78 \overline{)25272} \\
 \underline{234} \\
 187 \\
 \underline{156} \\
 312 \\
 \underline{312} \\
 0
 \end{array}$$

Since 78 is only a little less than 80, the first figure of the quotient is estimated by dividing 252 by 80, or 25 by 8, which gives 3. Multiplying 78 by 3 and subtracting the product from 252 gives a remainder *less than the divisor*. Then, 3 is the correct quotient figure.

The second figure of the quotient is estimated by dividing 18 by 8, giving 2, the correct figure, as is shown by multiplying and subtracting as before.

Since $31 \div 8$ is nearly 4, and the true divisor is a little less than 80, we estimate the last figure of the quotient to be 4. The test by multiplication and subtraction shows that 4 is the correct figure, and that the division is exact.

The quotient, then, is 324.

Divide:

- | | | |
|-------------------|----------------|------------------|
| 2. 575 by 23 | 14. 1218 by 29 | 26. 33,852 by 84 |
| 3. 648 by 18 | 15. 2842 by 49 | 27. 19,630 by 65 |
| 4. 780 by 15 | 16. 1080 by 36 | 28. 17,408 by 68 |
| 5. 952 by 28 | 17. 2214 by 54 | 29. 12,654 by 57 |
| 6. \$891 by 33 | 18. 7956 by 78 | 30. 15,756 by 78 |
| 7. \$888 by 24 | 19. 1449 by 69 | 31. 17,475 by 75 |
| 8. \$496 by 16 | 20. 8712 by 44 | 32. 70,016 by 64 |
| 9. \$676 by 13 | 21. 6867 by 63 | 33. 28,992 by 96 |
| 10. \$884 by \$17 | 22. 3738 by 89 | 34. 16,037 by 79 |
| 11. \$750 by \$25 | 23. 4425 by 59 | 35. 29,078 by 67 |
| 12. \$858 by \$39 | 24. 9890 by 46 | 36. 65,195 by 85 |
| 13. \$952 by \$34 | 25. 9635 by 47 | 37. 90,816 by 86 |

Written Exercises

1. If a man shears 15 sheep per hour, how many hours will it take him to shear 225 sheep?
2. A car containing 672 bushels of grain was unloaded in 16 minutes. How many bushels were unloaded per minute?
3. If 13 boys weigh 975 pounds, find their **average** weight, that is, the weight of each, supposing they are of equal weight.
4. A farmer bought 21 cows for \$ 966. Find the **average** amount paid for each cow.
5. Find the average rate per hour of an automobile that went 646 miles in 17 hours.
6. I have 954 strawberry plants set in 18 rows. How many plants are there in each row?
7. From 45 trees a fruit grower picked 540 bushels of apples. Find the average yield per tree.
8. There are 24 hours in one day. How many days are there in 840 hours?
9. A freight train ran 935 miles in 55 hours. What was its average rate per hour?
10. A bushel of wheat weighs 60 pounds. How many bushels are there in a bin that contains 3120 pounds?
11. A bottle-making machine turns out 75 bottles per hour. How long will it take to make 1800 bottles?
12. The cost of 25 village lots was \$ 3125. What was the average price per lot?

Written Exercises

Divide and test:

$$\begin{array}{r}
 38 \\
 19 \overline{)734} \\
 \underline{57} \\
 164 \\
 \underline{152} \\
 12, \text{ rem.}
 \end{array}$$

$$\begin{array}{r}
 340 \\
 28 \overline{)9543} \\
 \underline{84} \\
 114 \\
 \underline{112} \\
 23, \text{ rem.}
 \end{array}$$

$$\begin{array}{r}
 670 \\
 57 \overline{)38195} \\
 \underline{342} \\
 399 \\
 \underline{399} \\
 5, \text{ rem.}
 \end{array}$$

Test. — To the product of the quotient and the divisor add the remainder. The result should equal the dividend.

Find results and test:

- | | | |
|------------------|--------------------|----------------------|
| 4. $963 \div 17$ | 10. $5246 \div 53$ | 16. $18,179 \div 78$ |
| 5. $758 \div 23$ | 11. $4000 \div 19$ | 17. $21,310 \div 48$ |
| 6. $687 \div 39$ | 12. $8008 \div 29$ | 18. $41,683 \div 27$ |
| 7. $808 \div 16$ | 13. $9664 \div 46$ | 19. $40,000 \div 99$ |
| 8. $785 \div 24$ | 14. $4687 \div 59$ | 20. $23,601 \div 76$ |
| 9. $794 \div 38$ | 15. $5640 \div 28$ | 21. $71,111 \div 69$ |

Divide and test, timing yourself on the second column:

- | | | |
|---------------|----------------|------------------|
| 22. 565 by 14 | 29. 1250 by 37 | 36. 10,325 by 56 |
| 23. 430 by 25 | 30. 3824 by 49 | 37. 39,267 by 63 |
| 24. 674 by 18 | 31. 4382 by 15 | 38. 45,869 by 47 |
| 25. 930 by 32 | 32. 6792 by 22 | 39. 28,146 by 92 |
| 26. 748 by 26 | 33. 8006 by 58 | 40. 52,073 by 74 |
| 27. 809 by 33 | 34. 9263 by 64 | 41. 70,584 by 89 |
| 28. 918 by 24 | 35. 7340 by 38 | 42. 63,257 by 68 |

DIVIDENDS AND DIVISORS ENDING IN ZEROS**Oral Exercises**

1. Divide 90 by 10 ; 120 by 10 ; 200 by 10 ; 450 by 10.
How may a number ending in 0 be divided by 10 ?
2. How many times is 100 contained in 500 ? in 900 ?
How may a number ending in two 0's be divided by 100 ?
3. How many times is 1000 contained in 4000 ? How
may a number ending in three 0's be divided by 1000 ?

Divide :

- | | | |
|--------------|----------------|--------------------|
| 4. 470 by 10 | 7. 1600 by 100 | 10. 10,000 by 1000 |
| 5. 750 by 10 | 8. 8900 by 100 | 11. 53,000 by 1000 |
| 6. 800 by 10 | 9. 9000 by 100 | 12. 72,000 by 1000 |

Written Exercises

1. Divide 360 by 40 ; 1600 by 400 ; 76,000 by 4000.

$$\begin{array}{r} 4\cancel{0})36\cancel{0} \\ \underline{9} \end{array}$$

4 tens are contained in 36 tens as many times as 4 is contained in 36.

$$\begin{array}{r} 4\cancel{0}\cancel{0})16\cancel{0}\cancel{0} \\ \underline{4} \end{array}$$

4 hundreds are contained in 16 hundreds as many times as 4 is contained in 16.

$$\begin{array}{r} 4\cancel{0}\cancel{0}\cancel{0})76\cancel{0}\cancel{0}\cancel{0} \\ \underline{19} \end{array}$$

4 thousands are contained in 76 thousands as many times as 4 is contained in 76.

Divide :

- | | | |
|----------------|--------------------|--------------------------|
| 2. 920 by 40 | 7. 33,500 by 50 | 12. \$ 48,000 by \$ 240 |
| 3. 5760 by 80 | 8. 49,630 by 70 | 13. \$ 34,170 by \$ 170 |
| 4. 5280 by 110 | 9. 39,000 by 130 | 14. \$ 42,140 by \$ 140 |
| 5. 1080 by 120 | 10. 32,000 by 1600 | 15. \$ 54,000 by \$ 2000 |
| 6. 7680 by 120 | 11. 45,000 by 1500 | 16. \$ 72,000 by \$ 3000 |

DIVISORS OF THREE FIGURES

Written Exercises

Divide :

$$\begin{array}{r}
 35 \\
 243 \overline{)8505} \\
 \underline{729} \\
 1215 \\
 \underline{1215}
 \end{array}$$

$$\begin{array}{r}
 370 \\
 243 \overline{)89910} \\
 \underline{729} \\
 1701 \\
 \underline{1701} \\
 0
 \end{array}$$

Find quotients :

- | | | |
|----------------|------------------|------------------|
| 3. 1616 ÷ 101 | 20. 50,160 ÷ 114 | 37. 14,124 ÷ 428 |
| 4. 4687 ÷ 109 | 21. 27,375 ÷ 125 | 38. 54,216 ÷ 502 |
| 5. 2664 ÷ 111 | 22. 13,585 ÷ 209 | 39. 71,325 ÷ 317 |
| 6. 3927 ÷ 119 | 23. 30,885 ÷ 145 | 40. 34,720 ÷ 868 |
| 7. 4114 ÷ 121 | 24. 23,550 ÷ 314 | 41. 77,895 ÷ 577 |
| 8. 5246 ÷ 122 | 25. 40,986 ÷ 198 | 42. 86,944 ÷ 418 |
| 9. 6912 ÷ 108 | 26. 17,328 ÷ 304 | 43. 51,255 ÷ 603 |
| 10. 8979 ÷ 123 | 27. 37,120 ÷ 116 | 44. 74,060 ÷ 529 |
| 11. 6084 ÷ 117 | 28. 27,664 ÷ 133 | 45. 41,160 ÷ 392 |
| 12. 3100 ÷ 124 | 29. 17,845 ÷ 215 | 46. 31,755 ÷ 435 |
| 13. 7874 ÷ 127 | 30. 19,080 ÷ 424 | 47. 93,380 ÷ 805 |
| 14. 6864 ÷ 156 | 31. 33,872 ÷ 232 | 48. 48,174 ÷ 518 |
| 15. 8526 ÷ 203 | 32. 37,260 ÷ 405 | 49. 17,784 ÷ 342 |
| 16. 7420 ÷ 212 | 33. 37,830 ÷ 291 | 50. 95,427 ÷ 461 |
| 17. 9652 ÷ 508 | 34. 22,862 ÷ 322 | 51. 53,676 ÷ 639 |
| 18. 7775 ÷ 311 | 35. 52,312 ÷ 503 | 52. 39,820 ÷ 724 |
| 19. 6868 ÷ 404 | 36. 37,760 ÷ 236 | 53. 42,778 ÷ 586 |

Miscellaneous Exercises

Add and then subtract rapidly :

- | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 1. 38 | 2. 43 | 3. 55 | 4. 72 | 5. 69 | 6. 51 |
| <u>10</u> | <u>30</u> | <u>20</u> | <u>40</u> | <u>30</u> | <u>40</u> |
| 7. 65 | 8. 72 | 9. 78 | 10. 67 | 11. 56 | 12. 84 |
| <u>40</u> | <u>30</u> | <u>20</u> | <u>42</u> | <u>31</u> | <u>24</u> |

Give answers quickly :

- | | | | |
|--------------------|---------------------|-------------------|--------------------|
| 13. 48×10 | 16. 11×400 | 19. $250 \div 10$ | 22. $500 \div 100$ |
| 14. 21×20 | 17. 16×200 | 20. $400 \div 10$ | 23. $600 \div 200$ |
| 15. 12×30 | 18. 12×300 | 21. $180 \div 20$ | 24. $900 \div 300$ |

Tell quotients, reading rapidly across the page :

- | | | | | | | |
|-------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|----------------------|
| 25. $8 \overline{)40}$ | 7 $\overline{)56}$ | 9 $\overline{)54}$ | 6 $\overline{)48}$ | 5 $\overline{)45}$ | 9 $\overline{)81}$ | 12 $\overline{)144}$ |
| 26. $6 \overline{)60}$ | 8 $\overline{)72}$ | 8 $\overline{)56}$ | 9 $\overline{)63}$ | 4 $\overline{)36}$ | 8 $\overline{)96}$ | 11 $\overline{)132}$ |
| 27. $8 \overline{)480}$ | 6 $\overline{)540}$ | 3 $\overline{)150}$ | 9 $\overline{)450}$ | 7 $\overline{)630}$ | 6 $\overline{)660}$ | 12 $\overline{)840}$ |

28. Find the cost of 12 desks at \$100 each.

29. There are 100¢ in a dollar. How many dollars are there in 600¢? in 700¢? in 1000¢?

30. A farmer had 36 cows $\frac{1}{4}$ of which were Jerseys. How many Jersey cows did he have?

31. A fishing fleet was made up of 700 boats with 9 men to each boat. How many men were there in the fleet?

32. Edward stood halfway between two street lights that were 330 ft. apart. How far was he from each light?

Subtract these numbers from 8000 :

- | | | | |
|----------|----------|----------|----------|
| 33. 6040 | 35. 5076 | 37. 1668 | 39. 4305 |
| 34. 1838 | 36. 2979 | 38. 7260 | 40. 3141 |

Find answers :

- | | | | |
|--------------------|---------------------|------------------|--------------------|
| 41. 271×8 | 44. 735×12 | 47. $456 \div 6$ | 50. $1464 \div 12$ |
| 42. 509×7 | 45. 223×36 | 48. $904 \div 8$ | 51. $4650 \div 62$ |
| 43. 984×9 | 46. 119×59 | 49. $783 \div 9$ | 52. $8281 \div 91$ |

Add and test :

- | | | | |
|------------|-------------|-------------|---------------|
| 53. 234 | 54. 3288 | 55. 4961 | 56. 65,371 |
| 618 | 964 | 604 | 74,632 |
| 305 | 4891 | 5283 | 48,510 |
| 914 | 546 | 1859 | 56,909 |
| 876 | 2064 | 792 | 28,434 |
| <u>371</u> | <u>1487</u> | <u>2645</u> | <u>89,438</u> |

Perform the operations indicated, and test, keeping a record of the time required for each column :

- | | | |
|---------------------|----------------------|-------------------------|
| ✓ 57. $985 + 69$ | 66. $8190 \div 90$ | ✓ 75. $15,708 \div 231$ |
| 58. $880 \div 22$ | 67. 7658×96 | 76. $36,079 \times 400$ |
| 59. $723 - 76$ | 68. $3101 - 34$ | 77. $10,176 \div 848$ |
| 60. $489 + 98$ | 69. $6834 \div 67$ | 78. $83,750 \div 250$ |
| 61. $624 \div 39$ | 70. $8999 \div 78$ | 79. $70,012 \times 310$ |
| 62. 763×80 | 71. 4395×59 | 80. $48,400 \div 200$ |
| 63. $841 \div 29$ | 72. $7770 \div 37$ | 81. $75,993 \div 219$ |
| 64. $400 - 78$ | 73. $9856 \div 44$ | 82. $93,467 \times 500$ |
| 65. $952 \div 17$ | ✓ 74. $6759 \div 88$ | ✓ 83. $97,232 \div 412$ |

Chicken Problems

1. Earl has 64 hens $\frac{1}{2}$ of which are White Leghorns. How many White Leghorn hens has he?

2. Arthur fed his hens 4 quarts of corn per day. Find the cost, at 2¢ per quart, of the corn fed in a week (7 days).



3. Biddy laid 54 eggs while Dot laid 49. How many more eggs did Biddy lay than Dot?

4. Ralph has 112 chickens in one yard and 48 in another. How many chickens has he in both yards?

5. In a store Henry saw 14 boxes each containing 12 eggs. How many eggs were there in the boxes?

6. Mr. Howe had 480 chickens. If he sold $\frac{1}{4}$ of them, how many did he sell? How many had he left?

7. Frank's hens laid 24 eggs per day for a week. How many dozen eggs did they lay that week?

8. A chicken yard is 12 ft. by 20 ft. How many feet of fence are required to inclose it?

9. A flock of chickens was put in a yard 7 ft. by 41 ft. How many square feet of space were there in the yard?

DIVISORS AND FACTORS

A number that divides another number without a remainder is an **exact divisor** of that number.

Divisible means exactly divisible.

Thus, 9 is divisible by 3 but not by 2.

A number that is divisible by 2 is an **even number**.

A number that is not divisible by 2 is an **odd number**.

The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are **digits**.

TESTS OF DIVISIBILITY

Make a list of all the numbers from 2 to 40 that are divisible:

1. By 2 2. By 5 3. By 3 4. By 9 5. By 10

What units' figures have the numbers in list 1? in list 2?

Find whether the sum of the digits of each number in list 3 is divisible by 3; in list 4 is divisible by 9.

What is the units' figure of each number in list 5?

A number is divisible:

By 2, if the units' figure is 2, 4, 6, 8, or 0.

By 5, if the units' figure is 5 or 0.

By 3, if the sum of the digits is divisible by 3.

Thus, 321 is divisible by 3 because the sum of its digits $3+2+1=6$, and 6 is divisible by 3.

By 9, if the sum of its digits is divisible by 9.

Thus, 927 is divisible by 9 because the sum of its digits $9+2+7=18$, and 18 is divisible by 9.

By 10, if its units' figure is 0.

Oral Exercises

1. Which of the numbers from 10 to 100 are divisible by 2? by 5? by 3? by 9? by 10?

Select, from the following, the numbers that are divisible by 2; by 5; by 3; by 9; by 10:

- | | | | | |
|--------|--------|----------|----------|----------|
| 2. 260 | 6. 345 | 10. 792 | 14. 1235 | 18. 5373 |
| 3. 845 | 7. 723 | 11. 1600 | 15. 6786 | 19. 6030 |
| 4. 250 | 8. 426 | 12. 1440 | 16. 4041 | 20. 5080 |
| 5. 926 | 9. 866 | 13. 1385 | 17. 7644 | 21. 6240 |

FACTORING

An exact divisor of a number is a **factor** of it.

Thus, 2, 3, 4, and 6 are factors of 12.

Though 1 and 12 are exact divisors they are not usually regarded as factors.

A number that has no factors except itself and 1 is a **prime number**.

Memorize the prime numbers from 1 to 25:

1 2 3 5 7 11 13 17 19 23

A factor that is a prime number is a **prime factor**.

The prime factors of 12 are 2, 2, and 3.

Name the prime factors of:

4 6 9 10 16 . 28 30 49

The process of separating a number into its factors is **factoring**.

In separating a number into two factors, any exact divisor may be taken for one factor and the quotient for the other.

Oral Exercises

1. Separate into two factors in all possible ways:

10 12 16 21 32 36 28 40 42

2. Which of the following numbers are prime? Why?

7 9 14 19 22 27 29 33 48

Find the prime factors of:

3. 4 6. 18 9. 35 12. 24 15. 39 18. 60

4. 6 7. 20 10. 44 13. 30 16. 56 19. 75

5. 8 8. 25 11. 49 14. 50 17. 77 20. 90

Written Exercises

1. Find the prime factors of 615.

$$\begin{array}{r|l} 5 & 615 \\ 3 & 123 \\ & 41 \end{array}$$

Since the units' figure of 615 is 5, what number is a factor of 615? Divide by 5; what is the other factor?

Since the sum of the digits of 123 is divisible by 3, what number is a factor of 123? Divide by 3; what is the other factor?

Since 41 is prime, the prime factors of 615 are 3, 5, and 41.

Find the prime factors of:

2. 72 9. 51 16. 57 23. 160 30. 198 37. 345

3. 63 10. 68 17. 78 24. 220 31. 168 38. 456

4. 85 11. 87 18. 92 25. 270 32. 441 39. 567

5. 54 12. 64 19. 76 26. 225 33. 324 40. 648

6. 95 13. 96 20. 81 27. 180 34. 234 41. 780

7. 84 14. 88 21. 65 28. 375 35. 325 42. 891

8. 99 15. 69 22. 98 29. 126 36. 672 43. 912

MEASURES

LIQUID MEASURES

1. If you can get measures like these, test the truth of the following table of **liquid measures** by measuring water.

2. Learn the table :

2 pints (pt.) = 1 quart (qt.)

4 quarts = 1 gallon (gal.)

3. Name some liquids that are sold by the pint; by the quart; by the gallon.



Oral Exercises

1. How many quart bottles can a milkman fill out of a can that has 1 gal. of milk in it?
2. Tell the number of pint bottles that can be filled with 1 qt. of milk ; with 4 qt. of milk ; with 1 gal. of milk.
3. How many pints are there in $\frac{1}{2}$ qt. ?
4. To how many quarts is $\frac{1}{4}$ gal. equal? $\frac{1}{2}$ gal. ?
5. What part of a quart is 1 pt. ?
6. What part of a gallon is 1 qt. ? are 2 qt. ?
7. How many pints are 3 qt. ? 3 qt. + 1 pt. ?
8. How many quarts are 4 gal. ? 4 gal. 2 qt. ?
9. How many gallons and how many quarts over will a 10-quart pail hold ?

Written Exercises

Change to pints:

1. 14 qt. 3. 60 qt. 5. 45 qt. 7. 120 qt. 9. 340 qt.
 2. 25 qt. 4. 36 qt. 6. 72 qt. 8. 235 qt. 10. 525 qt.

Change to quarts:

11. 35 gal. 13. 31 gal. 15. 63 gal. 17. 124 gal. 19. 320 gal.
 12. 48 gal. 14. 54 gal. 16. 75 gal. 18. 240 gal. 20. 504 gal.

Change to quarts:

21. 28 pt. 23. 130 pt.
 22. 46 pt. 24. 264 pt.

Change to gallons:

25. 64 qt. 27. 368 qt.
 26. 96 qt. 28. 564 qt.

29. I bought a pint of milk each day for a month of 30 days. How many quarts of milk did I buy that month?

LENGTH MEASURES

1. If you have a foot rule and a yardstick verify the following table of length measures.

2. Learn the table:

<hr/>	
12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
<hr/>	

3. Name some things that are measured by the inch; by the foot; by the yard.

4. How many inches are there in 1 ft.? in 3 ft.? in 1 yd.? in $\frac{1}{4}$ yd.? in $\frac{1}{2}$ yd.?

Oral Exercises

1. How many inches are there in 1 ft. ? in $\frac{1}{2}$ ft. ?
2. To how many inches is $\frac{1}{4}$ ft. equal ?
3. What part of a foot are 6 in. ? 3 in. ?
4. How many feet are there in 3 yd. ? in 5 yd. ?
5. How many inches are there in 2 ft. ? in 3 ft. ? in 4 ft. ?
6. Mabel made a daisy chain 1 yd. long. How many feet long was it ? how many inches ?
7. How many feet are there in 4 yd. ? in 4 yd. 1 ft. ?
8. How many inches are there in 5 ft. ? in 5 ft. 6 in. ?
9. How many pieces of ribbon 2 yd. long can be cut from a bolt of ribbon that contains 10 yd. ?

Written Exercises

Change to feet :

- | | | | |
|-----------|-----------|-----------|-----------|
| 1. 78 yd. | 3. 46 yd. | 5. 67 yd. | 7. 38 yd. |
| 2. 96 yd. | 4. 75 yd. | 6. 85 yd. | 8. 65 yd. |

Change to inches :

- | | | | |
|------------|------------|------------|------------|
| 9. 59 ft. | 11. 76 ft. | 13. 48 ft. | 15. 64 ft. |
| 10. 87 ft. | 12. 84 ft. | 14. 79 ft. | 16. 89 ft. |

Change to yards :

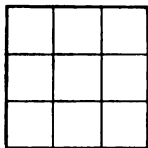
Change to feet :

- | | | | |
|------------|-------------|------------|-------------|
| 17. 84 ft. | 19. 579 ft. | 21. 72 in. | 23. 324 in. |
| 18. 78 ft. | 20. 894 ft. | 22. 96 in. | 24. 936 in. |

25. A football field is 100 yd. long. Express its length in feet.

AREA MEASURES

1. How many square inches are there in a square foot?
2. Draw full size on the blackboard a yard square, divided into foot squares. Draw the same to the scale $\frac{1}{2}$; to the scale $\frac{1}{4}$.
3. This is a yard square drawn to a smaller scale. Count the square feet in it.
4. How many square feet are there in a square yard?
5. Learn this table of area measures :



144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)

Written Exercises

How many square inches are there in

1. 2 sq. ft.?
3. 3 sq. ft.?
5. 1 sq. ft. + 48 sq. in.?
2. $\frac{1}{2}$ sq. ft.?
4. $\frac{1}{4}$ sq. ft.?
6. 2 sq. ft. + 72 sq. in.?

How many square feet are there in

7. 14 sq. yd.?
9. 24 sq. yd.?
11. 576 sq. in.?
8. 16 sq. yd.?
10. 32 sq. yd.?
12. 864 sq. in.?

13. How many square inches are there in a square yard?
14. Find the number of square inches in a rug whose area is 4 sq. ft.
15. How many square yards of carpet are required for a floor whose area is 216 sq. ft.?

AREA OF SQUARES AND RECTANGLES

Written Exercises

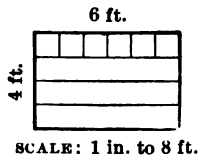
1. Find the area of a rectangle 6 ft. by 4 ft.

MODEL SOLUTION

In 1 row there are 6 sq. ft.

In 4 rows there are 4×6 sq. ft., or 24 sq. ft.

Then, the area = 24 sq. ft.



Draw to the given scale and find the area of:

2. A rectangle, 6 in. by 12 in., scale $\frac{1}{2}$.
3. A square, 9 in. by 9 in., scale 1 to 3.
4. A map, 4 ft. by 6 ft., scale 1 in. to 2 ft.
5. A rectangle, 10 in. by 15 in., scale 1 to 5.
6. A window, 4 ft. by 8 ft., scale 1 in. to 4 ft.
7. A flag, 20 ft. by 10 ft., scale 1 in. to 5 ft.
8. A table top, 36 in. by 60 in., scale 1 to 12.
9. A blackboard, 48 in. by 72 in., scale 1 to 12.
10. A school yard, 32 yd. by 40 yd., scale 1 in. to 8 yd.
11. Find the area of a baseball diamond which is a 90-foot square. What is the distance around the bases?
12. Find the area and the perimeter of a city lot 32 ft. wide and 150 ft. deep.

Find the perimeter and area of a rectangle:

13. 12 in. by 14 in. 16. 16 ft. by 19 ft. 19. 28 yd. by 32 yd.
14. 13 in. by 16 in. 17. 20 ft. by 24 ft. 20. 22 yd. by 45 yd.
15. 18 in. by 14 in. 18. 25 ft. by 17 ft. 21. 34 yd. by 48 yd.

REVIEW

Oral Exercises

1. Read $16 + 19 = 35$; $12 + 4 + 16 = 32$.
2. Tell what the sign $+$ indicates; the sign $=$.
3. Add 9 to 8. What are the numbers added called? What name is given to the result obtained?
4. Count by 5's from 1 to 96; from 4 to 99.
5. Find the value of 6 5's by addition; by multiplication. Which way is the shorter?
6. When the multiplier is 3, read $3 \times 21 = 63$; $21 \times 3 = 63$. Which number is the multiplicand? the product?

Add; then subtract:

7. $\begin{array}{r} 12 \\ 8 \\ \hline \end{array}$	8. $\begin{array}{r} 14 \\ 6 \\ \hline \end{array}$	9. $\begin{array}{r} 15 \\ 8 \\ \hline \end{array}$	10. $\begin{array}{r} 13 \\ 9 \\ \hline \end{array}$	11. $\begin{array}{r} 14 \\ 5 \\ \hline \end{array}$	12. $\begin{array}{r} 16 \\ 9 \\ \hline \end{array}$
13. $\begin{array}{r} 14 \\ 9 \\ \hline \end{array}$	14. $\begin{array}{r} 13 \\ 6 \\ \hline \end{array}$	15. $\begin{array}{r} 17 \\ 8 \\ \hline \end{array}$	16. $\begin{array}{r} 18 \\ 9 \\ \hline \end{array}$	17. $\begin{array}{r} 16 \\ 8 \\ \hline \end{array}$	18. $\begin{array}{r} 15 \\ 6 \\ \hline \end{array}$

19. Count by 6's from 2 to 98; from 5 to 95.
20. Read $15 - 7 = 8$. What does the sign $-$ indicate?
21. How much greater is 12 than 9? How do you find the answer? What is the answer called?
22. Subtract 8 from 17. Which number is the minuend? Which is the subtrahend? What is the remainder?
23. Repeat the multiplication tables of 4's, 6's, and 7's.

Read:

24. 296
25. 4165
26. 38,409
27. 800,293

28. Read $36 \div 9$. What does the sign \div indicate?
 29. Divide 48 by 6. Which number is the dividend?
 Which is the divisor? What is the quotient?
 30. What is the value of $24 \div 4$? of $\frac{1}{4}$ of 24?

Complete :

31. 1 yd. = — ft. 33. 1 gal. = — qt. 35. 1 qt. = — pt.
 32. 1 ft. = — in. 34. 8 gal. = — qt. 36. 1 gal. = — pt.
 37. Repeat the multiplication tables of 8's, 9's, 11's, and 12's; the corresponding division tables.
 38. How many things are there in a dozen? in a score?
 39. Give the prime numbers from 1 to 25.
 40. What are the prime factors of 18? of 48? of 60?

Give results quickly by rows; then by columns:

	52.	53.	54.	55.	56.
41.	8×3	$9 + 9$	$16 - 9$	$\frac{1}{2}$ of 20	$35 + 7$
42.	$9 + 4$	8×8	$14 + 7$	6×12	12×8
43.	$8 - 2$	6×9	$32 \div 4$	$\frac{1}{4}$ of 24	$17 - 9$
44.	$9 \div 3$	$9 - 3$	$49 \div 7$	$\frac{1}{2}$ of 18	$54 \div 6$
45.	5×7	6×6	$15 - 6$	7×11	$63 \div 7$
46.	6×4	7×8	$12 + 8$	5×12	$48 \div 4$
47.	$9 - 6$	9×5	11×6	$\frac{1}{4}$ of 16	12×7
48.	$8 + 7$	8×9	$40 \div 5$	$\frac{1}{2}$ of 12	$15 - 8$
49.	$6 + 9$	7×6	$17 - 8$	9×11	$42 \div 6$
50.	8×6	$8 + 8$	$72 \div 9$	$\frac{1}{4}$ of 36	$56 \div 7$
51.	7×9	9×9	$14 - 6$	$\frac{1}{4}$ of 28	12×9

Oral Exercises

1. How many quarts equal 1 gal.? $\frac{1}{4}$ gal.? $\frac{1}{2}$ gal.?
2. What is the scale when a line 1 in. long represents a line 6 in. long? a line 1 yd. long? a line 2 ft. long?
3. What part of a quart is 1 pt.? What part of a gallon is 1 qt.? are 2 qt.?
4. Find the area of a box cover 8 in. by 6 in.
5. Grace had 36 examples to solve. How many had she done, when she had solved $\frac{1}{4}$ of them?
6. What is the perimeter of a rug 6 ft. by 2 ft.?
7. After spending 5¢ for a box of torpedoes, Charles had 18¢ left. How much money had he at first?
8. Find the cost of $\frac{1}{2}$ yd. of lace at 24¢ a yard.
9. A case of rings contained 9 rows with 8 rings in each row. How many rings did the case contain?
10. In jumping the rope, Lucy failed 6 out of 25 times. How many times was she successful?
11. Emma has 5 red crayons, 9 blue ones, and 7 green ones. How many crayons has she?
12. A large milk can holds 32 qt. How many gallons does it hold?
13. Henry has a square garden, 10 ft. on each side, which is inclosed by a fence. How long is the fence?
14. On Friday our cook baked 6 pans of rolls, with 9 rolls in each pan. How many rolls did she bake?
15. Clyde has 16 jackstones and Ruth has 9. How many more jackstones has Clyde than Ruth?

Written Exercises

Add and test :

1.	2.	3.	4.	5.
39,636	23,809	98,437	7,348	28,398
14,684	7,634	4,689	42,789	76,495
49,871	19,872	57,923	16	48,973
27,588	56,391	37	8,547	52,798
89,663	4,589	12,463	96,873	86,698
<u>68,942</u>	<u>73,862</u>	<u>899</u>	<u>8,439</u>	<u>98,765</u>

Write in figures :

6. Eighty-seven thousand, one.
7. Twenty-six thousand, thirty.
8. Eight hundred thousand, nine.
9. Sixty thousand, four hundred eighteen.
10. Five hundred thirty thousand, eighty-three.
11. Forty-four thousand, eight hundred seventy.
12. One hundred seven thousand, three hundred ninety.
13. Six hundred two thousand, four hundred sixty-seven.
14. Multiply 45 by 10 ; 36 by 100 ; 725 by 1000.

How may a number be multiplied by 10, 100, or 1000 ?

Multiply :

- | | | |
|---------------|----------------|-----------------|
| 15. 365 by 40 | 19. 438 by 200 | 23. 925 by 4000 |
| 16. 507 by 60 | 20. 696 by 500 | 24. 786 by 6000 |
| 17. 794 by 90 | 21. 389 by 700 | 25. 673 by 8000 |
| 18. 976 by 80 | 22. 837 by 800 | 26. 998 by 9000 |

Write in words :

- | | | | |
|----------|----------|------------|-------------|
| 27. 2301 | 30. 1090 | 33. 43,625 | 36. 209,870 |
| 28. 8320 | 31. 7650 | 34. 82,005 | 37. 324,235 |
| 29. 6006 | 32. 8998 | 35. 60,406 | 38. 997,468 |

Perform the operations indicated :

- | | |
|-----------------------|---------------------------|
| 39. $826 - 487 + 85$ | 42. $1827 - 1222 + 4489$ |
| 40. $348 - 99 + 789$ | 43. $9408 + 1269 - 6220$ |
| 41. $273 + 465 - 382$ | 44. $684 - 42 + 938 - 76$ |
45. Multiply 347 by 265. Where is the right-hand figure of each partial product written ?
46. Multiply 274 by 204. Which partial product is not written ? How may the answer be tested ? Test it.
47. Divide 1000 by 77. Test the answer.
48. Divide 16,854 by 36. If the quotient figure should be taken too large, how would the error be discovered ? if it should be taken too small ? How can you tell when the proper quotient figure is taken ?

Find the answers and test :

- | | | |
|----------------------|------------------------|-------------------------|
| 49. 3927×38 | 56. $18,260 + 44$ | 63. $31,668 \div 203$ |
| 50. 5206×49 | 57. $42,703 \times 39$ | 64. $23,626 \div 524$ |
| 51. $4664 \div 22$ | 58. $25,200 \div 28$ | 65. $54,279 \times 108$ |
| 52. 6735×64 | 59. $36,945 \times 56$ | 66. $68,670 \div 315$ |
| 53. $8750 \div 35$ | 60. $53,072 \times 73$ | 67. $43,816 \times 820$ |
| 54. $5672 \div 79$ | 61. $65,782 \div 82$ | 68. $85,029 \times 647$ |
| 55. 9046×55 | 62. $81,536 \times 67$ | 69. $77,438 \div 832$ |

Use the tests of divisibility to find the prime factors of :

- | | | | | |
|--------|---------|---------|---------|----------|
| 70. 72 | 74. 125 | 78. 396 | 82. 630 | 86. 1872 |
| 71. 84 | 75. 144 | 79. 504 | 83. 486 | 87. 2310 |
| 72. 76 | 76. 330 | 80. 800 | 84. 594 | 88. 3136 |
| 73. 98 | 77. 432 | 81. 576 | 85. 864 | 89. 4350 |

Draw a diagram to the given scale, and find the area of :

90. A desk top, 4 ft. by 3 ft., scale 1 in. to 1 ft.
91. A rectangle, 8 yd. by 6 yd., scale 1 in. to 2 yd.
92. A magazine cover, 14 in. by 10 in., scale 1 to 2.

Find the answers and test :

- | | | |
|---------------|-----------------|----------------------|
| 93. 531 - 423 | 96. 9456 - 3567 | 99. 91,835 - 84,653 |
| 94. 327 - 159 | 97. 8759 - 5786 | 100. 42,931 - 28,734 |
| 95. 743 - 358 | 98. 7358 - 4929 | 101. 52,361 - 23,854 |

Find the cost :

102. At 20¢ a can, of 3 cans of cocoa ; of 4 cans.
103. At 25¢ a can, of 2 cans of sirup ; of 3 cans.
104. At 15¢ a dozen, of 3 dozen buns ; of 6 dozen.
105. At 42¢ a dozen, of 2 dozen eggs ; of $\frac{1}{2}$ dozen.
106. At 60¢ a pound, of $\frac{1}{2}$ pound of tea ; of $\frac{1}{4}$ pound.
107. At 40¢ a pound, of $\frac{1}{4}$ pound of cloves ; of $\frac{1}{2}$ pound.
108. At 18¢ a pound, of 4 pounds of lard ; of 5 pounds.
109. At 30¢ a pound, of 2 pounds of pepper ; of $\frac{1}{2}$ pound.
110. At 28¢ a pound, of 3 pounds of coffee ; of 2 pounds.
111. At 19¢ a cake, of 2 cakes of chocolate ; of 4 cakes.

Written Exercises

1. How many square inches are there in 9 sq. ft.?
2. Find the income from an acre of land yielding 7 tons of cucumbers that were sold at \$ 15 per ton.
3. In a month, the milkman left 56 qt. of milk at our house. How many gallons of milk did he leave?
4. A milliner had 325 hats and sold 89 of them on her opening day. How many hats had she left?
5. Ray's pony cost \$ 72 and his cart cost $\frac{1}{2}$ as much. How much did his cart cost?
6. If an addressing machine addresses 3000 envelopes per hour, how many can it address in 8 hours?
7. If 10 pounds of milk produce 1 pound of cheese, how much cheese will 1280 pounds of milk produce?
8. There are 75 eggs in a box. How many will be left after 4 dozen of them are sold?
9. Oliver counted the cars in six trains. There were 23, 37, 41, 38, 45, and 28. How many cars did he count?
10. How many oranges does 1 box contain, if a dozen boxes of the same grade contain 4320 oranges?
11. Mr. Ward had 996 sheep. If he sold $\frac{1}{4}$ of them, how many sheep did he sell?
12. A certain large electric sign is 85 ft. by 75 ft. Find its perimeter; its area.
13. In a political parade there were 234 men from the 10th ward, 142 from the 11th, 287 from the 14th, and 358 from the 16th. How many men were in line?

FOURTH YEAR — FIRST HALF

READING AND WRITING NUMBERS

ARABIC NUMERALS (REVIEW)

These ten figures are the **Arabic numerals** :

0 1 2 3 4 5 6 7 8 9

To help in reading numbers, commas are used to separate the figures into groups of three, beginning at the right. These groups are **periods**.

The left-hand period may have only one or two figures.

The units, tens, and hundreds of each period are the **orders of units**.

Names of orders :			Hundred-millions	Ter-millions	Millions	Hundred-thousands	Ter-thousands	Thousands	Hundreds	Tens	Units
			1	2	5,	6	3	8,	4	2	7
						2	0	7,	0	8	2
				7	3,	9	6	4,	0	0	0
			3	0	2,	0	0	0,	5	3	6

Names of periods : **MILLIONS THOUSANDS UNITS**

The numbers in the table in order are read :

“125 million, 638 thousand, 427.”

“207 thousand, 82.”

“73 million, 964 thousand.”

“302 million, 536.”

Each period beginning at the left is read with its name as if it stood alone, except that a period of 0's is passed over and the name of the units' period is omitted.

The word "and" is not used between periods.

Oral Exercises

Tell the number that is 1 greater than :

1. 9 2. 99 3. 999 4. 9999 5. 99,999

6. What is the largest number that can be written with 1 figure? with 2 figures? with 3 figures? with 4 figures? with 5 figures?

Tell the number that is 1 less than:

7. 800 8. 673 9. 5000 10. 12,601 11. 125,000

Read exercises 12-33 :

12. 47,622 17. 721,573 22. 9,403,216 27. 516,000,000
13. 19,505 18. 970,599 23. 7,644,302 28. 720,135,120
14. 37,410 19. 500,006 24. 11,676,000 29. 609,260,275
15. 10,025 20. 457,900 25. 87,904,076 30. 956,605,007
16. 42,200 21. 304,709 26. 90,000,125 31. 630,041,062

32. One year this government coined 80,719,163 one-cent pieces, which required 525,228 pounds of copper, 16,586 pounds of tin, and 11,257 pounds of zinc.

33. The Panama Canal workmen one year used 9,241,858 pounds of meat, 5,250,609 pounds of white potatoes, 787,881 pounds of sweet potatoes, 554,028 pounds of poultry, and 692,060 dozen eggs.

Written Exercises

Point off into periods, read, and then write in words :

- | | | | |
|----------|-----------|------------|---------------|
| 1. 96002 | 4. 153965 | 7. 1172816 | 10. 125872110 |
| 2. 36257 | 5. 110822 | 8. 8002998 | 11. 863967312 |
| 3. 80509 | 6. 105911 | 9. 1476200 | 12. 550300000 |

Write in figures :

13. 136 thousand, 15 (units).
14. 212 million, 19 thousand, 313.
15. 763 million, 210 thousand, 675.
16. Seventy-one million, nine thousand.
17. Eighteen thousand, six hundred forty.
18. Two hundred six million, four hundred.
19. Twenty-five thousand, four hundred sixteen.
20. Five hundred twenty-two thousand, eight hundred.
21. One hundred fifty-nine million, six hundred ninety-seven thousand.
22. Three hundred eighty-eight million, one hundred sixty-three thousand, seven.
23. Twenty-nine million, one hundred eighteen thousand, six hundred twenty-four.
24. Thirty million, seven hundred three thousand, eight hundred seven.
25. One hundred eight million, seventy-seven thousand, four hundred eighty-four.
26. Two hundred forty-five million, seven thousand, seven hundred thirty.

ROMAN NUMERALS

1. These seven letters are the Roman numerals :

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

They stand for the numbers written under them.

2. When you learned to tell time you learned how the first three of these Roman numerals are used to stand for the first twelve numbers, thus :

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1	2	3	4	5	6	7	8	9	10	11	12

3. Observe from these numbers that :

When a letter is followed by the same letter or a letter of less value, the values of the two letters are united.

Notice that II stands for 1 + 1, or 2; VI stands for 5 + 1, or 6.

When a letter is followed by a letter of greater value, the value of the less is taken from that of the greater.

Notice that IX stands for 1 less than 10, or 9; IV stands for 1 less than 5, or 4. Sometimes, as on clocks, 4 is written IIII.

4. Just as XII stands for 10 + 2, or 12, so XIV stands for 10 + 4, or 14.

Read: XIII; XV; XVI; XVII; XVIII; XIX.

5. Write the first 19 numbers with Roman numerals.

6. How many 10's are there in 20? How many X's are needed to write 20? Write 20 with letters.

Read: XXII; XXIV; XXV; XXVI; XXIX.

7. How many 10's are there in 30? Write 30 with letters. Use letters to write all the numbers from 20 to 39.

8. What Roman numeral stands for 50?
40 is 10 less than 50. Read: XL; XLI; XLIV.
9. Arrange the letters X and L to stand for 60.
Read: LX; LI; LIV; LV; LXV; LXVI; LXIX.
10. Write with letters the numbers from 40 to 69.
11. How many are 50 and 20? Read LXX.
How many are 50 and 30? Read LXXX.
Use letters to write the numbers from 75 to 85.
Read: LXXII; LXXXVII; LXXIV; LXXXIX.
12. What Roman numeral stands for 100? for 500? for 1000?
90 is 10 less than 100. Read: XC; XCII; XCIX.
400 is 100 less than 500. Read: CD; DC; DCC.
900 is 100 less than 1000. Write 900 with letters.

Oral Exercises

Read the following:

- | | | | | |
|-------|--------|---------|----------|------------|
| 1. CC | 4. CCC | 7. XXIX | 10. XLIX | 13. LXXXV |
| 2. CD | 5. DCC | 8. XLIV | 11. LXVI | 14. LXXIX |
| 3. CM | 6. XCV | 9. XCIX | 12. DCCC | 15. XCVIII |

Written Exercises

Write the following, using Roman numerals:

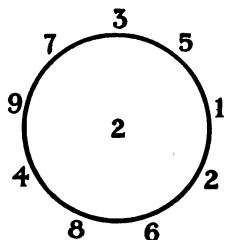
- | | | | | |
|-------|-------|--------|---------|----------|
| 1. 91 | 5. 88 | 9. 89 | 13. 400 | 17. 600 |
| 2. 73 | 6. 94 | 10. 97 | 14. 300 | 18. 900 |
| 3. 87 | 7. 86 | 11. 84 | 15. 500 | 19. 700 |
| 4. 74 | 8. 72 | 12. 99 | 16. 800 | 20. 1000 |

FUNDAMENTAL OPERATIONS (REVIEW)

ADDITION

Oral Exercises

1. Begin at 1 and going around the ring in either direction see how quickly you can add 2 to each number without *making a mistake*.



Begin at 4 and go in either direction. Begin at 6; at other numbers.

2. Instead of 2 put 3 in the ring and add as before.

3. Put 4 in the ring and add.

4. Put 5 in the ring and add.

5. Beginning at 7, add 6 to each of the numbers around the ring.

6. Beginning at 3, add 7 to each of the numbers.

7. Beginning at 5, add 8 to each of the numbers.

8. Put 9 in the ring and add it to each of the numbers.

Count

9. By 6's from 1 to 73; from 2 to 74; from 5 to 77.

10. By 7's from 0 to 84; from 1 to 85; from 2 to 86; from 3 to 87; from 4 to 88; from 5 to 89; from 6 to 90.

11. By 8's from 0 to 96; from 1 to 97; from 2 to 98; from 3 to 99; from 4 to 100; from 5 to 101.

12. By 9's from 0 to 108; from 2 to 110; from 5 to 113; from 6 to 114; from 7 to 115; from 8 to 116.

Written Exercises

Add

1.	2.	3.	4.	5.
178	6286	4578	40,285	278,718
279	940	726	64,047	352,387
<u>177</u>	<u>1122</u>	<u>1967</u>	<u>56,309</u>	<u>485,704</u>
6.	7.	8.	9.	10.
427	6489	5620	83,042	798,765
369	3758	4898	90,078	7,680
697	7925	2745	23,569	814,703
468	6375	3989	897	43,540
<u>961</u>	<u>842</u>	<u>2757</u>	<u>25,796</u>	<u>935,876</u>
11.	12.	13.	14.	15.
796	2564	7943	27,581	842,753
432	7892	1208	89,067	369,058
849	5406	796	43,540	4,569
89	297	4825	94,835	25,807
<u>625</u>	<u>6846</u>	<u>8642</u>	<u>48,369</u>	<u>732,479</u>

Add and test quickly, timing yourself:

16.	17.	18.	19.	20.
927	4238	8307	42,865	713,204
538	7346	1212	31,793	504,216
421	5374	7104	46,381	800,437
376	8363	899	86,045	4,036
673	7922	2793	73,912	13,459
<u>894</u>	<u>1718</u>	<u>8030</u>	<u>58,140</u>	<u>380,460</u>

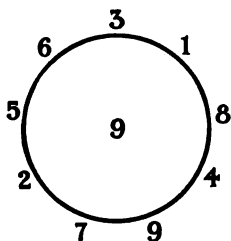
SUBTRACTION

Oral Exercises

1. Subtract, *correctly* and *rapidly*, each number around the ring, from the number within, beginning with 3 and going in either direction.

Begin with 8; with 5; with 6; with other numbers.

2. Put 10 in the ring instead of 9 and subtract from it the numbers outside the ring.



3. Put 11 in the ring and subtract from it the numbers outside.

4. Subtract each of the numbers around the ring from 12.

5. Subtract each of the numbers around the ring from 13.

6. Put 14 in the ring and subtract.

7. Subtract each of the numbers around the ring from 15; from 16.

8. Beginning at 9, subtract the numbers outside the ring from 17; from 18; from 19.

Subtract rapidly:

9.	61	10.	28	11.	35	12.	46	13.	98	14.	84
	<u>30</u>		<u>10</u>		<u>31</u>		<u>26</u>		<u>40</u>		<u>34</u>
15.	29	16.	77	17.	68	18.	59	19.	76	20.	97
	<u>12</u>		<u>25</u>		<u>44</u>		<u>33</u>		<u>24</u>		<u>54</u>
21.	700	22.	420	23.	350	24.	436	25.	589	26.	608
	<u>200</u>		<u>100</u>		<u>120</u>		<u>130</u>		<u>280</u>		<u>203</u>

Written Exercises

Subtract :

1. 166 <u>139</u>	2. 631 <u>408</u>	3. 2481 <u>1667</u>	4. 3427 <u>1908</u>	5. 5426 <u>2809</u>
6. 204 <u>78</u>	7. 907 <u>179</u>	8. 5209 <u>1872</u>	9. 2345 <u>1067</u>	10. 6472 <u>2428</u>
11. 546 <u>338</u>	12. 500 <u>183</u>	13. 3375 <u>2986</u>	14. 5316 <u>4684</u>	15. 4567 <u>3879</u>

Subtract each of these numbers from 100,000 :

16. 3467	20. 4567	24. 27,908	28. 87,698
17. 5886	21. 8956	25. 64,987	29. 75,467
18. 9924	22. 2875	26. 89,994	30. 49,654
19. 4798	23. 7693	27. 73,189	31. 37,713

Find answers and test, timing yourself on exercises 45-58:

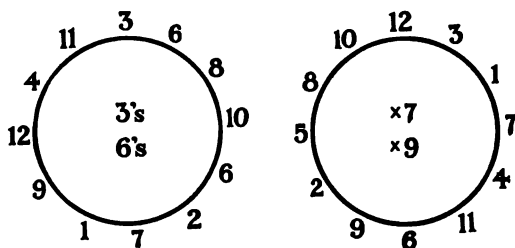
32. 4168 - 3985	41. 39,456 - 7824	50. 49,836 - 31,849
33. 3854 - 2198	42. 48,317 - 8545	51. 98,375 - 45,792
34. 5841 - 2976	43. 87,593 - 8349	52. 83,125 - 72,165
35. 4136 - 3972	44. 73,586 - 7525	53. 63,259 - 42,199
36. 6846 - 3879	45. 81,364 - 7816	54. 84,391 - 43,875
37. 5231 - 2865	46. 39,271 - 9315	55. 73,186 - 38,592
38. 8655 - 4948	47. 68,537 - 8871	56. 49,315 - 18,674
39. 7143 - 5976	48. 49,288 - 9199	57. 37,926 - 18,395
40. 6868 - 4939	49. 52,869 - 7951	58. 72,853 - 41,687

MULTIPLICATION**Oral Exercises**

1. How many are four 2's? seven 2's? 8×2 ? 9×2 ?

The numbers 2, 4, 6, 8, etc., are **multiples** of 2, because each is produced by multiplying 2 by some number.

2. Beginning with 3 and going in either direction around the first ring, name rapidly the multiples of 3, produced by multiplying it by each number around the ring.



3. Give the multiples of 6 in the same way.
4. Instead of 3 and 6 put 4 and 8 in the ring. Give the multiples of 4; of 8.
5. Now put 5 and 10 in the ring. Give as before the multiples of 5; of 10.
6. Give the multiples of 5 and 10 in pairs, beginning at the top of the ring, thus:
15, 30; 30, 60; 40, 80; and so on.
7. See how rapidly you can go around the second ring, multiplying 7 by each number; 9 by each number.
8. Put 11 and 12 in the ring. Multiply 11 by each number around the ring. Multiply 12 by each number.

Written Exercises

Multiply:

- | | | | | | |
|---|---|---|---|---|---|
| 1. $\begin{array}{r} 22 \\ 8 \end{array}$ | 2. $\begin{array}{r} 34 \\ 9 \end{array}$ | 3. $\begin{array}{r} 65 \\ 7 \end{array}$ | 4. $\begin{array}{r} 99 \\ 4 \end{array}$ | 5. $\begin{array}{r} 79 \\ 9 \end{array}$ | 6. $\begin{array}{r} 89 \\ 6 \end{array}$ |
| 7. $\begin{array}{r} 250 \\ 7 \end{array}$ | 8. $\begin{array}{r} 496 \\ 5 \end{array}$ | 9. $\begin{array}{r} 372 \\ 7 \end{array}$ | 10. $\begin{array}{r} 777 \\ 9 \end{array}$ | 11. $\begin{array}{r} 707 \\ 6 \end{array}$ | 12. $\begin{array}{r} 698 \\ 4 \end{array}$ |
| 13. $\begin{array}{r} 408 \\ 6 \end{array}$ | 14. $\begin{array}{r} 382 \\ 8 \end{array}$ | 15. $\begin{array}{r} 502 \\ 9 \end{array}$ | 16. $\begin{array}{r} 545 \\ 8 \end{array}$ | 17. $\begin{array}{r} 888 \\ 9 \end{array}$ | 18. $\begin{array}{r} 749 \\ 7 \end{array}$ |
| 19. $\begin{array}{r} 608 \\ 9 \end{array}$ | 20. $\begin{array}{r} 625 \\ 7 \end{array}$ | 21. $\begin{array}{r} 907 \\ 5 \end{array}$ | 22. $\begin{array}{r} 392 \\ 6 \end{array}$ | 23. $\begin{array}{r} 999 \\ 8 \end{array}$ | 24. $\begin{array}{r} 867 \\ 9 \end{array}$ |
| 25. $\begin{array}{r} 465 \\ 5 \end{array}$ | 26. $\begin{array}{r} 574 \\ 6 \end{array}$ | 27. $\begin{array}{r} 684 \\ 7 \end{array}$ | 28. $\begin{array}{r} 789 \\ 8 \end{array}$ | 29. $\begin{array}{r} 488 \\ 9 \end{array}$ | 30. $\begin{array}{r} 527 \\ 8 \end{array}$ |

Find products:

- | | | | |
|-------------------|--------------------|---------------------|---------------------|
| 31. 5×48 | 36. 2×128 | 41. 10×243 | 46. 5×4001 |
| 32. 4×64 | 37. 7×242 | 42. 11×127 | 47. 6×1025 |
| 33. 6×75 | 38. 5×344 | 43. 12×506 | 48. 7×1962 |
| 34. 3×59 | 39. 8×504 | 44. 11×627 | 49. 8×2888 |
| 35. 7×84 | 40. 9×465 | 45. 12×365 | 50. 9×5207 |

Multiply:

- | | | |
|---------------|----------------|-----------------|
| 51. 135 by 60 | 55. 435 by 400 | 59. 462 by 2000 |
| 52. 247 by 80 | 56. 169 by 700 | 60. 518 by 6000 |
| 53. 396 by 70 | 57. 625 by 900 | 61. 873 by 9000 |
| 54. 585 by 90 | 58. 756 by 800 | 62. 687 by 7000 |

Written Exercises

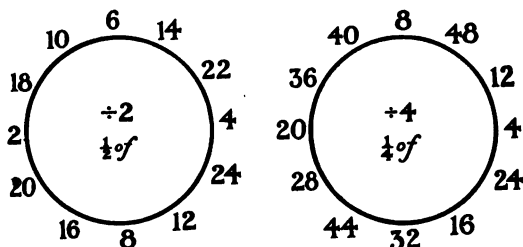
Multiply:

- | | | |
|---------------|----------------|-----------------|
| 1. 267 by 24 | 25. 3845 by 36 | 49. 3867 by 708 |
| 2. 836 by 27 | 26. 7186 by 14 | 50. 4230 by 692 |
| 3. 918 by 61 | 27. 5327 by 42 | 51. 7506 by 804 |
| 4. 309 by 38 | 28. 6284 by 51 | 52. 8342 by 620 |
| 5. 465 by 26 | 29. 4769 by 34 | 53. 8006 by 584 |
| 6. 732 by 55 | 30. 6873 by 50 | 54. 3816 by 602 |
| 7. 817 by 63 | 31. 2918 by 36 | 55. 5832 by 319 |
| 8. 398 by 80 | 32. 4873 by 27 | 56. 4923 by 909 |
| 9. 426 by 29 | 33. 9275 by 31 | 57. 3456 by 325 |
| 10. 938 by 71 | 34. 8462 by 82 | 58. 2189 by 216 |
| 11. 695 by 32 | 35. 7319 by 94 | 59. 4685 by 406 |
| 12. 346 by 46 | 36. 6685 by 73 | 60. 5816 by 243 |
| 13. 279 by 78 | 37. 6517 by 62 | 61. 4834 by 232 |
| 14. 518 by 92 | 38. 7395 by 41 | 62. 5819 by 390 |
| 15. 498 by 60 | 39. 3179 by 65 | 63. 6785 by 206 |
| 16. 691 by 49 | 40. 5234 by 26 | 64. 3849 by 387 |
| 17. 836 by 51 | 41. 3186 by 83 | 65. 2863 by 341 |
| 18. 527 by 28 | 42. 5279 by 91 | 66. 4185 by 416 |
| 19. 687 by 31 | 43. 3864 by 88 | 67. 3268 by 507 |
| 20. 867 by 76 | 44. 5279 by 71 | 68. 5673 by 690 |
| 21. 726 by 59 | 45. 8365 by 87 | 69. 6100 by 372 |
| 22. 968 by 90 | 46. 9182 by 96 | 70. 5239 by 425 |
| 23. 864 by 69 | 47. 8756 by 70 | 71. 6543 by 907 |
| 24. 528 by 81 | 48. 5698 by 68 | 72. 3468 by 687 |

DIVISION

Oral Exercises

1. Quickly divide each number around the first ring by 2; then find $\frac{1}{2}$ of each number.



2. Divide each number around the second ring by 4; then find $\frac{1}{4}$ of each number.
3. How many 3's are there in each of these numbers?
3, 9, 6, 12, 21, 15, 24, 18, 30, 27, 36, 33

Answer quickly:

4. 25	9. 18	14. 28	19. 40	24. 54
5. 15	10. 36	15. 42	20. 32	25. 45
6. 35	11. 24	16. 56	21. 64	26. 63
7. 55	12. 72	17. 84	22. 56	27. 81
8. 45	13. 48	18. 63	23. 96	28. 72

Answer quickly: thus, looking at 33 say "3 times 11."

Multiples of 11

Multiples of 12

29. 33	32. 44	35. 99	38. 24	41. 48	44. 84
30. 55	33. 66	36. 121	39. 36	42. 72	45. 132
31. 77	34. 88	37. 110	40. 60	43. 96	46. 144

Written Exercises

Divide and test:

1. 6) <u>306</u>	2. 4) <u>324</u>	3. 5) <u>155</u>	4. 2) <u>1484</u>	5. 3) <u>1569</u>	6. 2) <u>1668</u>
7. 7) <u>147</u>	8. 6) <u>552</u>	9. 5) <u>259</u>	10. 3) <u>2469</u>	11. 4) <u>1649</u>	12. 5) <u>4558</u>
13. 6) <u>504</u>	14. 7) <u>105</u>	15. 8) <u>272</u>	16. 4) <u>5263</u>	17. 6) <u>8508</u>	18. 2) <u>8433</u>
19. 5) <u>865</u>	20. 8) <u>752</u>	21. 4) <u>751</u>	22. 7) <u>5831</u>	23. 8) <u>6663</u>	24. 9) <u>5665</u>
25. 3) <u>419</u>	26. 7) <u>931</u>	27. 6) <u>492</u>	28. 8) <u>2005</u>	29. 9) <u>6840</u>	30. 6) <u>9041</u>

Find results:

31. $133 \div 7$	36. $577 \div 2$	41. $1610 \div 8$	46. $2760 \div 10$
32. $282 \div 6$	37. $935 \div 4$	42. $2010 \div 6$	47. $5592 \div 12$
33. $196 \div 7$	38. $866 \div 6$	43. $5400 \div 7$	48. $3080 \div 10$
34. $594 \div 6$	39. $689 \div 7$	44. $3780 \div 9$	49. $7370 \div 11$
35. $747 \div 3$	40. $708 \div 9$	45. $8425 \div 8$	50. $8424 \div 12$

Divide by 4; by 6; by 7; by 9:

51. 150	55. 5616	59. 7286	63. 77,223
52. 476	56. 6909	60. 1615	64. 47,337
53. 360	57. 2616	61. 4854	65. 89,648
54. 943	58. 1884	62. 8809	66. 61,950

Written Exercises

Divide :

- | | | |
|----------------|------------------|-------------------|
| 1. 1344 by 32 | 25. 38,409 by 21 | 49. 41,676 by 302 |
| 2. 6840 by 15 | 26. 94,640 by 28 | 50. 73,616 by 214 |
| 3. 4386 by 51 | 27. 64,130 by 55 | 51. 12,360 by 120 |
| 4. 4779 by 81 | 28. 45,658 by 37 | 52. 28,820 by 524 |
| 5. 4992 by 52 | 29. 65,472 by 33 | 53. 35,670 by 435 |
| 6. 5538 by 71 | 30. 84,732 by 46 | 54. 53,756 by 604 |
| 7. 6039 by 61 | 31. 91,280 by 35 | 55. 46,964 by 398 |
| 8. 4176 by 72 | 32. 20,544 by 64 | 56. 69,430 by 530 |
| 9. 7872 by 82 | 33. 71,099 by 49 | 57. 73,408 by 296 |
| 10. 8740 by 92 | 34. 40,442 by 73 | 58. 18,447 by 429 |
| 11. 1728 by 18 | 35. 35,786 by 29 | 59. 39,648 by 708 |
| 12. 4312 by 56 | 36. 12,644 by 58 | 60. 35,828 by 689 |
| 13. 6204 by 66 | 37. 88,920 by 76 | 61. 89,056 by 352 |
| 14. 2916 by 54 | 38. 46,800 by 39 | 62. 55,182 by 541 |
| 15. 2952 by 36 | 39. 88,998 by 78 | 63. 28,160 by 880 |
| 16. 1520 by 16 | 40. 55,269 by 23 | 64. 43,470 by 483 |
| 17. 3864 by 84 | 41. 35,910 by 19 | 65. 32,652 by 907 |
| 18. 5084 by 62 | 42. 44,268 by 34 | 66. 93,125 by 625 |
| 19. 1632 by 24 | 43. 28,925 by 89 | 67. 13,566 by 798 |
| 20. 3486 by 42 | 44. 29,775 by 75 | 68. 91,608 by 347 |
| 21. 4218 by 74 | 45. 50,832 by 48 | 69. 43,416 by 536 |
| 22. 7470 by 83 | 46. 72,504 by 57 | 70. 61,484 by 809 |
| 23. 8505 by 27 | 47. 95,974 by 94 | 71. 89,040 by 424 |
| 24. 9196 by 38 | 48. 34,385 by 65 | 72. 56,730 by 915 |

UNITED STATES MONEY

COINS



1. How many cents is a 5-cent piece worth? a dime?
How many 5-cent pieces equal a dime in value?



2. How many 10's are there in 100? To how many dimes are 100 cents (1 dollar) equal in value?
3. Learn this table of United States money:

10 cents = 1 dime

10 dimes = 1 dollar

Oral Exercises

- How many cents are $\frac{1}{2}$ of 100¢? $\frac{1}{4}$ of 100¢?
- How many cents are there in \$1? in $\$ \frac{1}{2}$? in $\$ \frac{1}{4}$?
- How many cents are there in \$2? in \$4? in \$3?
- How many dollars are there in 500¢? in 700¢?
- How many half dollars equal \$1? \$2? \$5? \$8?
- How many quarter dollars equal \$1? $\$ \frac{1}{2}$? $\$ \frac{1}{4}$? \$2?

MAKING CHANGE

Oral Exercises

1. If Ethel bought a 3-cent pencil, how much change did she receive from a 5-cent piece?

2. Edward's mother sent him to the store with a dime to buy a quart of milk, costing 7¢. How much change did he receive?

3. Mary paid 8¢ for a valentine. How much change did she receive from a dime?

4. If Bertha paid 10¢ for a ball, how much change did she receive from a quarter dollar?

5. Roger bought a knife that cost 40¢ and paid for it with a half dollar. How much change did he receive?

6. Helen's uncle gave her a silver dollar to buy a book that cost 90¢. How much change did she receive?

7. If I buy a pound of sugar for 6¢ and pay for it with a quarter dollar, how much change do I receive?

8. Margaret paid 24¢ for a set of toy dishes. How much change did she receive from a half dollar?

Using pencil (if necessary), find the change, when the coin at the top is paid for the purchase given below:

QUARTER DOLLAR	HALF DOLLAR	ONE DOLLAR	HALF DOLLAR	ONE DOLLAR
9. 15¢	14. 38¢	19. 67¢	24. 37¢	29. 83¢
10. 12¢	15. 27¢	20. 73¢	25. 26¢	30. 77¢
11. 18¢	16. 34¢	21. 56¢	26. 18¢	31. 64¢
12. 14¢	17. 23¢	22. 85¢	27. 32¢	32. 59¢
13. 16¢	18. 16¢	23. 48¢	28. 13¢	33. 36¢

READING AND WRITING UNITED STATES MONEY

1. Read: \$36; 18¢; \$50; 33¢; \$245.

Write, using the sign \$: 15 dollars; 400 dollars.

2. We write 4 dollars and 25 cents in this way: \$4.25.

The period that is used to separate the dollars from the cents is the **decimal point**.

Read the word *and* between dollars and cents.

Read: \$2.75; \$8.42; \$10.50; \$76.23; \$407.52.

Write, using the sign \$: 6 dollars and 31 cents; 12 dollars and 20 cents; 320 dollars and 82 cents.

3. We may write 75¢ in this way: \$.75.

Read: \$.23; \$.30; \$.45; \$.69; \$.50; \$.99.

Write, using the sign \$: 47¢; 60¢; 27¢; 80¢; 77¢.

4. We may write 3¢ in this way: \$.03.

Read: \$1.02; \$.05; \$3.01; \$.09; \$4.06; \$24.00.

Write, using the sign \$: 6¢; 7¢; 5 dollars and 8 cents.

Oral Exercises

Read:

- | | | | | |
|----------|----------|-----------|-----------|-------------|
| 1. \$.15 | 3. \$.05 | 5. \$1.60 | 7. \$4.84 | 9. \$50.08 |
| 2. \$.28 | 4. \$.09 | 6. \$3.45 | 8. \$6.05 | 10. \$78.06 |

Written Exercises

Write, using the sign \$:

- | | | |
|--------------|-------------|------------------------------|
| 1. 48 cents. | 5. 6 cents. | 9. 3 dollars and 8 cents. |
| 2. 36 cents. | 6. 4 cents. | 10. 8 dollars and 25 cents. |
| 3. 80 cents. | 7. 9 cents. | 11. 12 dollars and 50 cents. |
| 4. 75 cents. | 8. 7 cents. | 12. 25 dollars and 64 cents. |

ADDITION OF UNITED STATES MONEY

Oral Exercises

Add quickly:

1. \$41 26	6. 36¢ 42	11. 22¢ 56	16. 100¢ 50	21. \$3.40 .45
2. \$72 17	7. 19¢ 56	12. \$.22 .56	17. \$1.00 .50	22. \$9.30 .56
3. \$13 48	8. 38¢ 61	13. \$.43 .25	18. \$1.32 .27	23. \$7.22 .44
4. \$63 85	9. 25¢ 73	14. \$.72 .13	19. \$3.20 .45	24. \$8.37 .24
5. \$96 74	10. 84¢ 12	15. \$.58 .35	20. \$2.48 .37	25. \$6.47 .19

26. Horace bought a baseball for 25¢ and a glove for 80¢. How much did he pay for both?

27. How much did a newsboy receive for papers each day, if he sold his morning papers for \$.33 and his evening papers for \$.45?

28. After buying a chair that cost \$6.49 Clara's mother had \$.30 left. How much had she before the purchase?

29. On our class picnic we spent \$1.20 for lunches and \$.55 for car fares. How much did we spend in all?

30. Anna's mother spent \$27 for a dress and \$18 for a coat. How much did she pay for both?

31. From his garden one day Carl sold corn for 48¢ and beans for 12¢. How much did he receive for both?

Written Exercises

1. Find the sum of \$187.96, \$6.05, \$.57, and \$48.00.

\$187.96

6.05

.57

48.00

\$242.58

Arrange the numbers so that the decimal points are in a column.

Add as you have added other numbers.

Place the decimal point in the sum directly under the other decimal points. Read the sum.

Add and test, timing yourself on exercises 12-16:

2.	3.	4.	5.	6.
\$2.12	\$1.25	\$11.28	\$47.65	\$75.43
7.75	.39	23.84	4.98	98.25
5.73	.62	17.79	79.24	7.07
4.27	.14	8.50	7.79	79.89
<u>7.</u>	<u>8.</u>	<u>9.</u>	<u>10.</u>	<u>11.</u>
\$9.68	\$6.75	\$39.72	\$88.57	\$59.37
6.77	.89	57.58	94.83	7.26
7.06	1.27	6.98	29.77	87.98
3.92	3.98	66.45	3.98	68.55
8.45	7.25	78.63	74.65	26.43
<u>12.</u>	<u>13.</u>	<u>14.</u>	<u>15.</u>	<u>16.</u>
\$6.39	\$8.45	\$84.75	\$55.48	\$49.75
9.86	3.19	29.34	68.74	27.19
5.98	.98	73.96	9.25	8.37
4.96	2.57	5.89	83.57	61.83
2.63	7.64	46.67	7.87	99.05
3.47	.49	64.58	99.50	6.98

Written Exercises

1. Find the total cost of Mary's hat, if the frame cost \$1.63 and the trimmings \$1.75.
2. If it cost \$3.75 a week to keep a horse and \$.51 more to run a motor car, find the cost of running the car.
3. A dolls' house cost \$2.48 and the furnishings for it cost \$1.95. What was the total cost?
4. Harriet's father has a watch that cost \$58.50 and a chain that cost \$15.75. Find the cost of both.
5. The food for Ida's ducks one year cost \$4.64 and that for the ducklings, \$16.64. How much did it all cost?
6. George paid \$.25 for a ball, \$.25 for a bat, \$.50 for a mask, and \$.75 for a glove. Find the total cost.
7. Julia spent \$.79 for paints, \$.49 for brushes, and had \$1.62 left. How much money had she at first?
8. Carl bought a hammock for \$2.27, hammock hooks for \$.14, and a rope for \$.25. Find the total cost.
9. Henry has \$8.72, James has \$11.58, and Harold has \$14.36. How much money have they all together?
10. John has a magic lantern that cost \$1.49. He gave an entertainment with it and received \$1.26 more than the lantern cost. How much did he receive?
11. If the dresser in Ruth's room cost \$18.67, the bed \$24.74, and chairs \$8.45, find the cost of her furniture.
12. Roy's Indian suit cost \$1.74, his sister's \$.98, and their moccasins \$2.68. If their wigwam cost \$6.94, find the total cost of their Indian outfit.

SUBTRACTION OF UNITED STATES MONEY

Oral Exercises

Subtract, giving results quickly :

1. \$ 35	6. 58¢	11. 35¢	16. 100¢	21. \$ 3.65
<u>24</u>	<u>24</u>	<u>13</u>	<u>40</u>	<u>.20</u>

2. \$ 47	7. 85¢	12. \$.35	17. \$ 1.00	22. \$ 4.68
<u>22</u>	<u>42</u>	<u>.13</u>	<u>.40</u>	<u>.32</u>

3. \$ 54	8. 79¢	13. 69¢	18. 156¢	23. \$ 5.37
<u>31</u>	<u>53</u>	<u>25</u>	<u>33</u>	<u>.22</u>

4. \$ 66	9. 66¢	14. \$.69	19. \$ 1.56	24. \$ 5.96
<u>23</u>	<u>34</u>	<u>.25</u>	<u>.33</u>	<u>.43</u>

5. \$ 78	10. 94¢	15. \$.84	20. \$ 2.34	25. \$ 6.66
<u>42</u>	<u>41</u>	<u>.31</u>	<u>.25</u>	<u>.23</u>

26. Frank has \$.63 and wishes to buy a book costing \$.99. How much more money does he need ?

27. Mary had \$.78 and spent \$.25 for the use of a rowboat. How much money had she left ?

28. If John has \$.44 and Ellen has \$.67, how much less has John than Ellen ?

29. A boy bought skates for \$.60 and a hockey stick for \$.20. How much change did he receive out of \$1 ?

30. Roy earned \$.35 shoveling snow one week and \$.40 the next. How much less than \$1 did he earn in both weeks ?

Written Exercises

1. How much more is \$87.24 than \$28.56?

\$87.24 Subtract as you have subtracted other numbers,
 28.56 placing the decimal point in the result directly under
 the other decimal points.

\$58.68 Write the sign \$ before the result.

Subtract and test, timing yourself on exercises 26-33:

2. \$4.09	10. \$18.72	18. \$83.04	26. \$374.60
<u>2.75</u>	<u>9.38</u>	<u>24.96</u>	<u>98.73</u>

3. \$9.13	11. \$54.00	19. \$67.18	27. \$547.33
<u>5.64</u>	<u>.75</u>	<u>39.28</u>	<u>293.84</u>

4. \$7.36	12. \$80.00	20. \$91.07	28. \$200.00
<u>.98</u>	<u>14.37</u>	<u>45.45</u>	<u>173.20</u>

5. \$6.24	13. \$74.84	21. \$85.05	29. \$506.03
<u>4.85</u>	<u>6.95</u>	<u>56.31</u>	<u>289.45</u>

6. \$9.00	14. \$40.41	22. \$90.06	30. \$304.20
<u>8.46</u>	<u>9.66</u>	<u>38.39</u>	<u>92.50</u>

7. \$7.54	15. \$60.35	23. \$69.00	31. \$646.84
<u>5.75</u>	<u>39.76</u>	<u>2.46</u>	<u>375.96</u>

8. \$5.47	16. \$58.27	24. \$76.83	32. \$623.59
<u>.45</u>	<u>.85</u>	<u>32.48</u>	<u>45.39</u>

9. \$8.09	17. \$77.77	25. \$41.62	33. \$900.06
<u>4.86</u>	<u>8.88</u>	<u>29.88</u>	<u>285.29</u>

School Savings Bank Problems

The following table shows how much money the children in the fourth grade of a certain school deposited in their school savings bank from January to June, and how much they drew from the bank each month.

	A CLASS		B CLASS	
	DEPOSITED	WITHDRAWN	DEPOSITED	WITHDRAWN
JANUARY . .	\$ 1.36	\$.18	\$ 1.41	\$.37
FEBRUARY .	1.35	.42	1.19	.15
MARCH . . .	1.56	.28	1.48	.39
APRIL . . .	2.25	.50	1.62	.25
MAY	1.75	1.19	1.96	.37
JUNE	1.73	1.05	2.10	1.15

1. During January how much more did the A class deposit than they withdrew, or how much did they *save*?
2. Did the B class save more or less than the A class during January, and how much more or less?
3. How much more did the A class deposit during February than the B class?
4. Which class saved the greater amount of money during February, and how much greater?
5. Compare the savings of the two classes during March; during April; during May; during June.
6. At the end of January, how much money was there in the bank belonging to each class? to both classes?
7. Which class had the greater amount of money in the bank at the end of June, and how much greater?

MULTIPLICATION OF UNITED STATES MONEY

Oral Exercises

1. Julia bought 4 pounds of rice at 7¢ a pound. How much did the rice cost her?

2. Frank bought 8 4-cent stamps and a 2-cent stamp. How much money did he pay for stamps?

SOLUTION. — The sum he paid = $8 \times 4\text{¢} + 2\text{¢} = 32\text{¢} + 2\text{¢} = 34\text{¢}$.

3. A girl had 7 5-cent pieces and a 1-cent piece in her purse. How much money had she in her purse?

4. Andrew bought 9 pencils at 3¢ apiece, and had 4¢ left. How much money had he at first?

5. David has 4 dimes in his bank. How much less than half a dollar does his bank contain?

SOLUTION. — His bank contains $4 \times 10\text{¢}$, or 40¢. Then, it contains $50\text{¢} - 40\text{¢}$, or 10¢, less than half a dollar.

6. A boy bought 3 large marbles at 6¢ each. How much change should he receive from a quarter dollar?

Find the change out of a quarter dollar for each purchase:

7. 8 newspapers at 2¢ each; at 3¢ each.

8. 4 calendars at 3¢ each; at 5¢ each.

9. 3 papers of pins at 5¢ each; at 7¢ each.

10. At the baker's I bought 3 cakes at 20¢ each and a pie for 10¢. How much money did I spend there?

11. Find the cost of 4 pounds of sugar at 6¢ a pound, and a pound of ham at 20¢.

12. George has a dollar. If he buys 4 tickets for a ball game at 10¢ each, how much money will he have left?

Written Exercises

1. Multiply \$4.86 by 5.

$$\begin{array}{r} \$ 4.86 \\ \quad 5 \\ \hline \$ 24.30 \end{array}$$

Multiply as you multiply other numbers, placing a decimal point in the product directly under the one in the multiplicand.

Write the sign \$ before the product.

Multiply :

$$\begin{array}{r} 2. \$ 2.40 \\ \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \$ 3.25 \\ \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \$ 21.44 \\ \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \$ 157.75 \\ \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \$ 5.75 \\ \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \$ 9.89 \\ \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \$ 47.65 \\ \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \$ 309.27 \\ \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \$ 6.85 \\ \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \$ 4.95 \\ \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \$ 83.66 \\ \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \$ 918.24 \\ \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \$ 13.75 \\ \quad 45 \\ \hline 6875 \\ 5500 \\ \hline \$ 618.75 \end{array}$$

$$\begin{array}{r} 15. \$.47 \\ \quad 26 \\ \hline 282 \\ 94 \\ \hline \$ 12.22 \end{array}$$

$$\begin{array}{r} 16. \$.47 \\ \quad 206 \\ \hline 282 \\ 94 \\ \hline \$ 96.82 \end{array}$$

Multiply :

$$17. \$.08 \text{ by } 88$$

$$23. \$ 8.97 \text{ by } 16$$

$$29. \$ 45.64 \text{ by } 304$$

$$18. \$.27 \text{ by } 23$$

$$24. \$ 7.65 \text{ by } 27$$

$$30. \$ 34.37 \text{ by } 116$$

$$19. \$.25 \text{ by } 14$$

$$25. \$ 9.84 \text{ by } 38$$

$$31. \$ 83.95 \text{ by } 205$$

$$20. \$.65 \text{ by } 39$$

$$26. \$ 4.39 \text{ by } 49$$

$$32. \$ 68.33 \text{ by } 312$$

$$21. \$.75 \text{ by } 84$$

$$27. \$ 3.76 \text{ by } 65$$

$$33. \$ 53.61 \text{ by } 234$$

$$22. \$.96 \text{ by } 92$$

$$28. \$ 8.92 \text{ by } 78$$

$$34. \$ 72.89 \text{ by } 402$$

Written Exercises

1. Find the cost of 4 pairs of skates @ \$ 1.50.

This means "Find the cost of 4 pairs of skates at \$1.50 per pair."

Find the cost of:

- | | |
|--------------------------|--------------------------------|
| 2. 144 hats @ \$ 2.25. | 9. 56 watches @ \$ 15.85. |
| 3. 64 pianos @ \$ 475. | 10. 42 cameras @ \$ 22.50. |
| 4. 48 rings @ \$ 16.75. | 11. 66 shotguns @ \$ 14.10. |
| 5. 809 tickets @ \$.75. | 12. 200 hammocks @ \$ 1.35. |
| 6. 400 horses @ \$ 182. | 13. 55 automobiles @ \$ 675. |
| 7. 704 books @ \$ 1.10. | 14. 240 pairs gloves @ \$.95. |
| 8. 120 bicycles @ \$ 37. | 15. 288 pairs shoes @ \$ 3.15. |

16. How much must be paid for 50 crates of strawberries at \$ 2.85 per crate?

17. A man bought 44 crates of pineapples at \$ 1.45 per crate. How much did they cost him?

18. A grocer bought a carload of oranges, 362 boxes, at \$ 2.25 per box. How much did the oranges cost?

19. A dealer bought 36 crates of cantaloupes at \$ 2.50 a crate, and sold them at \$ 3.15 a crate. Find his gain.

20. Find the cost of 8 dozen boxes of writing paper at \$.27 per box.

21. How much is gained on a dozen canary birds costing \$ 12 and selling for \$ 2.50 each?

22. How much will it cost to make a sidewalk 4 yd. wide and 11 yd. long, at \$ 2.25 a square yard?

23. If 684 barrels of apples sold for \$ 1.82 per barrel, how much was received for the entire lot?

DIVISION OF UNITED STATES MONEY

Oral Exercises

1. How many boxes of Christmas candles costing 8¢ a box can be bought for 24¢?

2. Clara's father gave her 50¢ in dimes. How many dimes did he give her?

3. How many yards of ribbon costing 9¢ a yard can be bought for 27¢?

4. When a pound of almonds costs 28¢, how much does $\frac{1}{4}$ of a pound cost?

SUGGESTION. — $\frac{1}{4}$ of a pound of almonds costs $\frac{1}{4}$ of 28¢.

Find the cost of:

5. $\frac{1}{2}$ pint of cream, when a pint costs 16¢.

6. $\frac{1}{4}$ pound of tea, when a pound costs 44¢.

7. $\frac{1}{2}$ dozen bananas, when a dozen costs 18¢.

8. $\frac{1}{4}$ pound of coffee, when a pound costs 32¢.

9. Alfred has 42¢. How many rubber balls can he buy at 7¢ each?

10. If Helen has 36¢, how many Christmas toys can she buy at 9¢ each?

11. How many pans costing 6¢ each can be bought for 50¢, and how many cents will be left?

12. A street-car conductor exchanged a dollar for 5-cent pieces. How many 5-cent pieces did he receive?

13. A woman paid 60¢ for 3 dozen buttons. How much did they cost per dozen?

Written Exercises

1. Find $\frac{1}{4}$ of \$38.72, or divide \$38.72 by 4.

$$\begin{array}{r} 4 \overline{) \$38.72} \\ \$9.68 \end{array}$$

Divide as you divide other numbers, placing a decimal point in the quotient directly below the decimal point in the dividend.

Write the sign \$ before the quotient.

Find $\frac{1}{4}$ of:

Find $\frac{1}{2}$ of:

- | | | | |
|-----------|------------|------------|-------------|
| 2. \$3.72 | 4. \$31.68 | 6. \$12.72 | 8. \$534.24 |
| 3. \$7.44 | 5. \$48.96 | 7. \$59.86 | 9. \$927.28 |

Divide:

- | | | |
|------------------|------------------|-------------------|
| 10. \$55.44 by 6 | 14. \$47.34 by 9 | 18. \$973.55 by 5 |
| 11. \$64.75 by 7 | 15. \$31.12 by 8 | 19. \$786.56 by 8 |
| 12. \$52.88 by 8 | 16. \$96.30 by 9 | 20. \$808.47 by 9 |
| 13. \$74.52 by 6 | 17. \$99.92 by 8 | 21. \$797.84 by 8 |

22. If a gallon of choice maple sirup costs \$1.12, how much should a quart cost?

23. Mrs. Day bought half a dozen spoons for \$3.90. How much did they cost apiece?

24. Philip bought an 8-pound basket of cherries for \$1.20. How much did he pay for them per pound?

25. Mark had \$3.16. He spent $\frac{1}{4}$ of his money for a sled. How much did the sled cost?

26. How much do collars cost apiece, when the cost of a dozen is \$1.80?

27. A man boarded 12 days at a hotel and was charged \$30.00. How much did it cost him per day?

Written Exercises

Find quotients :

$$\begin{array}{r}
 \$4.36 \\
 1. \quad 34 \overline{) \$148.24} \\
 \underline{136} \\
 122 \\
 \underline{102} \\
 204 \\
 \underline{204} \\
 0000
 \end{array}$$

$$\begin{array}{r}
 \$20.92 \\
 2. \quad 89 \overline{) \$1861.88} \\
 \underline{178} \\
 818 \\
 \underline{801} \\
 178 \\
 \underline{178} \\
 0000
 \end{array}$$

Divide as you divide other numbers; placing the decimal point in the quotient directly above the decimal point in the dividend.

- | | | |
|----------------------|------------------------|-------------------------|
| 3. $\$98.21 \div 23$ | 10. $\$378.58 \div 46$ | 17. $\$427.04 \div 136$ |
| 4. $\$87.89 \div 17$ | 11. $\$469.16 \div 37$ | 18. $\$291.20 \div 520$ |
| 5. $\$84.00 \div 24$ | 12. $\$674.16 \div 53$ | 19. $\$431.97 \div 187$ |
| 6. $\$69.44 \div 32$ | 13. $\$859.14 \div 43$ | 20. $\$670.80 \div 312$ |
| 7. $\$74.88 \div 26$ | 14. $\$951.51 \div 69$ | 21. $\$733.86 \div 243$ |
| 8. $\$95.40 \div 36$ | 15. $\$598.92 \div 84$ | 22. $\$357.12 \div 496$ |
| 9. $\$59.09 \div 19$ | 16. $\$638.40 \div 95$ | 23. $\$486.25 \div 389$ |

24. A grocer sold 28 barrels of flour for \$189.00. Find the price per barrel.

25. A farmer sold 34 tons of hay and received \$722.50 for it. How much did he receive per ton?

26. An apple buyer paid \$218.75 for 125 barrels of Baldwin apples. What was the price per barrel?

27. A furniture dealer had 35 oak chairs in stock, valued at \$624.75. Find the average value per chair.

Miscellaneous Exercises

Find the number of dollars in :

- | | | | |
|---------|----------|----------|-----------|
| 1. 300¢ | 4. 6400¢ | 7. 2300¢ | 10. 1200¢ |
| 2. 500¢ | 5. 9200¢ | 8. 5600¢ | 11. 3600¢ |
| 3. 800¢ | 6. 8500¢ | 9. 9900¢ | 12. 5500¢ |

Find the number of cents in :

- | | | | | |
|----------|-----------|-----------|------------|------------|
| 13. \$ 5 | 16. \$ 26 | 19. \$ 48 | 22. \$ 120 | 25. \$ 569 |
| 14. \$ 7 | 17. \$ 34 | 20. \$ 83 | 23. \$ 203 | 26. \$ 843 |
| 15. \$ 9 | 18. \$ 55 | 21. \$ 97 | 24. \$ 528 | 27. \$ 678 |

Multiply by 3, 4, 5, 6, 7, 8, and 9 :

- | | | | | |
|-----------|-------------|-------------|--------------|--------------|
| 28. \$.76 | 30. \$ 4.85 | 32. \$ 6.98 | 34. \$ 19.72 | 36. \$ 76.25 |
| 29. \$.27 | 31. \$ 9.74 | 33. \$ 4.80 | 35. \$ 74.89 | 37. \$ 38.63 |

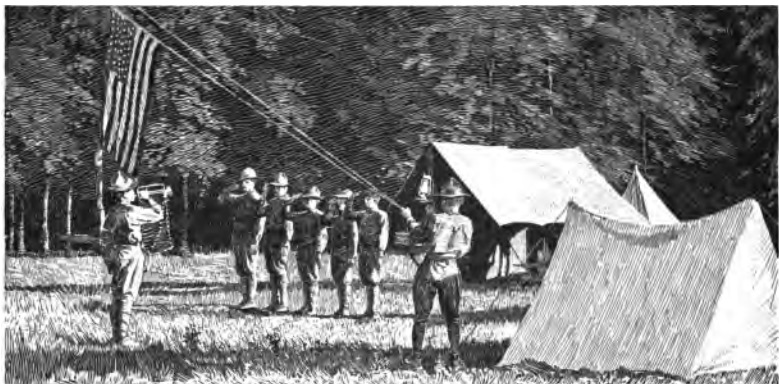
Subtract from \$ 5.00 :

- | | | | | |
|-------------|-------------|-------------|-----------|---------|
| 38. \$ 3.27 | 40. \$ 3.16 | 42. \$ 1.85 | 44. \$.63 | 46. 75¢ |
| 39. \$ 2.50 | 41. \$ 4.19 | 43. \$ 2.71 | 45. \$.82 | 47. 48¢ |

Find the answers quickly :

- | | | |
|------------------|-------------------|--------------------|
| 48. \$ 2.88 ÷ 18 | 53. \$ 21.65 × 78 | 58. \$ 732.72 ÷ 43 |
| 49. \$ 3.45 × 36 | 54. \$ 98.88 ÷ 32 | 59. \$ 539.89 × 39 |
| 50. \$ 5.60 × 45 | 55. \$ 56.34 × 57 | 60. \$ 507.52 ÷ 52 |
| 51. \$ 9.15 ÷ 61 | 56. \$ 87.25 × 64 | 61. \$ 478.86 × 69 |
| 52. \$ 7.38 × 53 | 57. \$ 66.42 ÷ 27 | 62. \$ 969.68 ÷ 46 |
63. Add \$ 150, \$ 17.85, \$ 42.60, \$ 984.10, and \$ 2012.
64. Add \$ 68.59, \$ 580, \$ 428.67, \$ 49.25, and \$ 591.38.
65. Add \$ 8420, \$ 397.50, \$ 56.49, \$ 650, and \$ 875.98.

Boy Scout Problems



1. How many members are there in a troop of Boy Scouts composed of 4 patrols of 8 boys each?

2. Before he can be a First-class Scout a boy must earn and deposit \$ 2 in a bank. How long will this take at 25¢ a week?

Find the cost of these Boy Scout articles of equipment:

- | | |
|----------------------|--------------------------------|
| 3. 8 axes @ \$.75. | 7. 12 knives @ \$.50. |
| 4. 7 hats @ \$.50. | 8. 10 haversacks @ \$.60. |
| 5. 6 belts @ \$.40. | 9. 14 first-aid kits @ \$.25. |
| 6. 9 coats @ \$1.35. | 10. 11 pairs leggings @ \$.55. |
11. Of the 72 arrows that a Boy Scout shot, $\frac{1}{4}$ missed the target. How many arrows missed the target?

12. A Boy Scout's signal flag was 24 in. square. Draw a diagram, scale 1 to 8, and find the area of the flag.

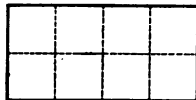
13. Find the cost of this three-meal ration for six scouts: bacon, 60¢; salmon, 45¢; potatoes, 12¢; butter, 35¢; eggs, 32¢; milk, 24¢; cocoa, 25¢; sugar, 6¢; flour, 15¢.

PROBLEM MAKING

Oral Exercises

Here are two problems about a rectangle 4 in. by 2 in.

1. What is the area of a rectangle 4 in. by 2 in.?



Area = 2×4 sq. in. = 8 sq. in.

2. What is the perimeter of an oblong 4 in. by 2 in.?

Perimeter = 4 in. + 2 in. + 4 in. + 2 in. = — in. = — ft.

Make and solve as many problems as you can about:

3. A window 5 ft. by 3 ft., with panes 1 ft. square.
4. A room, the floor of which is 6 yd. by 5 yd.
5. Paul has 60 marbles, George 10 marbles.
6. A wild rose has 5 petals.
7. A spider has 8 legs; a bee, 6 legs; a horse, 4 legs.
8. Julia planted 40 sweet peas. Some did not come up.
9. A horse can gallop 20 miles in 2 hours.
10. A cat has 5 toes on each fore paw and 4 toes on each hind paw.
11. The tail of Roy's kite was 6 yd. long at first. Afterward Roy cut off 4 ft. of it.
12. The kite string was 180 ft. long. We often measure string in yards.
13. Edna and Mabel colored 2 dozen eggs for Easter. $\frac{1}{2}$ of them were red and $\frac{1}{4}$ of them were blue.

The pupils should be encouraged to write other problems and solve them, the teacher suggesting material when necessary.

BILLS AND ACCOUNTS

1. Mr. Blair bought these groceries at Mr. Ford's store:
 April 20, 5 pounds of butter at \$.32 per pound;
 and April 24, 2 dozen oranges at \$.35 per dozen.

2. Mr. Blair did not pay for these things as he bought them, but had them *charged*, Mr. Ford keeping a record of them, called an **account**.

3. On April 24, Mr. Blair asked how much he owed and the grocer made out this **bill**:

		Utica, N.Y., <i>Apr 24, 1914.</i>	
<i>Mr Chas H Blair,</i>			
<i>385 Jefferson St.</i>			
OF A.B.Ford, 205 Genessee St.			
Dealer in Staple and Fancy Groceries			
Terms : Cash			
<i>April</i>	<i>20</i>	<i>5 lb butter @ \$.32</i>	<i>1 60</i>
<i>"</i>	<i>24</i>	<i>2 doz oranges @ .35</i>	<i>70 2 30</i>
		<i>Received payment</i>	
		<i>A. B. Ford.</i>	

4. Multiply the *price* per pound (or other *unit*) by the number of units and find whether the grocer made the **extensions** correctly.

5. Find whether \$ 2.30 is the correct amount, or **footing**.

6. When the bill was paid Mr. Ford **receipted** it by writing "Received payment" and his name.

If a clerk received the money, he would receipt by writing Mr. Ford's name and below it his own, preceded by "Per" or "By."

Written Exercises

Make out bills and receipt, supplying dates, names, and addresses:

1.

<i>(Name of town and state)</i> <i>(Date)</i> 191- <i>M</i> <i>(Name of buyer)</i> <i>(Buyer's address)</i> Bought of <i>(Name of seller and street address if any)</i>							
Date	—	2 lb. cinnamon	@ \$.40				
"	—	3 lb. mixed spices	@ .19				

- | | |
|--|--|
| <p>2. 2 boxes borax @ \$.12
 12 bars soap @ \$.05
 3 boxes starch @ \$.19</p> <p>3. 8 lb. rice @ \$.09
 6 lb. soda @ \$.08
 3 lb. tapioca @ \$.10</p> <p>4. 4 yd. lawn @ \$.19
 8 yd. linen @ \$.65
 12 yd. crepe @ \$.16
 6 yd. silk @ \$.85</p> <p>5. 4 lb. almonds @ \$.20
 2 lb. pecans @ \$.19
 3 lb. filberts @ \$.18
 5 lb. walnuts @ \$.25</p> | <p>6. 6 spools silk @ \$.09
 8 spools cotton @ \$.05
 5 doz. buttons @ \$.16</p> <p>7. 3 lb. coffee @ \$.35
 25 lb. sugar @ \$.06
 5 lb. crackers @ \$.09</p> <p>8. 5 lb. butter @ \$.33
 4 doz. eggs @ \$.42
 2 lb. cheese @ \$.25
 3 doz. oranges @ \$.35</p> <p>9. 2 rugs @ \$29.74
 3 blankets @ \$ 5.84
 1 dresser @ \$25.67
 2 bedsteads @ \$22.74</p> |
|--|--|

Suppose that you own a hardware store and have sold the following goods to several customers (your classmates or others). Make out a bill to each, and receipt it.

10. 2 pairs of scissors @ 75¢; 16 packages of tacks @ 4¢; 4 planes @ 80¢; 3 hammers @ 65¢.

11. 3 wrenches @ 25¢; 2 chisels @ 45¢; 8 doz. bolts @ 24¢; 2 lawn mowers @ \$4.75.

12. 2 tents @ \$8.25; 4 rubber blankets @ \$2.75; 2 camp stoves @ \$5.50.

13. 2 fishing rods @ \$2.50; 2 reels @ 75¢; 100 yd. fish line @ 2¢; 5 doz. trout flies @ 35¢.

Make out and foot bills for the following sales:

The seller may be a merchant known to you; the buyer, one of your classmates. Receipt the bills as though you were a clerk.

14. 5 doz. oranges @ 35¢; 8 doz. bananas @ 15¢; 12 qt. peanuts @ 5¢.

15. 9 collars @ 15¢; 6 pairs cuffs @ 25¢; 4 ties @ 50¢; 6 shirts @ \$1.50; 18 handkerchiefs @ 20¢.

16. 1 couch, \$25; 6 chairs @ \$1.75; 1 cot, \$5.25.

17. 3 hoes @ 25¢; 2 rakes @ 35¢; 1 spade, 75¢; 2 shovels @ 65¢; 100 ft. hose @ 15¢.

18. 1 croquet set, \$2.25; 2 lawn swings @ \$3.50; 50 ft. wire netting @ 6¢; 2 hammocks @ \$4.25.

19. 8 lb. lard @ 18¢; 12 muskmelons @ 8¢; 15 qt. raspberries @ 12¢; 50 lb. sugar @ 6¢.

20. 3 qt. varnish @ 75¢; 8 gal. paint @ \$1.85; 4 bottles liquid veneer @ 25¢; 3 cans floor wax @ 45¢.

CANCELLATION

1. You have learned that division, as $12 \div 6$, is sometimes indicated thus, $\frac{12}{6}$. What does $\frac{2 \times 6}{2 \times 3}$ mean?

2. What is the value of $\frac{2 \times 6}{2 \times 3}$? of $\frac{2 \times 6}{2 \times 3}$, or $\frac{6}{3}$?

3. Compare the results in exercise 2. Which is easier to find?

The process of shortening work in division by rejecting equal factors from dividend and divisor is **cancellation**.

Cancellation is based on the principle that:

Dividing both dividend and divisor by the same number does not change the quotient.

Oral Exercises

Tell what factor may be taken out of both dividend and divisor:

$$1. \frac{2 \times 3}{2 \times 4} \quad 3. \frac{5 \times 3}{7 \times 5} \quad 5. \frac{4 \times 7}{3 \times 7} \quad 7. \frac{6 \times 3}{3 \times 5} \quad 9. \frac{7 \times 8}{2 \times 8}$$

$$2. \frac{4 \times 5}{7 \times 4} \quad 4. \frac{2 \times 9}{9 \times 3} \quad 6. \frac{6 \times 2}{2 \times 7} \quad 8. \frac{5 \times 6}{6 \times 8} \quad 10. \frac{9 \times 7}{7 \times 5}$$

Take out the same factor from both dividend and divisor; then give the quotient:

$$11. \frac{9 \times 4}{3 \times 4} \quad 13. \frac{6 \times 3}{3 \times 2} \quad 15. \frac{4 \times 5}{5 \times 2} \quad 17. \frac{12 \times 7}{4 \times 7} \quad 19. \frac{3 \times 10}{2 \times 3}$$

$$12. \frac{7 \times 8}{2 \times 7} \quad 14. \frac{8 \times 6}{6 \times 4} \quad 16. \frac{6 \times 8}{8 \times 3} \quad 18. \frac{15 \times 9}{9 \times 3} \quad 20. \frac{5 \times 18}{5 \times 6}$$

Written Exercises

1. Divide
- $18 \times 25 \times 2$
- by
- $3 \times 5 \times 4$
- .

$$\begin{array}{r} 3 \\ 6 \quad 5 \\ \cancel{18} \times \cancel{25} \times 2 = 15 \\ \cancel{3} \times \cancel{5} \times \cancel{4} \\ 2 \end{array}$$

Write the dividend above the divisor with a line between them.

The factor 3 may be canceled (divided out) from dividend and divisor, leaving 6 in the dividend, and 1 (which need not be written) in the divisor:

Next the dividend and divisor may be divided by 5; then by 2.

- The remaining factors of the dividend are 6 and 5; of the divisor, 2. Of these the factor 2 may be canceled, leaving 3 and 5 in the dividend and only 1's (not written) in the divisor. $3 \times 5 = 15$, which divided by 1 equals 15, the required quotient.

Divide, using cancellation:

2. $\frac{5 \times 8}{4}$

6. $\frac{84}{6 \times 7}$

10. $\frac{12 \times 8}{2 \times 24}$

14. $\frac{9 \times 8}{2 \times 4 \times 3}$

3. $\frac{8 \times 9}{6}$

7. $\frac{150}{5 \times 6}$

11. $\frac{63 \times 5}{15 \times 7}$

15. $\frac{3 \times 2 \times 8}{2 \times 4}$

4. $\frac{6 \times 8}{4 \times 2}$

8. $\frac{168}{7 \times 3}$

12. $\frac{5 \times 32}{2 \times 40}$

16. $\frac{6 \times 7 \times 9}{3 \times 7}$

5. $\frac{8 \times 9}{3 \times 2}$

9. $\frac{216}{6 \times 9}$

13. $\frac{45 \times 9}{3 \times 15}$

17. $\frac{2 \times 9 \times 6}{6 \times 2 \times 3}$

18. $5 \times 12 \times 9$ by 18×6

22. 15×16 by $2 \times 5 \times 2$

19. $3 \times 32 \times 5$ by 15×8

23. 18×24 by $6 \times 8 \times 3$

20. $4 \times 44 \times 9$ by 33×2

24. 54×16 by $4 \times 6 \times 9$

21. $6 \times 72 \times 4$ by 24×8

25. 49×64 by $7 \times 4 \times 8$

26. Find 7 times $246 \div 6$; $475 \div 10$ multiplied by 12.

$$7 \times \frac{246}{6} = 287$$

$$\frac{475}{10} \times 12 = 570$$

Using cancellation, find results:

27. $6 \times \frac{470}{5}$

32. $\frac{100}{32} \times 8$

37. $12 \times \frac{205}{15}$

28. $5 \times \frac{741}{3}$

33. $\frac{220}{6} \times 9$

38. $\frac{850}{55} \times 22$

29. $4 \times \frac{536}{8}$

34. $\frac{486}{12} \times 6$

39. $15 \times \frac{252}{36}$

30. $3 \times \frac{954}{9}$

35. $\frac{340}{25} \times 5$

40. $\frac{738}{63} \times 49$

31. $7 \times \frac{528}{4}$

36. $\frac{425}{20} \times 8$

41. $64 \times \frac{534}{48}$

42. $7 \times \frac{\$86}{\cancel{7}} = \6.02

43. $\frac{\$15.29}{\cancel{20}} \times 16 = \61.16

44. $\frac{8 \times \$1.25}{4}$

49. $\frac{\$2.76}{8} \times 6$

54. $12 \times \frac{\$15.27}{6}$

45. $\frac{\$8.40 \times 6}{8}$

50. $\frac{\$7.63}{4} \times 8$

55. $14 \times \frac{\$21.05}{7}$

46. $\frac{3 \times \$7.70}{5}$

51. $\frac{\$4.62}{6} \times 9$

56. $24 \times \frac{\$38.40}{9}$

47. $\frac{6 \times \$2.91}{9}$

52. $\frac{\$9.06}{6} \times 4$

57. $18 \times \frac{\$82.04}{4}$

48. $\frac{8 \times \$3.75}{6}$

53. $\frac{\$8.48}{8} \times 6$

58. $30 \times \frac{\$44.12}{8}$

Written Exercises

Reason out the operations to be performed ; indicate all the work in a single statement ; and solve by cancellation :

1. The product of three numbers is 378. Two of the numbers are 9 and 6. What is the third number ?

REASONING

STATEMENT

Since 378 is the product of three numbers and two of them are 9 and 6, the third may be found by dividing 378 by 9 and that quotient by 6, or what is the same thing, by dividing 378 by 9×6 .

$$\frac{378}{9 \times 6} = ?$$

This may be indicated as in the statement and the result found by cancellation.

2. The product of three numbers is 224. If two of the numbers are 7 and 8, find the third number.

3. What number multiplied by 6×9 will give 270 ?

4. What number multiplied by 15 will give 100×6 ?

5. What number multiplied by 4×20 will give 480 ?

6. What number multiplied by 24 will give 120×12 ?

7. If the product is 540, and two of the factors are 5 and 18, what is the third factor ?

8. When 5 yd. of cloth cost \$ 3.75, how much do 14 yd. cost at the same rate ?

REASONING

STATEMENT

Since 5 yd. of cloth cost \$ 3.75, 1 yd. costs \$ $3.75 \div 5$, or $\frac{\$ 3.75}{5}$, and 14 yd. cost 14 times as much, as indicated in the statement.

$$14 \times \frac{\$ 3.75}{5} = ?$$

9. If 3 muskmelons cost \$.25, how much do 36 cost ?

Find the cost of :

10. 18 tops, when 6 tops cost \$ 2.94.
11. 10 books, when 2 books cost \$.90.
12. 24 dolls, when 4 dolls cost \$ 7.00.
13. 20 rugs, when 10 rugs cost \$ 19.80.
14. 15 lamps, when 6 lamps cost \$ 41.34.
15. 9 tables, when 2 tables cost \$ 46.48.
16. 15 rings, when 10 rings cost \$ 57.50.
17. 10 purses, when 5 purses cost \$ 27.05.
18. 16 clocks, when 6 clocks cost \$ 31.50.
19. 12 chairs, when 9 chairs cost \$ 41.31.
20. 18 kodaks, when 4 kodaks cost \$ 48.00.
21. 16 aprons, when 20 aprons cost \$ 9.80.
22. 14 trunks, when 7 trunks cost \$ 66.50.
23. 12 stoves, when 3 stoves cost \$ 73.80.
24. 8 watches, when 6 watches cost \$ 94.50.
25. 6 umbrellas, when 9 umbrellas cost \$ 17.82.
26. 15 yd. linen, when 12 yd. linen cost \$ 14.88.
27. 18 sweaters, when 12 sweaters cost \$ 59.52.
28. 15 fur caps, when 10 fur caps cost \$ 42.40.
29. 24 overcoats, when 4 overcoats cost \$ 74.00.
30. 15 bracelets, when 12 bracelets cost \$ 46.68.
31. 20 yd. ribbon, when 15 yd ribbon cost \$ 13.35.
32. 14 doz. knives, when 8 doz. knives cost \$ 67.84.
33. 12 doz. teaspoons, when 8 doz. teaspoons cost \$ 68.32.
34. 24 fountain pens, when 16 fountain pens cost \$ 42.72.

DRY MEASURES

1. Dry measures are used to measure dry and bulky things, as grain or vegetables.

2. Learn the table:

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)



PINT PECK QUART BUSHEL

3. Name things that are sold by the bushel; by other dry measures.

Oral Exercises

- How many pints are there in 1 qt. ? in 3 qt. ?
- Tell the number of quarts in 2 pt. ; in 12 pt.
- How many quarts are there in 1 pk. ? in 2 pk. ?
- Tell the number of pecks in 8 qt. ; in 24 qt.
- How many pecks are there in 1 bu. ? in 2 bu. ?
- Tell the number of bushels in 4 pk. ; in 20 pk.
- How many pecks are there in 1 bu. ? in $\frac{1}{4}$ bu. ?
- What part of a bushel is 1 pk. ?
- How many quarts are there in $\frac{1}{2}$ pk. ? in $\frac{1}{4}$ pk. ?
- Tell the number of pecks in 5 bu. ; in 7 bu. ; in $\frac{1}{2}$ bu.

Change to quarts:

Change to pecks:

- | | | | |
|-----------|------------|-----------|------------|
| 11. 4 pk. | 13. 14 pt. | 15. 4 bu. | 17. 40 qt. |
| 12. 7 pk. | 14. 18 pt. | 16. 8 bu. | 18. 64 qt. |

Written Exercises

1. How many quarts are there in 4 pk. 5 qt. ?

SOLUTION

$$1 \text{ pk.} = 8 \text{ qt.}$$

$$4 \text{ pk.} = 4 \times 8 \text{ qt.} = 32 \text{ qt.}$$

$$4 \text{ pk. } 5 \text{ qt.} = 32 \text{ qt.} + 5 \text{ qt.} = 37 \text{ qt.}$$

2. Express 50 pk. as bushels and pecks.

SOLUTION

$$4 \text{ pk.} = 1 \text{ bu.}$$

$$50 \text{ pk.} \div 4 \text{ pk.} = 12 (\text{bu.}), 2 (\text{pk.}) \text{ over.}$$

$$\text{Then, } 50 \text{ pk.} = 12 \text{ bu. } 2 \text{ pk.}$$

Exercise 1 illustrates *reduction to lower units*; exercise 2 illustrates *reduction to higher units*.

Reduce to quarts :

3. 22 pk.

7. 120 pt.

4. 30 pk.

8. 136 pt.

5. 45 pk.

9. 154 pt.

6. 53 pk.

10. 176 pt.

Reduce to pecks :

11. 44 bu.

15. 216 qt.

12. 92 bu.

16. 256 qt.

13. 76 bu.

17. 272 qt.

14. 89 bu.

18. 288 qt.

Reduce to the lower unit mentioned :

19. 3 qt. 1 pt.

24. 10 qt. 1 pt.

29. 16 bu. 2 pk.

20. 2 bu. 2 pk.

25. 12 pk. 5 qt.

30. 18 pk. 6 qt.

21. 3 pk. 4 qt.

26. 11 bu. 3 pk.

31. 20 bu. 1 pk.

22. 5 qt. 1 pt.

27. 15 qt. 1 pt.

32. 24 bu. 3 pk.

23. 3 bu. 1 pk.

28. 14 pk. 7 qt.

33. 30 pk. 5 qt.

Express in the next higher and the given unit :

34. 63 pt.

38. 102 qt.

42. 275 pt.

46. 321 pt.

35. 70 pk.

39. 115 pt.

43. 204 qt.

47. 287 pk.

36. 91 pk.

40. 156 qt.

44. 198 pk.

48. 338 qt.

37. 87 pt.

41. 174 pk.

45. 268 qt.

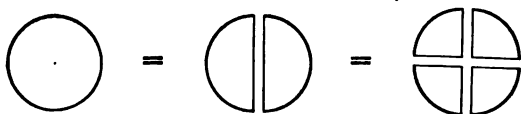
49. 345 pk.

FRACTIONS

HALVES AND FOURTHS

Oral and Written Exercises

1. One whole circle is equal to how many half circles?
to how many quarter circles?



1 circle = 2 half circles = 4 quarter circles.

1 = 2 halves = 4 fourths.

1 = $\frac{2}{2}$ = $\frac{4}{4}$.

2. How many halves are 1 half + 1 half? how many wholes?

3. How many fourths are 1 fourth + 1 fourth? How many halves are 2 fourths?

4. $\frac{1}{2} + \frac{1}{2} = ?$ 5. $\frac{1}{4} + \frac{1}{4} = ?$ 6. $\frac{2}{4} = \frac{?}{2}$ 7. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = ?$

Write and complete :

8. $1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} =$ 11. $\frac{3}{4} + \frac{1}{4} = \frac{?}{4} =$ 14. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} =$

9. $1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} =$ 12. $\frac{3}{4} - \frac{1}{4} = \frac{?}{4} = \frac{?}{2}$ 15. $\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} =$

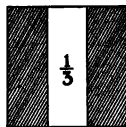
10. $1 - \frac{3}{4} = \frac{4}{4} - \frac{3}{4} =$ 13. $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} =$ 16. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$

17. Ruth gave $\frac{1}{2}$ of an apple to her mother and $\frac{1}{4}$ of it to her sister. What part of the apple did Ruth give away?

18. John walked $\frac{3}{4}$ of a mile Monday and $\frac{1}{2}$ of a mile Tuesday. How much farther did he walk the first day than the second?

THIRDS AND SIXTHS

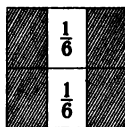
1. Into how many equal parts is this square divided?
One of the *three equal* parts of a square, or of anything, is **one third** of it.



2. How many thirds are shaded?
One third is written $\frac{1}{3}$. Write two thirds.

3. If each third of the square is divided into two equal parts, into how many equal parts will the square be divided?

One of the *six equal* parts of the square, or of anything, is **one sixth** of it.



4. How many sixths are shaded?
One sixth is written $\frac{1}{6}$. Write two sixths; four sixths.

Oral and Written Exercises

1. Looking at the squares, tell how many sixths there are in $\frac{1}{3}$; in $\frac{2}{3}$; in $\frac{3}{3}$; in $\frac{1}{2}$.

2. Change to thirds: $\frac{4}{6}$; $\frac{6}{6}$; $\frac{2}{6}$.

3. How many halves are $\frac{3}{6}$?

4. Change 1 to thirds; to sixths; $\frac{1}{2}$ to sixths.

Do as the signs indicate:

5. $\frac{1}{3} + \frac{1}{3}$

7. $\frac{2}{3} + \frac{1}{3}$

9. $1 - \frac{2}{3}$

11. $1 - \frac{5}{6}$

6. $\frac{2}{3} - \frac{1}{3}$

8. $1 - \frac{1}{3}$

10. $\frac{1}{6} + \frac{5}{6}$

12. $1 - \frac{1}{6}$

Change halves or thirds to sixths before adding or subtracting, but in the answers give $\frac{1}{3}$ for $\frac{2}{6}$, $\frac{1}{2}$ for $\frac{3}{6}$, and $\frac{2}{3}$ for $\frac{4}{6}$.

13. $\frac{1}{6} + \frac{1}{6}$

15. $\frac{1}{6} + \frac{1}{2}$

17. $\frac{1}{6} + \frac{2}{3}$

19. $\frac{1}{3} - \frac{1}{6}$

14. $\frac{5}{6} - \frac{1}{6}$

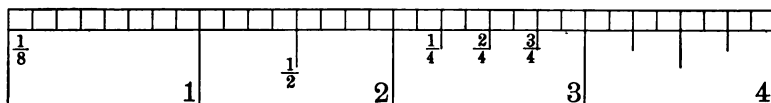
16. $\frac{1}{6} + \frac{1}{3}$

18. $\frac{1}{2} - \frac{1}{6}$

20. $\frac{2}{3} - \frac{1}{6}$

EIGHTHS

This rule is 4 inches long. Into how many equal parts is the first inch divided?



One of the *eight equal* parts of anything is one **eighth** of it. Write one eighth; three eighths.

Oral and Written Exercises

1. Look at the second inch of the rule and tell how many eighths are equal to $\frac{1}{2}$; to $\frac{2}{2}$, or 1.

2. Look at the third inch and tell how many eighths are equal to $\frac{1}{4}$; to $\frac{2}{4}$, or $\frac{1}{2}$; to $\frac{3}{4}$; to $\frac{4}{4}$, or 1.

3. In the right-hand inch point to $\frac{1}{2}$ in.; $\frac{1}{4}$ in.; $\frac{1}{8}$ in.

Give results:

4. $\frac{1}{8} + \frac{7}{8}$ 6. $1 - \frac{7}{8}$ 8. $1 - \frac{3}{8}$ 10. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

5. $1 - \frac{1}{8}$ 7. $\frac{3}{8} + \frac{5}{8}$ 9. $1 - \frac{5}{8}$ 11. $\frac{3}{8} + \frac{1}{8} + \frac{1}{8}$

Do as the signs indicate:

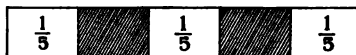
Change halves or fourths to eighths before adding or subtracting, but in your answers give $\frac{1}{4}$ for $\frac{2}{8}$, $\frac{1}{2}$ for $\frac{4}{8}$, and $\frac{3}{4}$ for $\frac{6}{8}$.

12. $\frac{1}{8} + \frac{1}{8}$ 17. $\frac{5}{8} - \frac{1}{8}$ 22. $\frac{1}{8} + \frac{1}{4}$ 27. $\frac{7}{8} - \frac{1}{4}$ 32. $\frac{3}{4} - \frac{5}{8}$
 13. $\frac{7}{8} - \frac{3}{8}$ 18. $\frac{1}{8} + \frac{3}{8}$ 23. $\frac{7}{8} - \frac{1}{2}$ 28. $\frac{1}{4} + \frac{3}{8}$ 33. $\frac{1}{2} - \frac{3}{8}$
 14. $\frac{5}{8} + \frac{1}{8}$ 19. $\frac{3}{8} - \frac{1}{8}$ 24. $\frac{5}{8} - \frac{1}{4}$ 29. $\frac{1}{2} - \frac{1}{8}$ 34. $\frac{5}{8} + \frac{1}{4}$
 15. $\frac{7}{8} - \frac{5}{8}$ 20. $\frac{5}{8} - \frac{3}{8}$ 25. $\frac{3}{8} + \frac{1}{2}$ 30. $\frac{7}{8} - \frac{3}{4}$ 35. $\frac{3}{4} - \frac{3}{8}$
 16. $\frac{3}{8} + \frac{3}{8}$ 21. $\frac{7}{8} - \frac{1}{8}$ 26. $\frac{1}{8} + \frac{3}{4}$ 31. $\frac{1}{8} + \frac{1}{2}$ 36. $\frac{5}{8} - \frac{1}{2}$

FIFTHS AND TENTHS

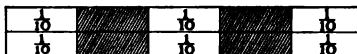
1. Into how many equal parts is this rectangle divided?

One of the *five equal* parts of anything is **one fifth** of it.



2. How many fifths are shaded? Write two fifths.

3. If each fifth of the rectangle is divided into two equal parts, into how many equal parts will the rectangle be divided?



One of the *ten equal* parts of anything is **one tenth** of it.

4. How many tenths of this rectangle are light?

Oral and Written Exercises

1. Look at the rectangles and tell how many tenths there are in $\frac{1}{5}$; in $\frac{2}{5}$; in $\frac{3}{5}$; in $\frac{4}{5}$; in $\frac{5}{5}$, or 1; in $\frac{1}{2}$.

2. Change to fifths: 1; $\frac{4}{10}$; $\frac{2}{10}$; $\frac{10}{10}$, or 1; $\frac{6}{10}$; $\frac{8}{10}$.

3. Change 1 to tenths; $\frac{1}{2}$ to tenths; $\frac{5}{10}$ to halves.

Do as the signs indicate:

- | | | | | |
|--------------------------------|--------------------------------|---------------------------------|---------------------------------|-----------------------------------|
| 4. $\frac{1}{5} + \frac{1}{5}$ | 7. $\frac{1}{5} + \frac{3}{5}$ | 10. $\frac{4}{5} - \frac{2}{5}$ | 13. $\frac{2}{5} + \frac{3}{5}$ | 16. $\frac{3}{10} + \frac{7}{10}$ |
| 5. $\frac{2}{5} - \frac{1}{5}$ | 8. $\frac{4}{5} - \frac{1}{5}$ | 11. $1 - \frac{4}{5}$ | 14. $1 - \frac{2}{5}$ | 17. $1 - \frac{3}{10}$ |
| 6. $\frac{1}{5} + \frac{2}{5}$ | 9. $\frac{3}{5} - \frac{2}{5}$ | 12. $\frac{4}{5} - \frac{3}{5}$ | 15. $1 - \frac{3}{5}$ | 18. $\frac{9}{10} + \frac{1}{10}$ |

Change halves and fifths to tenths before adding or subtracting, and give answers in fifths, when possible.

- | | | | | |
|-----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 19. $\frac{1}{10} + \frac{7}{10}$ | 23. $\frac{1}{10} + \frac{3}{5}$ | 27. $\frac{1}{2} - \frac{1}{10}$ | 31. $\frac{4}{5} - \frac{3}{10}$ | 35. $\frac{9}{10} - \frac{1}{5}$ |
| 20. $\frac{7}{10} - \frac{3}{10}$ | 24. $\frac{3}{10} - \frac{1}{5}$ | 28. $\frac{1}{5} + \frac{1}{10}$ | 32. $\frac{2}{5} + \frac{1}{10}$ | 36. $\frac{7}{10} - \frac{1}{2}$ |
| 21. $\frac{9}{10} - \frac{7}{10}$ | 25. $\frac{1}{10} + \frac{4}{5}$ | 29. $\frac{1}{2} - \frac{3}{10}$ | 33. $\frac{3}{5} - \frac{1}{10}$ | 37. $\frac{3}{10} + \frac{1}{5}$ |
| 22. $\frac{1}{10} + \frac{3}{10}$ | 26. $\frac{7}{10} - \frac{2}{5}$ | 30. $\frac{1}{5} + \frac{7}{10}$ | 34. $\frac{1}{2} + \frac{3}{10}$ | 38. $\frac{9}{10} - \frac{1}{2}$ |

TWELFTHS

1. One of the *twelve equal* parts of anything is one **twelfth** of it. One twelfth is written $\frac{1}{12}$.

2. How many twelfths of this rectangle are shaded? how many halves? $\frac{1}{12} = \frac{1}{2}$.

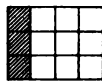


3. How many twelfths of this rectangle are shaded? how many thirds? $\frac{1}{12} = \frac{1}{3}$.

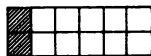


Look at the rectangle again and tell how many twelfths are equal to $\frac{2}{3}$.

4. Look at this rectangle and tell how many twelfths are equal to $\frac{1}{4}$; to $\frac{3}{4}$.



5. How many twelfths are equal to $\frac{1}{6}$? how many are equal to $\frac{5}{6}$?



Written Exercises

Draw lines and divide them to show that:

1. $\frac{1}{2} = \frac{6}{12}$

4. $\frac{1}{6} = \frac{2}{12}$

7. $\frac{6}{12} = \frac{1}{2}$

10. $\frac{2}{12} = \frac{1}{6}$

2. $\frac{1}{3} = \frac{4}{12}$

5. $\frac{2}{3} = \frac{8}{12}$

8. $\frac{4}{12} = \frac{1}{3}$

11. $\frac{9}{12} = \frac{3}{4}$

3. $\frac{1}{4} = \frac{3}{12}$

6. $\frac{3}{4} = \frac{9}{12}$

9. $\frac{3}{12} = \frac{1}{4}$

12. $\frac{10}{12} = \frac{5}{6}$

13. Find the sum and the difference of $\frac{7}{12}$ and $\frac{1}{3}$.

SOLUTIONS. — $\frac{7}{12} + \frac{1}{3} = \frac{7}{12} + \frac{4}{12} = \frac{11}{12}$, the sum.

$\frac{7}{12} - \frac{1}{3} = \frac{7}{12} - \frac{4}{12} = \frac{3}{12} = \frac{1}{4}$, the difference.

Find the sum and the difference of:

14. $\frac{5}{12}, \frac{1}{12}$

17. $\frac{1}{2}, \frac{1}{12}$

20. $\frac{5}{12}, \frac{1}{6}$

23. $\frac{2}{3}, \frac{1}{12}$

15. $\frac{7}{12}, \frac{5}{12}$

18. $\frac{1}{4}, \frac{1}{12}$

21. $\frac{7}{12}, \frac{1}{2}$

24. $\frac{5}{6}, \frac{1}{12}$

16. $\frac{11}{12}, \frac{1}{12}$

19. $\frac{1}{3}, \frac{1}{12}$

22. $\frac{5}{12}, \frac{1}{3}$

25. $\frac{3}{4}, \frac{1}{12}$

HALVES, THIRDS, AND FOURTHS

1. Can you add the fractions $\frac{1}{2}$ and $\frac{1}{3}$ as they stand?
Can you subtract one from the other as they stand?
2. How many sixths are there in $\frac{1}{2}$? in $\frac{1}{3}$?
3. Add $\frac{3}{6}$ and $\frac{2}{6}$. Subtract $\frac{2}{6}$ from $\frac{3}{6}$.
4. What must be done to *different kinds* of fractions before they can be added or subtracted?
5. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = ?$
7. $\frac{2}{3} + \frac{1}{2} = ?$
9. $\frac{4}{3} - \frac{1}{2} = ?$
6. $\frac{1}{2} - \frac{1}{3} = \text{---} - \text{---} = ?$
8. $\frac{2}{3} - \frac{1}{2} = ?$
10. $\frac{3}{2} - \frac{2}{3} = ?$

Written Exercises

1. Find the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

SOLUTION. — $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$, the sum.

2. Subtract $\frac{2}{3}$ from $\frac{3}{4}$.

SOLUTION. — $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$, the difference.

Copy, complete, and read :

3. $\frac{1}{4} + \frac{2}{3} =$
5. $\frac{3}{4} - \frac{1}{3} =$
7. $\frac{1}{3} + \frac{3}{4} =$
9. $\frac{5}{4} - \frac{2}{3} =$
4. $\frac{1}{3} - \frac{1}{4} =$
6. $\frac{2}{3} - \frac{1}{4} =$
8. $\frac{5}{4} - \frac{1}{3} =$
10. $\frac{4}{3} - \frac{3}{4} =$

11. A man sold $\frac{1}{2}$ of his sheep at one time and $\frac{1}{3}$ of them at another time. What part of his sheep did he sell?
12. From a piece of cloth containing $\frac{3}{4}$ yd., I used $\frac{1}{3}$ yd. What part of a yard was left?
13. Mr. Hale plowed $\frac{1}{3}$ of a field one day, and $\frac{1}{4}$ of it the next day. What part of the field did he plow?
14. Nora bought $\frac{3}{4}$ yd. of lawn and used $\frac{2}{3}$ yd. for an apron. How much lawn did she have left?

SEVENTHS, NINTHS, AND ELEVENTHS

One of the *seven equal* parts of anything is **one seventh**; one of the *nine equal* parts, **one ninth**; one of the *eleven equal* parts, **one eleventh**.

Oral and Written Exercises

Give answers promptly:

1. $\frac{3}{7} + \frac{1}{7}$

3. $\frac{6}{7} - \frac{2}{7}$

5. $\frac{4}{11} + \frac{3}{11}$

7. $\frac{9}{11} - \frac{4}{11}$

2. $\frac{5}{7} - \frac{2}{7}$

4. $\frac{4}{7} + \frac{3}{7}$

6. $\frac{7}{11} - \frac{2}{11}$

8. $\frac{6}{11} + \frac{5}{11}$

Looking at this line,

--	--	--	--	--	--	--

, complete:

9. $\frac{3}{9} = \frac{?}{9}$

10. $\frac{6}{9} = \frac{?}{9}$

11. $\frac{9}{9} = ?$

12. $\frac{1}{3} = \frac{?}{9}$

13. $\frac{2}{3} = \frac{?}{9}$

Find results:

14. $\frac{4}{9} + \frac{5}{9}$

17. $\frac{5}{9} - \frac{2}{9}$

20. $\frac{8}{9} - \frac{5}{9}$

23. $\frac{7}{9} - \frac{1}{3}$

26. $\frac{2}{3} - \frac{1}{9}$

15. $\frac{1}{9} + \frac{2}{9}$

18. $\frac{8}{9} + \frac{1}{9}$

21. $\frac{1}{3} - \frac{1}{9}$

24. $\frac{2}{9} + \frac{4}{9}$

27. $\frac{8}{9} - \frac{2}{3}$

16. $\frac{4}{9} - \frac{1}{9}$

19. $\frac{7}{9} - \frac{4}{9}$

22. $\frac{7}{9} + \frac{2}{9}$

25. $\frac{8}{9} - \frac{2}{9}$

28. $\frac{9}{9} - \frac{1}{3}$

PARTS OF NUMBERS

Just as $\frac{1}{2}$ of 6 means $6 \div 2$ and $\frac{1}{4}$ of 8 means $8 \div 4$, so $\frac{1}{3}$ of 9 means $9 \div 3$, $\frac{1}{5}$ of 10 means $10 \div 5$, and so on.

Oral and Written Exercises

Tell quickly the value of:

1. $\frac{1}{2}$ of 24

5. $\frac{1}{3}$ of 3

9. $\frac{1}{3}$ of 21

13. $\frac{1}{3}$ of 27

2. $\frac{1}{2}$ of 16

6. $\frac{1}{3}$ of 9

10. $\frac{1}{3}$ of 15

14. $\frac{1}{3}$ of 33

3. $\frac{1}{4}$ of 28

7. $\frac{1}{3}$ of 6

11. $\frac{1}{3}$ of 24

15. $\frac{1}{3}$ of 30

4. $\frac{1}{4}$ of 36

8. $\frac{1}{3}$ of 12

12. $\frac{1}{3}$ of 18

16. $\frac{1}{3}$ of 36

In the following, find the parts indicated :

$\frac{1}{5}$ of	$\frac{1}{6}$ of	$\frac{1}{7}$ of	$\frac{1}{8}$ of	$\frac{1}{9}$ of	$\frac{1}{10}$ of
17. 20	25. 24	33. 35	41. 32	49. 27	57. 40
18. 40	26. 30	34. 42	42. 40	50. 18	58. 80
19. 25	27. 48	35. 28	43. 64	51. 45	59. 50
20. 35	28. 36	36. 56	44. 48	52. 36	60. 90
21. 30	29. 60	37. 49	45. 80	53. 90	61. 70
22. 45	30. 42	38. 77	46. 96	54. 54	62. 110
23. 60	31. 54	39. 63	47. 72	55. 99	63. 100
24. 55	32. 72	40. 84	48. 88	56. 81	64. 120

65. Find $\frac{1}{11}$ of: 33; 55; 44; 66; 99; 121; 132.

66. Find $\frac{1}{12}$ of: 48; 36; 60; 96; 108; 120; 144.

67. An orchard contained 80 apple trees and $\frac{1}{10}$ as many pear trees. How many pear trees were there?

68. Helen picked 36 roses $\frac{1}{3}$ of which were white. How many white roses did she pick?

69. One Saturday Hiram saw 24 robins and $\frac{1}{6}$ as many bluebirds. How many bluebirds did he see?

70. Frank made a trip of 48 miles $\frac{1}{8}$ of which was by boat. How many miles did he travel by boat?

71. Elsie had 60¢ and Agnes $\frac{1}{5}$ as much. How much money did Agnes have?

72. I paid \$45 for a large rug and $\frac{1}{9}$ as much for a small one. How much did the small rug cost?

73. Mr. Myers had a farm of 96 acres $\frac{1}{12}$ of which was woodland. How many acres of woodland had he?

READING AND WRITING FRACTIONS

1. Any single thing is a **unit**.
2. One or more of the equal parts of a unit is a **fraction**.
3. The fraction $\frac{7}{8}$ stands for 7 of the 8 equal parts of 1.
In this fraction 7 is the **numerator**, and 8 the **denominator**; 7 and 8 are the **terms** of the fraction.

4. You learned on page 47 that $\frac{6}{2}$ means $6 \div 2$.

A fraction may be regarded as an *indicated division*, with the dividend above, and the divisor below, the line.

Thus, $\frac{7}{8}$ indicates $7 \div 8$.

5. The *numerator* of a fraction, then, may be thought of as a *dividend* and the *denominator* as a *divisor*.

The **value** of a fraction is the quotient of its numerator divided by its denominator.

Written Exercises

Read; then write the name of each fraction:

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|---------------------------------|-----------------------------------|
| 1. $\frac{1}{2}, \frac{2}{2}$ | 3. $\frac{1}{8}, \frac{2}{8}$ | 5. $\frac{1}{7}, \frac{4}{7}$ | 7. $\frac{1}{9}, \frac{5}{9}$ | 9. $\frac{1}{11}, \frac{6}{11}$ |
| 2. $\frac{1}{4}, \frac{3}{4}$ | 4. $\frac{1}{6}, \frac{5}{6}$ | 6. $\frac{1}{8}, \frac{7}{8}$ | 8. $\frac{1}{10}, \frac{7}{10}$ | 10. $\frac{1}{12}, \frac{11}{12}$ |

Notice how the name of the fraction is formed from the name of its numerator and of its denominator.

Write with figures:

- | | |
|----------------------|------------------------------|
| 11. Five twelfths. | 13. Eleven twenty-fourths. |
| 12. Nine sixteenths. | 14. Thirteen thirty-seconds. |

Write the name of each fraction and tell its meaning:

- | | | | | | |
|---------------------|--------------------|---------------------|---------------------|---------------------|---------------------|
| 15. $\frac{11}{20}$ | 16. $\frac{4}{15}$ | 17. $\frac{13}{24}$ | 18. $\frac{21}{32}$ | 19. $\frac{12}{25}$ | 20. $\frac{19}{36}$ |
|---------------------|--------------------|---------------------|---------------------|---------------------|---------------------|

PRINCIPLES OF FRACTIONS

The value of a fraction depends upon that of its numerator and its denominator. Consider the effect on the fraction of the following changes made in its terms :

1. $\frac{12 \times 2}{4} = \frac{24}{4}$ What is the value of $\frac{12}{4}$? of $\frac{24}{4}$? Multiplying the numerator by 2, *multiplies* the fraction by 2.

2. $\frac{12}{4 \div 2} = \frac{12}{2}$ What is the value of $\frac{12}{4}$? of $\frac{12}{2}$? Dividing the denominator by 2, *multiplies* the fraction by 2.

3. $\frac{12 \div 2}{6} = \frac{6}{6}$ What is the value of $\frac{12}{6}$? of $\frac{6}{6}$? Dividing the numerator by 2, *divides* the fraction by 2.

4. $\frac{12}{6 \times 2} = \frac{12}{12}$ What is the value of $\frac{12}{6}$? of $\frac{12}{12}$? Multiplying the denominator by 2, *divides* the fraction by 2.

5. $\frac{6 \times 2}{3 \times 2} = \frac{12}{6}$ What is the value of $\frac{6}{3}$? of $\frac{12}{6}$? Multiplying both numerator and denominator by 2, *does not change the value of the fraction*.

6. $\frac{12 \div 2}{6 \div 2} = \frac{6}{3}$ Dividing both numerator and denominator by 2, *does not change the value of the fraction*.

The foregoing illustrates the following *principles* :

1. *Multiplying the numerator or dividing the denominator of a fraction by any number, multiplies the fraction by that number.*

2. *Dividing the numerator or multiplying the denominator of a fraction by any number, divides the fraction by that number.*

3. *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

REDUCTION OF FRACTIONS

The process of changing the form of fractions without changing their value is **reduction of fractions**.

Thus, when we change $\frac{1}{8}$ to $3\frac{1}{8}$, $2\frac{1}{2}$ to $\frac{5}{2}$, $\frac{1}{3}$ to $\frac{2}{3}$, $\frac{3}{8}$ to $\frac{1}{2}$, or 1 to $\frac{5}{5}$, we perform a reduction.

Reduction of fractions to higher terms.

1. You have learned that $\frac{1}{2} = \frac{4}{8}$. Which fraction has the larger, or *higher*, terms?

Changing $\frac{1}{2}$ to the *equal* fraction $\frac{4}{8}$ is *reducing* $\frac{1}{2}$ to *higher* terms.

2. Multiply both terms of $\frac{1}{2}$ by 4. To what fraction does this change $\frac{1}{2}$? How may $\frac{1}{2}$ be reduced to eighths?

3. To reduce $\frac{1}{2}$ to *sixths* you must multiply both terms by the quotient $6 \div 2$, or 3. Reduce $\frac{1}{2}$ to sixths.

How may $\frac{1}{4}$ be reduced to eighths? $\frac{4}{5}$ to tenths?

Oral and Written Exercises

1. Reduce to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$.
2. Reduce to sixteenths: $\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{3}{8}$.
3. Reduce to eighteenths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{9}$; $\frac{5}{6}$; $\frac{4}{9}$.
4. Reduce to twentieths: $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{10}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{9}{10}$.
5. Reduce to twenty-fourths: $\frac{1}{6}$; $\frac{1}{8}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{5}{12}$.

Reduce:

- | | | |
|---------------------------|----------------------------|-----------------------------|
| 6. $\frac{1}{5}$ to 15ths | 10. $\frac{5}{6}$ to 18ths | 14. $\frac{1}{12}$ to 24ths |
| 7. $\frac{2}{9}$ to 18ths | 11. $\frac{7}{8}$ to 16ths | 15. $\frac{7}{10}$ to 20ths |
| 8. $\frac{4}{5}$ to 20ths | 12. $\frac{3}{5}$ to 20ths | 16. $\frac{7}{12}$ to 24ths |
| 9. $\frac{3}{8}$ to 24ths | 13. $\frac{5}{8}$ to 24ths | 17. $\frac{1}{12}$ to 24ths |

Reduction of fractions to lower terms.

1. You have learned that $\frac{4}{10} = \frac{2}{5}$.

Changing $\frac{4}{10}$ to $\frac{2}{5}$ is *reducing* $\frac{4}{10}$ to *lower terms*.

2. What number will exactly divide both terms of $\frac{4}{10}$?
What fraction is obtained by dividing the terms by 2?

Then, how may $\frac{4}{10}$ be reduced to lower terms?

3. Can you reduce $\frac{2}{5}$ to lower terms?

Then, $\frac{4}{10}$ reduced to *lowest terms* is $\frac{2}{5}$.

4. Reduce to lowest terms: $\frac{2}{4}$; $\frac{6}{8}$; $\frac{3}{6}$; $\frac{6}{9}$; $\frac{2}{8}$.

A fraction is expressed in its **lowest terms** when its terms have no *common divisor* except 1.

Oral and Written Exercises

Quickly express each fraction in its lowest terms:

- | | | | | | |
|------------------|------------------|-------------------|--------------------|--------------------|---------------------|
| 1. $\frac{2}{6}$ | 4. $\frac{4}{8}$ | 7. $\frac{2}{10}$ | 10. $\frac{2}{12}$ | 13. $\frac{4}{12}$ | 16. $\frac{10}{12}$ |
| 2. $\frac{2}{8}$ | 5. $\frac{4}{6}$ | 8. $\frac{3}{12}$ | 11. $\frac{4}{10}$ | 14. $\frac{6}{10}$ | 17. $\frac{8}{10}$ |
| 3. $\frac{3}{9}$ | 6. $\frac{6}{8}$ | 9. $\frac{5}{10}$ | 12. $\frac{6}{12}$ | 15. $\frac{8}{12}$ | 18. $\frac{9}{12}$ |

Reduce to lowest terms:

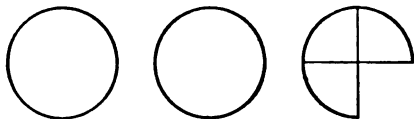
- | | | | | | |
|---------------------|---------------------|--------------------|---------------------|---------------------|---------------------|
| 19. $\frac{2}{16}$ | 26. $\frac{8}{16}$ | 33. $\frac{3}{24}$ | 40. $\frac{10}{15}$ | 47. $\frac{10}{16}$ | 54. $\frac{12}{16}$ |
| 20. $\frac{7}{14}$ | 27. $\frac{5}{20}$ | 34. $\frac{4}{20}$ | 41. $\frac{8}{24}$ | 48. $\frac{15}{20}$ | 55. $\frac{12}{20}$ |
| 21. $\frac{9}{18}$ | 28. $\frac{2}{24}$ | 35. $\frac{3}{18}$ | 42. $\frac{11}{22}$ | 49. $\frac{12}{18}$ | 56. $\frac{16}{24}$ |
| 22. $\frac{4}{16}$ | 29. $\frac{5}{15}$ | 36. $\frac{4}{24}$ | 43. $\frac{12}{24}$ | 50. $\frac{9}{24}$ | 57. $\frac{12}{15}$ |
| 23. $\frac{3}{15}$ | 30. $\frac{10}{20}$ | 37. $\frac{5}{25}$ | 44. $\frac{6}{20}$ | 51. $\frac{15}{18}$ | 58. $\frac{16}{20}$ |
| 24. $\frac{20}{20}$ | 31. $\frac{6}{18}$ | 38. $\frac{6}{24}$ | 45. $\frac{9}{15}$ | 52. $\frac{15}{24}$ | 59. $\frac{20}{24}$ |
| 25. $\frac{6}{15}$ | 32. $\frac{6}{16}$ | 39. $\frac{8}{20}$ | 46. $\frac{14}{16}$ | 53. $\frac{18}{20}$ | 60. $\frac{18}{24}$ |

Reduction of integers and mixed numbers to fractions.

How many *whole* circles do you see here? how many fourths of a circle over?

Two and three fourths
is written $2\frac{3}{4}$.

This means $2 + \frac{3}{4}$.



A number that stands for one or more *whole* units is a **whole number**, or an **integer**.

A number expressed by an integer and a fraction is a **mixed number**.

Oral and Written Exercises

1. How many half dollars are there in 1 dollar? How many times 2 half dollars are there in 3 dollars?
2. How many halves are there in 1? in 3? in 4?
3. How many halves are there in $1\frac{1}{2}$? in $3\frac{1}{2}$? in $4\frac{1}{2}$?
4. How many fourths are there in 1? in 2? in $2\frac{3}{4}$?
5. Change 2 to halves; 4 to thirds; 3 to fourths.
6. Reduce $2\frac{1}{2}$ to halves; $4\frac{2}{3}$ to thirds; $3\frac{4}{5}$ to fifths.

Reduce each of these integers to a fraction whose denominator is 2; 3; 4; 5; 6; 8:

7. 4 8. 7 9. 5 10. 8 11. 6 12. 9 13. 10

Reduce to a fraction:

- | | | | | | |
|--------------------|--------------------|--------------------|---------------------|---------------------|---------------------|
| 14. $7\frac{1}{2}$ | 18. $3\frac{1}{6}$ | 22. $5\frac{3}{8}$ | 26. $7\frac{1}{10}$ | 30. $10\frac{1}{4}$ | 34. $10\frac{4}{5}$ |
| 15. $5\frac{1}{3}$ | 19. $9\frac{1}{4}$ | 23. $7\frac{5}{6}$ | 27. $3\frac{7}{12}$ | 31. $11\frac{1}{2}$ | 35. $12\frac{3}{4}$ |
| 16. $4\frac{2}{5}$ | 20. $8\frac{2}{3}$ | 24. $6\frac{7}{8}$ | 28. $4\frac{3}{10}$ | 32. $10\frac{1}{3}$ | 36. $11\frac{3}{5}$ |
| 17. $2\frac{5}{8}$ | 21. $6\frac{3}{4}$ | 25. $9\frac{4}{5}$ | 29. $8\frac{5}{12}$ | 33. $12\frac{3}{4}$ | 37. $12\frac{7}{8}$ |

Reduction of improper fractions to integers or mixed numbers.

1. Which is less, the numerator of $\frac{3}{4}$ or the denominator? the numerator or the denominator of $\frac{4}{5}$? of $\frac{3}{8}$?

A fraction whose numerator is less than its denominator is a **proper fraction**.

2. Which is less, $\frac{3}{4}$ or 1? $\frac{4}{5}$ or 1? $\frac{3}{8}$ or 1? $\frac{5}{6}$ or 1?

A *proper fraction* is less than 1.

3. How does the numerator compare with the denominator in $\frac{3}{2}$? in $\frac{3}{2}$? in $\frac{4}{3}$? in $\frac{5}{4}$? in $\frac{6}{5}$?

A fraction whose numerator is equal to or greater than its denominator is an **improper fraction**.

4. How does $\frac{2}{2}$ compare with 1? $\frac{3}{2}$ with 1? $\frac{4}{3}$ with 1?

An *improper fraction* is equal to or greater than 1.

Oral and Written Exercises

1. To how many dollars are 4 quarter dollars equal? 8 quarters? \$ $\frac{4}{4}$? \$ $\frac{8}{4}$? \$ $\frac{12}{4}$? \$ $\frac{16}{4}$?

2. How many ones and how many thirds over are $\frac{7}{3}$? $\frac{10}{3}$? $\frac{14}{3}$? $\frac{16}{3}$? $\frac{20}{3}$?

3. Change to an integer: $\frac{6}{3}$; $\frac{10}{2}$; $\frac{15}{5}$; $\frac{12}{3}$; $\frac{20}{4}$; $\frac{25}{5}$.

4. Reduce to a mixed number: $\frac{7}{2}$; $\frac{10}{3}$; $\frac{13}{4}$; $\frac{14}{5}$; $\frac{17}{6}$; $\frac{21}{8}$.

5. What is the value of $\frac{4}{2}$? $\frac{5}{2}$? $\frac{9}{3}$? $\frac{8}{3}$? $\frac{12}{6}$? $\frac{15}{4}$?

Reduce to an integer or a mixed number:

6. $\frac{7}{3}$ 9. $\frac{18}{6}$ 12. $\frac{32}{4}$ 15. $\frac{35}{6}$ 18. $\frac{45}{8}$ 21. $\frac{28}{5}$

7. $\frac{8}{2}$ 10. $\frac{24}{5}$ 13. $\frac{37}{10}$ 16. $\frac{48}{8}$ 19. $\frac{54}{5}$ 22. $\frac{69}{10}$

8. $\frac{9}{4}$ 11. $\frac{27}{8}$ 14. $\frac{41}{6}$ 17. $\frac{17}{2}$ 20. $\frac{36}{4}$ 23. $\frac{72}{12}$

Reduction of fractions to least common denominator.

1. Change $\frac{1}{2}$ and $\frac{2}{3}$ each to 6ths; to 12ths; to some other *common* denominator. Which is the smallest, or *least*?

2. What is the least number that will exactly contain the denominator of each of the fractions $\frac{1}{3}$ and $\frac{3}{4}$?

Reduce $\frac{1}{3}$ and $\frac{3}{4}$ each to twelfths.

Fractions that have the same denominator are said to have a **common denominator**.

The smallest *common* denominator to which two or more fractions can be reduced is their **least common denominator** (l. c. d.).

Oral and Written Exercises

1. Reduce $\frac{1}{4}$ and $\frac{1}{8}$ to eighths; $\frac{1}{2}$ and $\frac{3}{4}$ to fourths.

Reduce:

2. $\frac{1}{2}, \frac{1}{4}$ to 4ths 5. $\frac{1}{4}, \frac{1}{3}$ to 12ths 8. $\frac{1}{4}, \frac{1}{5}$ to 20ths

3. $\frac{1}{3}, \frac{1}{6}$ to 6ths 6. $\frac{1}{2}, \frac{2}{5}$ to 10ths 9. $\frac{2}{3}, \frac{2}{5}$ to 15ths

4. $\frac{1}{2}, \frac{1}{3}$ to 6ths 7. $\frac{2}{3}, \frac{3}{4}$ to 12ths 10. $\frac{3}{4}, \frac{3}{5}$ to 20ths

Reduce to fractions having the l. c. d.:

SUGGESTION. — For the l. c. d., take the *least* multiple of the *largest* denominator that will exactly contain each of the others.

11. $\frac{1}{2}$ and $\frac{1}{6}$ 17. $\frac{1}{2}$ and $\frac{1}{10}$ 23. $\frac{2}{3}$ and $\frac{2}{5}$ 29. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$

12. $\frac{3}{8}$ and $\frac{1}{2}$ 18. $\frac{1}{4}$ and $\frac{1}{12}$ 24. $\frac{1}{2}$ and $\frac{1}{7}$ 30. $\frac{1}{3}, \frac{3}{4}, \frac{5}{12}$

13. $\frac{2}{3}$ and $\frac{5}{6}$ 19. $\frac{1}{2}$ and $\frac{5}{12}$ 25. $\frac{3}{5}$ and $\frac{1}{4}$ 31. $\frac{2}{5}, \frac{1}{2}, \frac{9}{10}$

14. $\frac{1}{4}$ and $\frac{1}{8}$ 20. $\frac{2}{5}$ and $\frac{3}{10}$ 26. $\frac{1}{4}$ and $\frac{1}{6}$ 32. $\frac{1}{2}, \frac{2}{3}, \frac{7}{12}$

15. $\frac{1}{3}$ and $\frac{4}{9}$ 21. $\frac{2}{3}$ and $\frac{7}{12}$ 27. $\frac{2}{3}$ and $\frac{3}{4}$ 33. $\frac{1}{2}, \frac{5}{6}, \frac{1}{12}$

16. $\frac{3}{8}$ and $\frac{3}{4}$ 22. $\frac{3}{4}$ and $\frac{11}{12}$ 28. $\frac{2}{3}$ and $\frac{5}{8}$ 34. $\frac{2}{3}, \frac{1}{9}, \frac{7}{18}$

ADDITION AND SUBTRACTION OF FRACTIONS

1. How many fifths are $\frac{3}{5}$ and $\frac{2}{5}$? how many ones?
2. How many fourths are $\frac{3}{4} - \frac{1}{4}$? how many halves?
3. How many eighths are $\frac{1}{2} + \frac{3}{8}$? $\frac{1}{2} - \frac{3}{8}$? $\frac{5}{8} + \frac{1}{4}$? $\frac{5}{8} - \frac{1}{4}$?

What must be done to fractions that have different denominators before they can be added or subtracted?

4. How many eighths are $\frac{3}{8}$ and $\frac{7}{8}$? how many ones and how many eighths over? how many fourths over?

$$\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}.$$

5. Add $\frac{3}{4}$ and $\frac{1}{2}$; $\frac{5}{6}$ and $\frac{1}{3}$. Subtract $\frac{1}{2}$ from $\frac{3}{4}$; $\frac{1}{6}$ from $\frac{1}{4}$.

Fractions must have a common denominator before they can be added or subtracted.

Oral and Written Exercises

In fractions, unless otherwise specified, express the answer as an integer, a proper fraction in its lowest terms, or as a mixed number with the fraction in its lowest terms, as the case may be.

Find the sum of:

- | | | | | |
|-------------------------------|--------------------------------|-----------------------------------|--------------------------------|---------------------------------|
| 1. $\frac{2}{3}, \frac{1}{3}$ | 9. $\frac{3}{5}, \frac{2}{5}$ | 17. $\frac{3}{10}, \frac{1}{10}$ | 25. $\frac{1}{4}, \frac{1}{8}$ | 33. $\frac{3}{4}, \frac{1}{2}$ |
| 2. $\frac{2}{5}, \frac{1}{5}$ | 10. $\frac{5}{8}, \frac{3}{8}$ | 18. $\frac{7}{10}, \frac{3}{10}$ | 26. $\frac{3}{4}, \frac{1}{8}$ | 34. $\frac{5}{6}, \frac{1}{2}$ |
| 3. $\frac{3}{4}, \frac{1}{4}$ | 11. $\frac{4}{5}, \frac{2}{5}$ | 19. $\frac{9}{10}, \frac{1}{10}$ | 27. $\frac{2}{3}, \frac{1}{6}$ | 35. $\frac{1}{3}, \frac{1}{6}$ |
| 4. $\frac{3}{5}, \frac{1}{5}$ | 12. $\frac{5}{6}, \frac{1}{6}$ | 20. $\frac{7}{10}, \frac{1}{10}$ | 28. $\frac{1}{2}, \frac{1}{8}$ | 36. $\frac{5}{6}, \frac{2}{3}$ |
| 5. $\frac{5}{8}, \frac{1}{8}$ | 13. $\frac{4}{5}, \frac{3}{5}$ | 21. $\frac{9}{10}, \frac{3}{10}$ | 29. $\frac{3}{8}, \frac{1}{4}$ | 37. $\frac{2}{5}, \frac{1}{10}$ |
| 6. $\frac{4}{5}, \frac{1}{5}$ | 14. $\frac{7}{8}, \frac{3}{8}$ | 22. $\frac{9}{10}, \frac{7}{10}$ | 30. $\frac{3}{4}, \frac{3}{8}$ | 38. $\frac{1}{2}, \frac{1}{10}$ |
| 7. $\frac{3}{8}, \frac{1}{8}$ | 15. $\frac{7}{8}, \frac{5}{8}$ | 23. $\frac{7}{12}, \frac{5}{12}$ | 31. $\frac{1}{2}, \frac{3}{8}$ | 39. $\frac{4}{5}, \frac{3}{10}$ |
| 8. $\frac{7}{8}, \frac{1}{8}$ | 16. $\frac{8}{9}, \frac{2}{9}$ | 24. $\frac{11}{12}, \frac{7}{12}$ | 32. $\frac{5}{6}, \frac{1}{3}$ | 40. $\frac{7}{10}, \frac{1}{2}$ |

41-80. Find the difference in exercises 1-40.

Written Exercises

1. Find the sum and the difference of
- $\frac{7}{8}$
- and
- $\frac{11}{24}$
- .

$$\frac{7}{8} + \frac{11}{24} = \frac{21}{24} + \frac{11}{24} = \frac{32}{24} = 1\frac{8}{24} = 1\frac{1}{3}, \text{ the sum.}$$

$$\frac{7}{8} - \frac{11}{24} = \frac{21}{24} - \frac{11}{24} = \frac{10}{24} = \frac{5}{12}, \text{ the difference.}$$

You have learned (p. 169) that the fractions must be reduced to fractions having a *common* denominator before they can be added or subtracted. It is best to reduce them to their l.c.d., which in this case is 24; then, $\frac{7}{8} = \frac{21}{24}$ and $\frac{11}{24} = \frac{11}{24}$; the sum of these fractions is $1\frac{1}{3}$, and their difference is $\frac{5}{12}$.

Find the sum of:

2. $\frac{1}{2}, \frac{7}{12}$ 7. $\frac{1}{2}, \frac{7}{16}$ 12. $\frac{1}{4}, \frac{9}{20}$ 17. $\frac{1}{2}, \frac{2}{3}$ 22. $\frac{3}{4}, \frac{1}{6}$

3. $\frac{1}{3}, \frac{5}{12}$ 8. $\frac{5}{6}, \frac{5}{12}$ 13. $\frac{2}{5}, \frac{14}{15}$ 18. $\frac{1}{3}, \frac{3}{4}$ 23. $\frac{5}{6}, \frac{1}{8}$

4. $\frac{1}{2}, \frac{11}{12}$ 9. $\frac{1}{4}, \frac{9}{16}$ 14. $\frac{5}{6}, \frac{7}{24}$ 19. $\frac{1}{4}, \frac{1}{5}$ 24. $\frac{1}{9}, \frac{5}{6}$

5. $\frac{1}{6}, \frac{7}{12}$ 10. $\frac{3}{4}, \frac{11}{12}$ 15. $\frac{4}{5}, \frac{9}{20}$ 20. $\frac{2}{3}, \frac{3}{4}$ 25. $\frac{5}{6}, \frac{3}{10}$

6. $\frac{2}{3}, \frac{7}{12}$ 11. $\frac{2}{3}, \frac{14}{15}$ 16. $\frac{5}{8}, \frac{13}{24}$ 21. $\frac{3}{4}, \frac{3}{5}$ 26. $\frac{7}{8}, \frac{5}{12}$

27. How many quarts of sirup are
- $\frac{1}{2}$
- qt. and
- $\frac{5}{8}$
- qt.?

Find the difference of:

28. $\frac{1}{4}, \frac{1}{12}$ 33. $\frac{1}{2}, \frac{3}{16}$ 38. $\frac{11}{12}, \frac{1}{6}$ 43. $\frac{1}{2}, \frac{1}{3}$ 48. $\frac{3}{4}, \frac{1}{6}$

29. $\frac{1}{3}, \frac{1}{12}$ 34. $\frac{1}{4}, \frac{1}{16}$ 39. $\frac{15}{16}, \frac{3}{4}$ 44. $\frac{1}{3}, \frac{1}{4}$ 49. $\frac{7}{8}, \frac{5}{6}$

30. $\frac{3}{4}, \frac{5}{12}$ 35. $\frac{5}{6}, \frac{7}{12}$ 40. $\frac{17}{18}, \frac{5}{6}$ 45. $\frac{2}{3}, \frac{1}{2}$ 50. $\frac{5}{6}, \frac{4}{9}$

31. $\frac{5}{6}, \frac{1}{12}$ 36. $\frac{3}{4}, \frac{5}{16}$ 41. $\frac{19}{20}, \frac{1}{5}$ 46. $\frac{3}{4}, \frac{2}{5}$ 51. $\frac{5}{12}, \frac{1}{8}$

32. $\frac{2}{3}, \frac{5}{12}$ 37. $\frac{3}{5}, \frac{4}{15}$ 42. $\frac{17}{24}, \frac{3}{8}$ 47. $\frac{4}{5}, \frac{1}{3}$ 52. $\frac{9}{10}, \frac{1}{6}$

53. From
- $\frac{5}{6}$
- yd. of lace
- $\frac{1}{3}$
- yd. is cut. How much is left?

Find results:

54. $\frac{5}{8} + \frac{1}{4} + \frac{1}{2}$

56. $\frac{1}{2} + \frac{4}{5} + \frac{9}{10}$

58. $\frac{3}{8} + \frac{1}{6} - \frac{5}{12}$

55. $\frac{2}{3} + \frac{1}{2} + \frac{1}{6}$

57. $\frac{5}{6} + \frac{1}{3} + \frac{7}{12}$

59. $\frac{2}{3} - \frac{2}{5} + \frac{14}{15}$

FOURTH YEAR—SECOND HALF

DIVISORS AND MULTIPLES

GREATEST COMMON DIVISOR

A number that is an exact divisor of two or more numbers is a **common divisor** or a **common factor** of those numbers.

The *largest* number that is an exact divisor of two or more numbers is their **greatest common divisor** (g. c. d.).

Oral Exercises

Tell by inspection the greatest common divisor of:

- | | | | |
|------------|----------|------------|--------------|
| 1. 3 and 6 | 5. 6, 12 | 9. 10, 20 | 13. 2, 4, 8 |
| 2. 4 and 8 | 6. 8, 12 | 10. 10, 15 | 14. 3, 6, 9 |
| 3. 4 and 6 | 7. 4, 10 | 11. 12, 15 | 15. 4, 8, 10 |
| 4. 6 and 9 | 8. 9, 18 | 12. 15, 18 | 16. 4, 8, 16 |
17. What is the greatest number of boys among whom I can divide 9 apples and 12 pears equally?

Written Exercises

1. Find the greatest common divisor of 18 and 30.

$18 = 2 \times 3 \times 3$	By factoring 18 and 30, as on p. 98,
$30 = 2 \times 3 \times 5$	it is seen that the <i>common</i> divisors of
g. c. d. = $2 \times 3 = 6$	these numbers are 2, 3, and 2×3 , or 6;
	hence, their <i>greatest</i> common divisor is 6.

Find the product of all the common prime factors.

Find the greatest common divisor of:

- | | | | |
|--------------|------------|------------|---------------|
| 2. 12 and 16 | 8. 14, 35 | 14. 27, 36 | 20. 5, 15, 20 |
| 3. 16 and 20 | 9. 16, 40 | 15. 30, 48 | 21. 8, 16, 24 |
| 4. 12 and 18 | 10. 25, 35 | 16. 25, 45 | 22. 6, 18, 27 |
| 5. 15 and 25 | 11. 24, 32 | 17. 24, 60 | 23. 6, 24, 30 |
| 6. 20 and 30 | 12. 18, 30 | 18. 32, 48 | 24. 8, 24, 32 |
| 7. 18 and 27 | 13. 35, 42 | 19. 36, 54 | 25. 9, 27, 45 |

26. In a parade 32 pupils from the 4th grade, 36 from the 3d, and 40 from the 2d marched by grades. Find the greatest number that could march abreast, having all rows full.

LEAST COMMON MULTIPLE

A number that is divisible by another number is a **multiple** of that number.

A number that is divisible by each of several numbers is a **common multiple** of those numbers.

The *least* number that is divisible by each of several numbers is their **least common multiple** (l. c. m.).

Oral Exercises

Tell by inspection the least common multiple of:

- | | | | |
|------------|----------|-----------|-------------|
| 1. 2 and 3 | 7. 2, 8 | 13. 4, 12 | 19. 2, 4, 8 |
| 2. 3 and 6 | 8. 3, 7 | 14. 6, 12 | 20. 3, 6, 9 |
| 3. 2 and 5 | 9. 4, 8 | 15. 2, 10 | 21. 2, 4, 6 |
| 4. 6 and 2 | 10. 3, 9 | 16. 4, 10 | 22. 4, 3, 2 |
| 5. 4 and 6 | 11. 8, 6 | 17. 8, 12 | 23. 6, 2, 8 |
| 6. 3 and 5 | 12. 9, 6 | 18. 9, 12 | 24. 4, 5, 2 |

Written Exercises

1. Find the least common multiple of 4, 6, and 14.

$$4 = 2 \times 2$$

$$6 = 2 \times 3$$

$$14 = 2 \times 7$$

$$\text{l. c. m.} = 2 \times 2 \times 3 \times 7 = 84$$

The l. c. m. must contain each of the numbers 4, 6, and 14; that is, it must contain each of the *different* prime factors of these numbers and no other factors.

Then, the factors of the l. c. m. are 2, 2 (the greatest number of 2's found in any of the numbers), 3, and 7 (the only factors of any of them not already taken).

Hence, the l. c. m. of 4, 6, and $14 = 2 \times 2 \times 3 \times 7$, or 84.

Find the product of all the different prime factors, each used the greatest number of times it occurs in any of the numbers.

Find the least common multiple of:

- | | | |
|--------------|---------------|------------------|
| 2. 9 and 18 | 12. 36 and 12 | 22. 4, 6, and 12 |
| 3. 6 and 10 | 13. 20 and 30 | 23. 2, 8, and 16 |
| 4. 4 and 18 | 14. 10 and 15 | 24. 6, 9, and 18 |
| 5. 6 and 15 | 15. 12 and 18 | 25. 5, 4, and 10 |
| 6. 8 and 10 | 16. 16 and 12 | 26. 6, 8, and 12 |
| 7. 6 and 14 | 17. 15 and 20 | 27. 2, 3, and 10 |
| 8. 9 and 15 | 18. 20 and 50 | 28. 4, 9, and 12 |
| 9. 6 and 16 | 19. 16 and 40 | 29. 5, 6, and 15 |
| 10. 8 and 14 | 20. 24 and 16 | 30. 4, 6, and 18 |
| 11. 9 and 21 | 21. 18 and 24 | 31. 6, 4, and 10 |

Find the g. c. d. and the l. c. m. of:

- | | | |
|--------------|---------------|------------------|
| 32. 6 and 24 | 34. 30 and 45 | 36. 4, 8, and 12 |
| 33. 8 and 36 | 35. 48 and 72 | 37. 6, 9, and 36 |

FRACTIONS

REDUCTION TO HIGHER OR LOWER TERMS

1. Change to tenths: $\frac{1}{2}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{2}{20}$; $\frac{3}{30}$.
2. Change to twelfths: $\frac{1}{4}$; $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{2}{24}$; $\frac{6}{36}$.
3. Reduce to lowest terms: $\frac{6}{8}$; $\frac{4}{6}$; $\frac{6}{9}$; $\frac{9}{12}$; $\frac{10}{15}$; $\frac{12}{20}$; $\frac{8}{24}$.

Written Exercises

1. Change $\frac{3}{8}$ to thirty-seconds.

$$\frac{3 \times 4}{8 \times 4} = \frac{12}{32}$$

To produce 32, the denominator 8 must be multiplied by $32 \div 8$, or 4.

Then, (p. 163, 3) we multiply *both* terms by 4.

2. Change to fortieths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{5}{8}$; $\frac{3}{10}$.
3. Change to forty-eighths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{8}$; $\frac{7}{12}$; $\frac{9}{16}$.
4. Reduce $\frac{18}{30}$ to its lowest terms.

$$\frac{2)18}{2)30} = \frac{9}{15}$$

We first divide *both* terms of $\frac{18}{30}$ by 2 (p. 163, 3); the result is $\frac{9}{15}$.

$$\frac{3)9}{3)15} = \frac{3}{5}$$

Then, we divide *both* terms of $\frac{9}{15}$ by 3; the result is $\frac{3}{5}$.

$$\frac{6)18}{6)30} = \frac{3}{5}$$

Or, we may directly divide *both* terms of the fraction $\frac{18}{30}$ by their *greatest common divisor* (p. 171); the result is $\frac{3}{5}$.

Reduce to lowest terms:

5. $\frac{10}{30}$	10. $\frac{20}{40}$	15. $\frac{25}{30}$	20. $\frac{15}{25}$	25. $\frac{24}{30}$	30. $\frac{24}{40}$
6. $\frac{8}{32}$	11. $\frac{10}{25}$	16. $\frac{12}{32}$	21. $\frac{12}{18}$	26. $\frac{25}{60}$	31. $\frac{27}{36}$
7. $\frac{7}{28}$	12. $\frac{18}{24}$	17. $\frac{18}{36}$	22. $\frac{40}{60}$	27. $\frac{16}{48}$	32. $\frac{24}{60}$
8. $\frac{8}{40}$	13. $\frac{12}{30}$	18. $\frac{15}{40}$	23. $\frac{16}{40}$	28. $\frac{24}{32}$	33. $\frac{18}{45}$
9. $\frac{20}{30}$	14. $\frac{14}{28}$	19. $\frac{16}{32}$	24. $\frac{24}{48}$	29. $\frac{14}{42}$	34. $\frac{48}{60}$

REDUCTION TO LEAST COMMON DENOMINATOR

Written Exercises

1. Reduce $\frac{5}{8}$ and $\frac{7}{12}$ to fractions having the least common denominator.

The l. c. d. of two or more fractions is the same as the l. c. m. of their denominators, *when the fractions are given in lowest terms.*

It is readily seen that 24 is the least number that is divisible by both 8 and 12; that is, 24 is the l. c. m. of 8 and 12.

Then, reduce each of the given fractions to 24ths as on page 174. Since $24 \div 8 = 3$, both terms of $\frac{5}{8}$ are multiplied by 3, giving $\frac{15}{24}$, and since $24 \div 12 = 2$, both terms of $\frac{7}{12}$ are multiplied by 2, giving $\frac{14}{24}$.

When the *least common multiple* of the given denominators cannot readily be discovered by inspection, it may be found as on page 173, and the fractions may then be reduced to fractions having the least common denominator as shown above.

Reduce to fractions having the l. c. d.:

- | | | | |
|------------------------------------|----------------------------------|-----------------------------------|---|
| 2. $\frac{3}{4}$ and $\frac{4}{5}$ | 10. $\frac{3}{8}, \frac{5}{16}$ | 18. $\frac{1}{12}, \frac{5}{36}$ | 26. $\frac{1}{2}, \frac{3}{4}, \frac{1}{6}$ |
| 3. $\frac{3}{5}$ and $\frac{5}{8}$ | 11. $\frac{4}{5}, \frac{5}{12}$ | 19. $\frac{7}{10}, \frac{3}{20}$ | 27. $\frac{1}{4}, \frac{5}{8}, \frac{7}{16}$ |
| 4. $\frac{3}{4}$ and $\frac{2}{9}$ | 12. $\frac{3}{4}, \frac{9}{10}$ | 20. $\frac{3}{20}, \frac{1}{40}$ | 28. $\frac{4}{5}, \frac{2}{3}, \frac{8}{15}$ |
| 5. $\frac{2}{3}$ and $\frac{7}{8}$ | 13. $\frac{3}{8}, \frac{11}{12}$ | 21. $\frac{3}{10}, \frac{7}{15}$ | 29. $\frac{3}{4}, \frac{1}{8}, \frac{1}{12}$ |
| 6. $\frac{3}{4}$ and $\frac{5}{6}$ | 14. $\frac{5}{6}, \frac{8}{15}$ | 22. $\frac{7}{20}, \frac{11}{30}$ | 30. $\frac{1}{5}, \frac{2}{3}, \frac{9}{10}$ |
| 7. $\frac{5}{6}$ and $\frac{2}{5}$ | 15. $\frac{2}{9}, \frac{7}{12}$ | 23. $\frac{5}{12}, \frac{3}{16}$ | 31. $\frac{2}{5}, \frac{1}{8}, \frac{11}{20}$ |
| 8. $\frac{3}{8}$ and $\frac{1}{6}$ | 16. $\frac{1}{6}, \frac{5}{16}$ | 24. $\frac{4}{15}, \frac{9}{20}$ | 32. $\frac{7}{8}, \frac{7}{32}, \frac{9}{16}$ |
| 9. $\frac{5}{6}$ and $\frac{5}{9}$ | 17. $\frac{1}{9}, \frac{8}{15}$ | 25. $\frac{5}{16}, \frac{7}{24}$ | 33. $\frac{3}{4}, \frac{1}{12}, \frac{5}{24}$ |

ADDITION AND SUBTRACTION OF FRACTIONS

Oral Exercises

1. When two fractions have the *same* denominator, how do you find their sum? their difference?

How should the answer be expressed?

2. What do you do to fractions having *different* denominators before you can find their sum or their difference?

Add or subtract as the signs indicate:

3. $\frac{1}{4} + \frac{3}{8}$

7. $\frac{1}{2} + \frac{3}{4}$

11. $\frac{5}{6} - \frac{1}{3}$

15. $\frac{1}{5} - \frac{1}{10}$

4. $\frac{1}{2} - \frac{1}{4}$

8. $\frac{1}{3} - \frac{1}{6}$

12. $\frac{3}{4} + \frac{3}{8}$

16. $\frac{1}{2} - \frac{3}{10}$

5. $\frac{3}{8} + \frac{1}{2}$

9. $\frac{1}{6} + \frac{2}{3}$

13. $\frac{2}{3} + \frac{5}{6}$

17. $\frac{1}{2} + \frac{7}{10}$

6. $\frac{3}{4} - \frac{1}{8}$

10. $\frac{7}{8} - \frac{1}{4}$

14. $\frac{5}{8} + \frac{1}{2}$

18. $\frac{4}{5} - \frac{3}{10}$

19. Find the sum of $\frac{1}{2}$ gal. and $\frac{7}{8}$ gal.

20. If I bought $\frac{1}{2}$ lb. of one kind of tea and $\frac{3}{4}$ lb. of another kind, how much tea did I buy?

21. In a grove $\frac{5}{8}$ of the trees were maple and $\frac{1}{4}$ beech. What part of the trees were of these kinds?

22. I sold A $\frac{3}{5}$ of a ton of hay and B $\frac{1}{10}$ of a ton. How much more did I sell A than B?

23. In one field there is $\frac{3}{4}$ of an acre and in another $\frac{5}{8}$ of an acre. How many acres are there in both fields?

24. From a pound of sugar, $\frac{1}{4}$ lb. was used for coffee and $\frac{1}{2}$ lb. for a pudding. What part of a pound was left?

25. Ralph paid $\frac{1}{3}$ of his money for a knife and $\frac{1}{4}$ for some pencils. What part of his money did he spend?

26. David earned \$ $\frac{1}{2}$ one day and \$ $\frac{2}{5}$ the next. How much did he earn in both days?

Written Exercises

Perform the operations indicated:

- | | | | |
|--------------------------------|----------------------------------|-------------------------------------|---|
| 1. $\frac{1}{2} + \frac{2}{3}$ | 10. $\frac{1}{4} + \frac{3}{10}$ | 19. $\frac{9}{10} - \frac{3}{20}$ | 28. $\frac{1}{3} + \frac{3}{4} - \frac{5}{12}$ |
| 2. $\frac{3}{4} - \frac{1}{3}$ | 11. $\frac{8}{9} - \frac{7}{12}$ | 20. $\frac{7}{12} + \frac{7}{36}$ | 29. $\frac{1}{4} + \frac{5}{6} - \frac{7}{12}$ |
| 3. $\frac{2}{3} + \frac{3}{4}$ | 12. $\frac{1}{6} + \frac{1}{16}$ | 21. $\frac{17}{20} - \frac{9}{40}$ | 30. $\frac{3}{4} + \frac{7}{8} + \frac{1}{16}$ |
| 4. $\frac{3}{5} - \frac{1}{4}$ | 13. $\frac{5}{8} - \frac{1}{10}$ | 22. $\frac{5}{12} + \frac{5}{8}$ | 31. $\frac{2}{3} - \frac{3}{8} + \frac{11}{16}$ |
| 5. $\frac{2}{3} + \frac{5}{8}$ | 14. $\frac{7}{9} - \frac{8}{15}$ | 23. $\frac{7}{15} - \frac{3}{10}$ | 32. $\frac{4}{5} - \frac{3}{4} + \frac{7}{10}$ |
| 6. $\frac{2}{5} + \frac{5}{6}$ | 15. $\frac{7}{8} + \frac{5}{12}$ | 24. $\frac{9}{16} + \frac{13}{24}$ | 33. $\frac{7}{8} - \frac{1}{6} + \frac{5}{12}$ |
| 7. $\frac{8}{9} - \frac{5}{6}$ | 16. $\frac{5}{6} - \frac{7}{10}$ | 25. $\frac{19}{20} - \frac{11}{30}$ | 34. $\frac{4}{5} + \frac{5}{6} - \frac{3}{10}$ |
| 8. $\frac{4}{5} + \frac{3}{8}$ | 17. $\frac{3}{5} + \frac{5}{12}$ | 26. $\frac{7}{15} + \frac{9}{20}$ | 35. $\frac{8}{9} - \frac{3}{4} + \frac{17}{18}$ |
| 9. $\frac{5}{6} - \frac{5}{8}$ | 18. $\frac{5}{6} - \frac{8}{15}$ | 27. $\frac{5}{12} + \frac{8}{15}$ | 36. $\frac{1}{6} + \frac{3}{4} + \frac{3}{20}$ |

37. Find the sum of $\frac{1}{2}$ ft., $\frac{1}{3}$ ft., and $\frac{1}{6}$ ft.
38. Find the perimeter of a rug $\frac{3}{4}$ yd. by $\frac{1}{2}$ yd.
39. One day Clyde spent $\$ \frac{3}{4}$ for a book, $\$ \frac{1}{2}$ for a cap, and $\$ \frac{1}{4}$ for lunch. How much did he spend that day?
40. From a lot containing $\frac{1}{6}$ of an acre, Mr. King sold $\frac{3}{8}$ of an acre. What part of an acre did he have left?
41. Find the distance around a triangle whose sides are $\frac{2}{3}$ ft., $\frac{3}{4}$ ft., and $\frac{5}{6}$ ft. long.
42. Earl caught three trout, weighing $\frac{1}{4}$ lb., $\frac{3}{8}$ lb., and $\frac{9}{16}$ lb. Find their total weight.
43. Mary traveled $\frac{5}{8}$ of a mile by boat and $\frac{3}{10}$ of a mile by car. How much farther did she go by boat than by car?
44. Ella spent $\frac{2}{5}$ of her vacation with her aunt, $\frac{1}{3}$ with her grandmother, and the rest at home. What part of her vacation did she spend at home?

ADDITION AND SUBTRACTION OF MIXED NUMBERS

Oral Exercises

Add:

$$\begin{array}{r} 1. \quad 4\frac{3}{5} \\ 1\frac{1}{5} \\ \hline 5\frac{4}{5} \end{array} \quad \begin{array}{l} \frac{1}{5} + \frac{3}{5} = ? \\ 1 + 4 = ? \\ 5 + 4 = ? \end{array}$$

$$\begin{array}{r} 10. \quad 5\frac{3}{4} \\ 2\frac{1}{4} \\ \hline 8 \end{array} \quad \begin{array}{l} \frac{1}{4} + \frac{3}{4} = ? \\ 2 + 5 = ? \\ 7 + 1 = ? \end{array}$$

$$\begin{array}{r} 19. \quad 6\frac{1}{4} = 6\frac{1}{4} \\ 3\frac{1}{2} = 3\frac{2}{4} \\ \hline 9\frac{3}{4} \end{array}$$

$$\begin{array}{r} 2. \quad 2\frac{1}{3} \\ 3\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} 6. \quad 5\frac{1}{5} \\ 4\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 5\frac{1}{2} \\ 3\frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} 15. \quad 4\frac{1}{5} \\ 8\frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 1\frac{1}{4} \\ 3\frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} 24. \quad 4\frac{1}{2} \\ 5\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 4\frac{2}{5} \\ 5\frac{2}{5} \\ \hline \end{array} \quad \begin{array}{r} 7. \quad 6\frac{2}{9} \\ 7\frac{5}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 4\frac{1}{4} \\ 2\frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} 16. \quad 6\frac{3}{7} \\ 7\frac{4}{7} \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 2\frac{1}{2} \\ 4\frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} 25. \quad 8\frac{2}{3} \\ 3\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 7\frac{1}{9} \\ 3\frac{4}{9} \\ \hline \end{array} \quad \begin{array}{r} 8. \quad 9\frac{4}{7} \\ 5\frac{2}{7} \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 3\frac{3}{5} \\ 6\frac{2}{5} \\ \hline \end{array} \quad \begin{array}{r} 17. \quad 5\frac{1}{8} \\ 8\frac{5}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 5\frac{1}{8} \\ 3\frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} 26. \quad 6\frac{2}{3} \\ 7\frac{2}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 4\frac{2}{7} \\ 6\frac{3}{7} \\ \hline \end{array} \quad \begin{array}{r} 9. \quad 8\frac{4}{9} \\ 7\frac{4}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 7\frac{1}{6} \\ 5\frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} 18. \quad 9\frac{2}{9} \\ 7\frac{4}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 6\frac{1}{3} \\ 2\frac{1}{9} \\ \hline \end{array} \quad \begin{array}{r} 27. \quad 9\frac{1}{4} \\ 5\frac{5}{8} \\ \hline \end{array}$$

Subtract:

$$\begin{array}{r} 28. \quad 4\frac{3}{5} \\ 1\frac{1}{5} \\ \hline 3\frac{2}{5} \end{array} \quad \begin{array}{l} \frac{3}{5} - \frac{1}{5} = ? \\ 4 - 1 = ? \\ 3 + \frac{2}{5} = ? \end{array}$$

$$\begin{array}{r} 35. \quad 5\frac{3}{4} \\ 2\frac{1}{4} \\ \hline 3\frac{1}{2} \end{array} \quad \begin{array}{l} \frac{3}{4} - \frac{1}{4} = ? \\ 5 - 2 = ? \\ 3 + \frac{1}{2} = ? \end{array}$$

$$\begin{array}{r} 42. \quad 7\frac{3}{4} = 7\frac{6}{8} \\ 2\frac{3}{8} = 2\frac{3}{8} \\ \hline 5\frac{3}{8} \end{array}$$

$$\begin{array}{r} 29. \quad 6\frac{2}{3} \\ 3\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} 32. \quad 8\frac{4}{5} \\ 2\frac{1}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 36. \quad 7\frac{5}{6} \\ 3\frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} 39. \quad 7\frac{5}{8} \\ 4\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 43. \quad 8\frac{1}{2} \\ 3\frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} 46. \quad 4\frac{1}{3} \\ 4\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 2\frac{3}{5} \\ 2\frac{1}{5} \\ \hline \end{array} \quad \begin{array}{r} 33. \quad 7\frac{8}{9} \\ 2\frac{1}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 37. \quad 5\frac{3}{8} \\ 4\frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} 40. \quad 8\frac{7}{9} \\ 6\frac{1}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 44. \quad 5\frac{5}{6} \\ 4\frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} 47. \quad 7\frac{5}{8} \\ 2\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 31. \quad 5\frac{4}{7} \\ 2\frac{2}{7} \\ \hline \end{array} \quad \begin{array}{r} 34. \quad 9\frac{4}{5} \\ 7\frac{2}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 38. \quad 6\frac{5}{9} \\ 2\frac{2}{9} \\ \hline \end{array} \quad \begin{array}{r} 41. \quad 9\frac{7}{8} \\ 3\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 45. \quad 7\frac{3}{4} \\ 5\frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} 48. \quad 9\frac{8}{9} \\ 6\frac{2}{3} \\ \hline \end{array}$$

Oral Exercises

1. What is the sum of $\$3\frac{1}{4}$ and $\$1\frac{1}{4}$?
2. Find the sum of $4\frac{5}{8}$ yd. and $2\frac{1}{8}$ yd.
3. How much more is $3\frac{3}{4}$ lb. than $2\frac{1}{2}$ lb.?
4. What number added to $5\frac{1}{2}$ will give $7\frac{3}{4}$?
5. How many pints of cream are $2\frac{1}{2}$ pt. less $1\frac{1}{8}$ pt.?
6. How many bushels of apples are $2\frac{1}{4}$ bu. and $5\frac{1}{8}$ bu.?
7. One week Ralph earned $\$2\frac{3}{4}$ and his brother $\$1\frac{1}{4}$.
How much did both boys earn?
8. A druggist made $2\frac{1}{2}$ gal. of strawberry sirup and $2\frac{1}{4}$ gal. of pineapple sirup. How much sirup did he make?
9. Edna had $1\frac{3}{4}$ qt. of lemonade. When $\frac{1}{2}$ qt. of it was used, how much was left?
10. A grocer bought strawberries at 10¢ a box and sold them at $12\frac{1}{2}$ ¢ a box. How much did he gain on a box?
11. Mabel picked $4\frac{1}{3}$ doz. violets and $1\frac{1}{8}$ doz. tulips. How many dozen flowers did she pick?
12. When Rose had used $3\frac{3}{8}$ lb. of butter from the $8\frac{1}{2}$ lb. that she bought, how many pounds were left?
13. From a piece of cheese weighing $6\frac{3}{4}$ lb. a grocer sold $2\frac{1}{8}$ lb. How many pounds of cheese were left?
14. From a bunch of bananas containing $8\frac{5}{8}$ doz. a dealer sold $2\frac{1}{2}$ doz. How many dozen bananas had he left?
15. Helen's height is $4\frac{5}{12}$ ft. and her mother's is $5\frac{1}{2}$ ft. How much taller is Helen's mother than Helen?
16. John drove $6\frac{3}{8}$ miles in the morning and $7\frac{1}{8}$ miles in the afternoon. How far did he drive that day?

Written Exercises

1. Add
- $3\frac{1}{2}$
- ,
- $9\frac{5}{8}$
- , and
- $6\frac{3}{4}$
- .

$$3\frac{1}{2} = 3\frac{6}{12}$$

$$9\frac{5}{8} = 9\frac{10}{12}$$

$$6\frac{3}{4} = 6\frac{9}{12}$$

$$\underline{\quad\quad\quad}$$

$$20\frac{1}{12}$$

Reducing the fractions to their l.c.d., we have $\frac{6}{12}$, $\frac{10}{12}$, and $\frac{9}{12}$, whose sum is $\frac{25}{12}$, or $2\frac{1}{12}$.

Writing $\frac{1}{12}$ under the fractions and adding the 2 to the integers, we have for the whole sum $20\frac{1}{12}$.

Add:

$$\begin{array}{r} 2. \quad 3\frac{1}{2} \\ \underline{4\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 3. \quad 5\frac{3}{4} \\ \underline{3\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 4. \quad 6\frac{7}{8} \\ \underline{1\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 5. \quad 1\frac{3}{8} \\ \underline{3\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 6. \quad 3\frac{2}{3} \\ \underline{2\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 7. \quad 6\frac{7}{8} \\ \underline{9\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 8. \quad 4\frac{1}{2} \\ \underline{2\frac{1}{3}} \\ \underline{8\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 9. \quad 6\frac{5}{6} \\ \underline{7\frac{1}{2}} \\ \underline{4\frac{1}{3}} \end{array}$$

$$\begin{array}{r} 10. \quad 3\frac{7}{8} \\ \underline{2\frac{1}{4}} \\ \underline{7\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 11. \quad 6\frac{1}{3} \\ \underline{4\frac{1}{2}} \\ \underline{3\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 12. \quad 5\frac{1}{2} \\ \underline{8\frac{5}{6}} \\ \underline{7\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 13. \quad 3\frac{1}{4} \\ \underline{5\frac{2}{3}} \\ \underline{4\frac{1}{6}} \end{array}$$

14. From
- $9\frac{1}{8}$
- subtract
- $6\frac{3}{4}$
- .

$$9\frac{1}{8} = 9\frac{1}{8} = 8\frac{9}{8}$$

$$\begin{array}{r} 6\frac{3}{4} = 6\frac{6}{8} = 6\frac{6}{8} \\ \underline{2\frac{3}{8}} \end{array}$$

Reducing the fractions to their l. c. d., we see that $\frac{3}{8}$ cannot be subtracted from $\frac{1}{8}$; hence, 1 is taken from 9, changed to eighths, and combined with $\frac{1}{8}$; then, $\frac{6}{8}$ is subtracted from $\frac{9}{8}$ and 6 from 8, giving the remainder $2\frac{3}{8}$.

Subtract:

$$\begin{array}{r} 15. \quad 5\frac{1}{5} \\ \underline{2\frac{2}{5}} \end{array}$$

$$\begin{array}{r} 16. \quad 8\frac{1}{4} \\ \underline{6\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 17. \quad 9\frac{1}{3} \\ \underline{3\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 18. \quad 7\frac{1}{2} \\ \underline{4\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 19. \quad 8\frac{3}{4} \\ \underline{2\frac{7}{8}} \end{array}$$

$$\begin{array}{r} 20. \quad 6\frac{1}{3} \\ \underline{1\frac{5}{12}} \end{array}$$

$$\begin{array}{r} 21. \quad 8\frac{1}{3} \\ \underline{4\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 22. \quad 7\frac{3}{8} \\ \underline{2\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 23. \quad 6\frac{1}{6} \\ \underline{3\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 24. \quad 9\frac{1}{2} \\ \underline{5\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 25. \quad 7\frac{1}{4} \\ \underline{3\frac{1}{3}} \end{array}$$

$$\begin{array}{r} 26. \quad 8\frac{1}{5} \\ \underline{4\frac{3}{10}} \end{array}$$

$$\begin{array}{r} 27. \quad 7\frac{1}{5} \\ \underline{4\frac{1}{2}} \end{array}$$

$$\begin{array}{r} 28. \quad 9\frac{1}{3} \\ \underline{5\frac{3}{4}} \end{array}$$

$$\begin{array}{r} 29. \quad 8\frac{1}{9} \\ \underline{4\frac{2}{3}} \end{array}$$

$$\begin{array}{r} 30. \quad 6\frac{1}{2} \\ \underline{2\frac{4}{5}} \end{array}$$

$$\begin{array}{r} 31. \quad 7\frac{3}{4} \\ \underline{3\frac{5}{6}} \end{array}$$

$$\begin{array}{r} 32. \quad 9\frac{3}{4} \\ \underline{5\frac{1}{12}} \end{array}$$

- 33-50. Add the numbers in exercises 15-32.

Find the sum :

51. $15\frac{1}{2}$ <u>8\frac{1}{2}</u>	52. $17\frac{1}{3}$ <u>10\frac{2}{3}</u>	53. $24\frac{1}{6}$ <u>11\frac{5}{6}</u>	54. $18\frac{1}{2}$ <u>12\frac{3}{4}</u>	55. $42\frac{3}{10}$ <u>26\frac{4}{5}</u>
56. $26\frac{1}{3}$ <u>14\frac{5}{6}</u>	57. $25\frac{1}{2}$ <u>11\frac{5}{8}</u>	58. $32\frac{1}{2}$ <u>18\frac{5}{6}</u>	59. $58\frac{3}{8}$ <u>32\frac{3}{4}</u>	60. $68\frac{1}{8}$ <u>35\frac{5}{12}</u>
61. $44\frac{1}{2}$ <u>29\frac{2}{3}</u>	62. $95\frac{1}{2}$ <u>54\frac{3}{5}</u>	63. $38\frac{3}{5}$ <u>15\frac{3}{4}</u>	64. $82\frac{1}{4}$ <u>64\frac{5}{6}</u>	65. $97\frac{1}{3}$ <u>42\frac{1}{12}</u>
66. $77\frac{1}{3}$ <u>49\frac{4}{5}</u>	67. $36\frac{1}{4}$ <u>17\frac{4}{5}</u>	68. $71\frac{1}{3}$ <u>43\frac{7}{8}</u>	69. $54\frac{1}{6}$ <u>25\frac{7}{8}</u>	70. $59\frac{3}{4}$ <u>21\frac{1}{12}</u>

71-90. Find the difference in exercises 51-70.

Add :

91. $19\frac{1}{4}$ <u>28\frac{1}{2}</u> <u>32\frac{3}{4}</u>	92. $37\frac{2}{3}$ <u>91</u> <u>84\frac{2}{3}</u>	93. $58\frac{1}{6}$ <u>45\frac{1}{2}</u> <u>62\frac{5}{6}</u>	94. $28\frac{3}{8}$ <u>56\frac{1}{4}</u> <u>36\frac{7}{8}</u>	95. $22\frac{7}{10}$ <u>18\frac{1}{2}</u> <u>19\frac{3}{10}</u>
96. $44\frac{1}{2}$ <u>68\frac{1}{3}</u> <u>47\frac{1}{6}</u>	97. $56\frac{1}{6}$ <u>75\frac{3}{4}</u> <u>94\frac{1}{2}</u>	98. $23\frac{1}{2}$ <u>49\frac{3}{8}</u> <u>36\frac{3}{4}</u>	99. 89 <u>54\frac{5}{8}</u> <u>63\frac{3}{4}</u>	100. $35\frac{1}{4}$ <u>26\frac{5}{12}</u> <u>84\frac{7}{12}</u>
101. $38\frac{4}{5}$ <u>27</u> <u>42\frac{1}{2}</u>	102. $76\frac{1}{2}$ <u>24\frac{7}{8}</u> <u>35\frac{1}{4}</u>	103. $25\frac{2}{3}$ <u>38\frac{5}{6}</u> <u>74\frac{1}{4}</u>	104. $65\frac{4}{5}$ <u>29\frac{3}{4}</u> <u>87\frac{7}{10}</u>	105. $15\frac{1}{8}$ <u>16\frac{3}{4}</u> <u>18\frac{5}{12}</u>

Find correct answers :

106. $3\frac{1}{4} - 1\frac{3}{4} + 2\frac{7}{8}$	109. $3\frac{5}{6} + 3\frac{1}{2} + 2\frac{2}{3}$	112. $6\frac{1}{4} - 2\frac{5}{8} + 1\frac{3}{8}$
107. $5\frac{5}{8} + 4\frac{1}{8} - 5\frac{1}{2}$	110. $4\frac{3}{4} - 2\frac{3}{8} + 4\frac{1}{2}$	113. $4\frac{2}{3} - 1\frac{1}{8} + 3\frac{5}{8}$
108. $7\frac{1}{6} - 2\frac{5}{6} + 2\frac{2}{3}$	111. $5\frac{1}{2} + 1\frac{2}{3} - 4\frac{3}{4}$	114. $2\frac{3}{4} + 3\frac{1}{2} + 1\frac{5}{8}$

Written Exercises

1. From five and one sixth take two and two thirds.
2. I had $\$12\frac{3}{4}$ and earned $\$3\frac{3}{4}$. How much had I then?
3. Mrs. Morse bought $15\frac{1}{2}$ yd. of silk and used all but $1\frac{5}{8}$ yd. of it. How many yards did she use?
4. Ellen saved $\$25\frac{1}{2}$ and her brother saved $\$15\frac{3}{4}$. How much did both save?
5. From a piece of dress goods $40\frac{1}{2}$ yd. long a merchant sold $16\frac{3}{4}$ yd. How many yards were left in the piece?
6. Louise is $28\frac{1}{2}$ years younger than her mother. How old is her mother, if Louise is $9\frac{1}{6}$ years old?
7. Richard could jump $31\frac{1}{2}$ ft., but by running he could jump $6\frac{1}{3}$ ft. How much farther could he jump by running?
8. John weighs $71\frac{1}{2}$ lb. and Edward $64\frac{5}{8}$ lb. When both ride on their pony, what weight does the pony carry?
9. A tank contained $98\frac{1}{4}$ gal. of kerosene. How many gallons were left after $22\frac{1}{2}$ gal. had been sold?
10. If it takes $14\frac{3}{4}$ yd. of carpet for our hall floor and $10\frac{2}{3}$ yd. for the stairs, how many yards are needed for both?
11. Guy threw a baseball $40\frac{2}{3}$ yd. and Max threw it $38\frac{1}{8}$ yd. How much farther did Guy throw it than Max?
12. How far is it around a flower bed in the shape of a triangle whose sides are $22\frac{1}{3}$ ft., $19\frac{5}{8}$ ft., and $21\frac{2}{3}$ ft. long?
13. An ice house is $92\frac{2}{3}$ ft. long and $56\frac{5}{8}$ ft. wide. How much greater is its length than its width?
14. Carl had on his cart $24\frac{1}{2}$ lb. flour, 16 lb. sugar, $2\frac{1}{4}$ lb. bacon, and $\frac{3}{8}$ lb. ginger. How much did the load weigh?

Oral Exercises

Find results quickly:

- | | | | |
|-----------------------|----------------------------------|----------------------------------|--|
| 1. $5 - 2\frac{1}{2}$ | 5. $6\frac{1}{2} + 1\frac{1}{2}$ | 9. $8\frac{1}{3} - \frac{5}{6}$ | 13. $2\frac{1}{4} + 1\frac{1}{4} + 3\frac{1}{4}$ |
| 2. $3 + 4\frac{2}{3}$ | 6. $3\frac{1}{3} - 2\frac{2}{3}$ | 10. $5\frac{1}{2} + \frac{7}{8}$ | 14. $5\frac{1}{3} + 4\frac{2}{3} + 2\frac{1}{3}$ |
| 3. $7 - 6\frac{1}{4}$ | 7. $9\frac{1}{4} - 3\frac{3}{4}$ | 11. $9\frac{1}{4} - \frac{5}{8}$ | 15. $4\frac{1}{6} + 3\frac{1}{6} + 1\frac{1}{6}$ |
| 4. $8 + 5\frac{1}{6}$ | 8. $4\frac{4}{5} + 1\frac{3}{5}$ | 12. $6\frac{2}{3} + \frac{1}{2}$ | 16. $6\frac{1}{8} + 2\frac{3}{8} + 4\frac{7}{8}$ |

17. Find the perimeter of a book 9 in. by $5\frac{1}{2}$ in.
18. How many yards are 6 yd. less $2\frac{1}{3}$ yd.?
19. How many dozen are $4\frac{1}{2}$ doz. and $1\frac{2}{3}$ doz.?
20. How many feet are $12\frac{1}{3}$ ft. less $6\frac{2}{3}$ ft.?
21. How many gallons are $5\frac{1}{4}$ gal. less $2\frac{1}{2}$ gal.?
22. How many hours are $11\frac{1}{2}$ hours and $3\frac{1}{3}$ hours?
23. How much is left, if \$ $1\frac{3}{4}$ is spent out of \$ $3\frac{1}{4}$?
24. If it takes $2\frac{1}{2}$ yd. of cloth for a waist and $3\frac{3}{4}$ yd. for a skirt, how many yards are needed for both?
25. Robert studied $2\frac{2}{3}$ hours at school one day, and $1\frac{2}{3}$ hours at home. How many hours did he study that day?
26. Edna bought $6\frac{1}{4}$ yd. of ribbon and used $3\frac{1}{2}$ yd. of it for her dress. How many yards did she have left?
27. I picked $5\frac{3}{8}$ bu. of apples from one tree and $4\frac{7}{8}$ bu. from another. How many bushels of apples did I pick?
28. Harriet is $12\frac{1}{3}$ years old and Grace is $7\frac{5}{6}$ years old. How much older is Harriet than Grace?
29. I bought two remnants of cloth containing $4\frac{5}{8}$ yd. and $2\frac{1}{2}$ yd. How many yards of cloth did I buy?
30. One week Harold put \$ $2\frac{2}{5}$ in the savings bank and the next week, \$ $1\frac{4}{5}$. How much did he put in the bank?

Written Exercises

Add:

- | | | | | |
|---|---|---|--|---|
| 1. $15\frac{7}{8}$
<u>$12\frac{3}{4}$</u> | 2. $24\frac{5}{6}$
<u>$11\frac{2}{3}$</u> | 3. $42\frac{1}{4}$
<u>$20\frac{1}{3}$</u> | 4. $18\frac{1}{3}$
<u>$9\frac{5}{12}$</u> | 5. $25\frac{4}{5}$
<u>$12\frac{3}{10}$</u> |
| 6. $26\frac{1}{2}$
<u>$14\frac{2}{3}$</u> | 7. $54\frac{3}{4}$
<u>$28\frac{5}{6}$</u> | 8. $52\frac{2}{3}$
<u>$19\frac{4}{5}$</u> | 9. $38\frac{5}{6}$
<u>$20\frac{1}{12}$</u> | 10. $46\frac{1}{2}$
<u>$27\frac{7}{10}$</u> |

11-20. Subtract in exercises 1-10.

Add $15\frac{3}{4}$ to each of the following numbers:

- | | | | | |
|---------------------|---------------------|---------------------|---------------------|----------------------|
| 21. $18\frac{1}{2}$ | 23. $16\frac{3}{4}$ | 25. $25\frac{3}{8}$ | 27. $28\frac{1}{6}$ | 29. $42\frac{1}{12}$ |
| 22. $32\frac{5}{6}$ | 24. $48\frac{7}{8}$ | 26. $24\frac{2}{3}$ | 28. $36\frac{5}{6}$ | 30. $50\frac{5}{24}$ |

31-40. From each number in exercises 21-30 subtract $9\frac{1}{2}$.

Add:

- | | | | | |
|--|--|--|--|---|
| 41. $14\frac{2}{3}$
<u>$18\frac{1}{6}$</u>
<u>$10\frac{1}{3}$</u> | 42. $12\frac{1}{2}$
<u>$11\frac{3}{4}$</u>
<u>$15\frac{1}{2}$</u> | 43. $15\frac{1}{6}$
<u>$13\frac{1}{2}$</u>
<u>$18\frac{2}{3}$</u> | 44. $20\frac{3}{8}$
<u>$14\frac{3}{4}$</u>
<u>$17\frac{5}{8}$</u> | 45. $31\frac{1}{3}$
<u>$22\frac{3}{4}$</u>
<u>$16\frac{5}{6}$</u> |
| 46. $32\frac{1}{3}$
<u>$20\frac{1}{6}$</u>
<u>$18\frac{1}{2}$</u>
<u>$24\frac{2}{3}$</u> | 47. $20\frac{1}{2}$
<u>$15\frac{1}{8}$</u>
<u>$31\frac{3}{4}$</u>
<u>$18\frac{5}{8}$</u> | 48. $18\frac{1}{2}$
<u>$23\frac{1}{4}$</u>
<u>$17\frac{1}{2}$</u>
<u>$25\frac{1}{6}$</u> | 49. $24\frac{1}{2}$
<u>$85\frac{3}{4}$</u>
<u>$62\frac{1}{3}$</u>
<u>$70\frac{1}{6}$</u> | 50. $41\frac{1}{3}$
<u>$9\frac{1}{2}$</u>
<u>$58\frac{1}{6}$</u>
<u>$37\frac{3}{8}$</u> |

51. Find the sum of $6\frac{1}{2}$ yd., $4\frac{2}{3}$ yd., and $8\frac{5}{6}$ yd.52. From $20\frac{1}{4}$ gal. of milk, a milkman sold $5\frac{3}{8}$ gal. How much was left?53. The sum of two numbers is $16\frac{3}{10}$. If one of the numbers is $7\frac{1}{2}$, what is the other number?

54. A wagon with its load weighs $2\frac{1}{2}$ tons. The wagon alone weighs $\frac{3}{8}$ of a ton. Find the weight of the load.

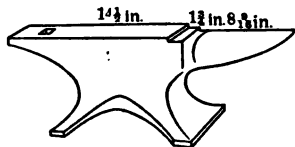
55. John rowed down the river for $2\frac{3}{4}$ hours and it took him $3\frac{5}{8}$ hours to row back. How long was he rowing?

56. It takes $3\frac{1}{3}$ hours to drive to Glen and $\frac{3}{4}$ of an hour to go by train. How much time is saved in going by train?

57. Mr. Fox had a farm of $86\frac{4}{5}$ acres and he bought $57\frac{7}{10}$ acres more. How many acres of land had he then?

58. How long is this anvil?

59. The top is $3\frac{3}{4}$ in. wide, and the square hole is $1\frac{5}{16}$ in. from each side. How wide is the hole?



60. A 135-pound anvil after several years' use weighed $132\frac{5}{8}$ pounds. Find the loss of weight due to wear.

61. Add twenty-four and one half, thirty-six and four fifths, and forty-eight and seven tenths.

62. A flagstaff $50\frac{1}{8}$ ft. high was broken off by a storm so that it measured $43\frac{3}{4}$ ft. How much was broken off?

63. A kite string was broken into three pieces, $32\frac{1}{2}$ yd., $57\frac{1}{4}$ yd., and $41\frac{7}{12}$ yd. long. How long was the string?

64. Find the distance around a room $20\frac{3}{8}$ ft. by $14\frac{1}{4}$ ft.

65. A miller shoveled $38\frac{1}{2}$ bu. of corn from a bin containing $75\frac{3}{8}$ bu. How many bushels of corn were left?

66. From the sum of thirty-nine and one sixth and sixty-two and three eighths subtract their difference.

67. Mr. Carr burned $12\frac{3}{4}$ tons of coal last winter, $11\frac{7}{10}$ tons the winter before, and $11\frac{4}{5}$ tons the winter before that. How much coal did he use in the three winters?

MULTIPLICATION OF FRACTIONS

Multiplication of a fraction by an integer.

1. How many pins are 2 times 1 pin? $2 \times 1 = ?$

How many thirds are 2 times 1 third? $2 \times \frac{1}{3} = ?$

How many fifths are 2 times 2 fifths? $2 \times \frac{2}{5} = ?$

2. *Multiply the numerator of $\frac{1}{4}$ by 2 and reduce to lowest terms. Divide the denominator of $\frac{1}{4}$ by 2. Compare results. You have multiplied $\frac{1}{4}$ by 2 in two ways.*

3. Multiply in two ways: $\frac{1}{6}$ by 3; $\frac{1}{8}$ by 4; $\frac{3}{8}$ by 2.

From the preceding and from p. 163, 1, observe that:

To multiply a fraction by an integer, *multiply the numerator or divide the denominator by the integer.*

Oral Exercises

Multiply:

Find:

1. $\frac{1}{6}$ by 2

5. $\frac{3}{8}$ by 2

9. 5 times $\frac{2}{3}$

13. $5 \times \frac{3}{10}$

2. $\frac{3}{4}$ by 2

6. $\frac{4}{5}$ by 5

10. 2 times $\frac{7}{8}$

14. $2 \times \frac{5}{12}$

3. $\frac{1}{3}$ by 4

7. $\frac{5}{6}$ by 2

11. 3 times $\frac{7}{9}$

15. $6 \times \frac{7}{12}$

4. $\frac{5}{6}$ by 3

8. $\frac{5}{8}$ by 4

12. 2 times $\frac{6}{7}$

16. $4 \times \frac{9}{16}$

Find the cost of:

17. 8 yd. silk @ $\$ \frac{1}{2}$.

20. 2 lb. candy @ $\$ \frac{3}{5}$.

18. 2 yd. lace @ $\$ \frac{3}{4}$.

21. 4 yd. satin @ $\$ \frac{7}{8}$.

19. 3 doz. eggs @ $\$ \frac{2}{5}$.

22. 5 lb. butter @ $\$ \frac{3}{10}$.

23. At $\$ \frac{1}{4}$ a pound, find the cost of 8 lb. of nuts.

24. A family uses $\frac{3}{8}$ lb. of butter at each meal. How much butter is used at the table in 4 meals?

Written Exercises

1. Find 8 times
- $7\frac{1}{2}$
- .

$$8 \times \frac{11}{12} = \frac{\cancel{8}^2 \times 11}{\underset{3}{\cancel{12}_3}} = \frac{22}{3} = 7\frac{1}{3}$$

You have learned that to multiply $7\frac{1}{2}$ by 8 you may multiply the numerator 11 by 8.

Indicating this as in the process, canceling, and reducing, you find the result to be $7\frac{1}{3}$.

Cancel whenever possible.

Find:

Multiply:

- | | | | |
|---------------------------|----------------------------|--------------------------|---------------------------|
| 2. $6 \times \frac{3}{4}$ | 6. $12 \times \frac{2}{3}$ | 10. $\frac{3}{10}$ by 15 | 14. $\frac{7}{12}$ by 20 |
| 3. $8 \times \frac{1}{6}$ | 7. $14 \times \frac{3}{4}$ | 11. $\frac{5}{12}$ by 18 | 15. $\frac{5}{18}$ by 24 |
| 4. $6 \times \frac{7}{8}$ | 8. $15 \times \frac{4}{5}$ | 12. $\frac{9}{16}$ by 24 | 16. $\frac{4}{15}$ by 36 |
| 5. $4 \times \frac{5}{6}$ | 9. $16 \times \frac{5}{8}$ | 13. $\frac{2}{15}$ by 30 | 17. $\frac{11}{20}$ by 45 |

Multiply by 15:

- | | | | | | |
|-------------------|-------------------|--------------------|--------------------|--------------------|---------------------|
| 18. $\frac{3}{5}$ | 20. $\frac{3}{4}$ | 22. $\frac{3}{10}$ | 24. $\frac{7}{12}$ | 26. $\frac{9}{20}$ | 28. $\frac{11}{18}$ |
| 19. $\frac{5}{6}$ | 21. $\frac{7}{8}$ | 23. $\frac{2}{15}$ | 25. $\frac{5}{16}$ | 27. $\frac{7}{30}$ | 29. $\frac{13}{24}$ |

Multiply each of the fractions in exercises 18-29:

- 30-41. By 21 42-53. By 25 54-65. By 36 66-77. By 40

78. If an electric car moves at the rate of $\frac{1}{6}$ of a mile a minute, how many miles will it travel in 50 minutes?

79. John has been away $7\frac{1}{2}$ of a year. Paul has been away 6 times as long. How long has Paul been away?

80. If I walk a mile in $\frac{3}{10}$ of an hour, how long will it take me to walk 12 miles at the same rate?

81. On Saturday Charles earned $\$1\frac{3}{20}$ and his father earned 5 times as much. How much did his father earn?

Oral Exercises

1. How many are $3 \times 2\frac{1}{2}$?

SUGGESTION.— $3 \times 2\frac{1}{2}$ means the sum of 3×2 and $3 \times \frac{1}{2}$.

2. Find: $2 \times 1\frac{1}{2}$; $3 \times 1\frac{2}{3}$; $4 \times 2\frac{1}{4}$; $6 \times 2\frac{1}{3}$; $8 \times 2\frac{1}{2}$.
 3. Multiply: $3\frac{1}{5}$ by 5; $2\frac{3}{4}$ by 4; $3\frac{2}{3}$ by 6; $6\frac{1}{4}$ by 8.
 4. Find the cost of 6 pencils at $2\frac{1}{2}$ ¢ apiece.
 5. What is the cost of 4 tons of coal at \$ $6\frac{1}{4}$ per ton?
 6. Find the perimeter of a square whose side is $1\frac{1}{2}$ ft.

Written Exercises

1. Multiply $24\frac{7}{8}$ by 6.

$$\begin{array}{r} 24\frac{7}{8} \\ 6 \\ \hline \frac{7}{8} \text{ multiplied by } 6 = 5\frac{1}{4} \\ 24 \text{ multiplied by } 6 = 144 \\ 24\frac{7}{8} \text{ multiplied by } 6 = 149\frac{1}{4} \end{array}$$

Multiply:

- | | | |
|------------------------|--------------------------|--------------------------|
| 2. $7\frac{2}{3}$ by 6 | 8. $10\frac{1}{2}$ by 6 | 14. $25\frac{3}{4}$ by 2 |
| 3. $6\frac{3}{4}$ by 8 | 9. $11\frac{2}{3}$ by 3 | 15. $30\frac{1}{5}$ by 5 |
| 4. $8\frac{1}{2}$ by 7 | 10. $14\frac{1}{4}$ by 8 | 16. $26\frac{3}{8}$ by 8 |
| 5. $9\frac{5}{8}$ by 4 | 11. $12\frac{1}{6}$ by 4 | 17. $42\frac{3}{8}$ by 4 |
| 6. $5\frac{3}{5}$ by 9 | 12. $15\frac{1}{8}$ by 6 | 18. $28\frac{2}{3}$ by 6 |
| 7. $7\frac{3}{8}$ by 5 | 13. $16\frac{2}{5}$ by 5 | 19. $35\frac{5}{8}$ by 9 |
20. How much will 8 tons of hay cost at \$ $18\frac{1}{4}$ per ton?
 21. How far will a train run in 10 hours at the rate of $33\frac{1}{8}$ miles an hour?

Multiplication of an integer by a fraction.

1. Point to $\frac{1}{3}$ of these 12 marks.

Now point to $\frac{2}{3}$ of the 12 marks.



2. $\frac{2}{3}$ of 12 marks = — times $\frac{1}{3}$ of 12 marks = —.

3. How many are $\frac{1}{3}$ of 12? $\frac{2}{3}$ of 12? $\frac{1}{4}$ of 12? $\frac{3}{4}$ of 12?

Finding $\frac{2}{4}$ of 12 is **multiplying** 12 by $\frac{2}{4}$.

The word “of” is equivalent to the sign “ \times ”; thus, $\frac{2}{4}$ of 12 means $\frac{2}{4} \times 12$.

Oral Exercises

1. Find $\frac{2}{5}$ of 20.

SOLUTION. — $\frac{1}{5}$ of 20 = 4; $\frac{2}{5}$ of 20 = 2 times 4 = 8.

Find:

- | | | | |
|------------------------|------------------------|-------------------------|-------------------------|
| 2. $\frac{1}{3}$ of 21 | 6. $\frac{1}{5}$ of 15 | 10. $\frac{2}{3}$ of 18 | 14. $\frac{3}{4}$ of 32 |
| 3. $\frac{2}{3}$ of 21 | 7. $\frac{4}{5}$ of 15 | 11. $\frac{5}{6}$ of 30 | 15. $\frac{2}{3}$ of 36 |
| 4. $\frac{1}{4}$ of 28 | 8. $\frac{1}{6}$ of 24 | 12. $\frac{3}{4}$ of 36 | 16. $\frac{5}{8}$ of 48 |
| 5. $\frac{3}{4}$ of 28 | 9. $\frac{5}{6}$ of 24 | 13. $\frac{2}{8}$ of 40 | 17. $\frac{5}{6}$ of 60 |

Find the cost of:

18. $\frac{1}{2}$ of a yard of lawn at 24¢ a yard.
19. $\frac{2}{3}$ of a dozen napkins at \$6 a dozen.
20. $\frac{1}{2}$ of a pound of tea at 50¢ a pound.
21. $\frac{5}{8}$ of a pound of figs at 16¢ a pound.
22. $\frac{3}{8}$ of a yard of ribbon at 40¢ a yard.
23. $\frac{5}{6}$ of a yard of oilcloth at 30¢ a yard.
24. $\frac{3}{4}$ of a pound of butter at 36¢ a pound.
25. $\frac{7}{8}$ of a gallon of vinegar at 24¢ a gallon.

Written Exercises

Find, as on page 189 :

1. $\frac{1}{2}$ of 34 5. $\frac{1}{6}$ of 78 9. $\frac{2}{3}$ of 174 13. $\frac{1}{2}$ of \$1.52
2. $\frac{2}{3}$ of 42 6. $\frac{4}{5}$ of 85 10. $\frac{3}{5}$ of 260 14. $\frac{2}{3}$ of \$2.25
3. $\frac{3}{4}$ of 60 7. $\frac{2}{3}$ of 81 11. $\frac{3}{4}$ of 336 15. $\frac{1}{6}$ of \$1.74
4. $\frac{2}{5}$ of 65 8. $\frac{5}{6}$ of 90 12. $\frac{5}{8}$ of 272 16. $\frac{3}{5}$ of \$3.50
17. Find $\frac{5}{6}$ of 21; that is, *multiply* 21 by $\frac{5}{6}$.

$$\frac{1}{6} \text{ of } 21 = \frac{21}{6}; \quad \frac{5}{6} \text{ of } 21 = 5 \times \frac{21}{6} = \frac{105}{6} = 17\frac{3}{6} = 17\frac{1}{2}.$$

It is shorter to indicate the work and cancel, thus :

$$\begin{array}{c} 5 \\ \cancel{6} \\ 2 \end{array} \times \begin{array}{c} 7 \\ \cancel{21} \\ 2 \end{array} = \frac{35}{2} = 17\frac{1}{2}$$

Multiply in the shortest way:

18. 6 by $\frac{3}{4}$ 24. 10 by $\frac{5}{6}$ 30. 22 by $\frac{3}{4}$ 36. 114 by $\frac{1}{4}$
19. 4 by $\frac{5}{6}$ 25. 18 by $\frac{1}{4}$ 31. 54 by $\frac{2}{3}$ 37. 160 by $\frac{1}{6}$
20. 6 by $\frac{2}{9}$ 26. 20 by $\frac{2}{3}$ 32. 26 by $\frac{3}{8}$ 38. 210 by $\frac{1}{8}$
21. 8 by $\frac{5}{6}$ 27. 12 by $\frac{5}{8}$ 33. 39 by $\frac{5}{9}$ 39. 182 by $\frac{3}{4}$
22. 4 by $\frac{3}{8}$ 28. 25 by $\frac{1}{6}$ 34. 46 by $\frac{5}{6}$ 40. 260 by $\frac{3}{8}$
23. 6 by $\frac{7}{8}$ 29. 32 by $\frac{3}{5}$ 35. 60 by $\frac{7}{8}$ 41. 231 by $\frac{2}{9}$

Make and solve a problem about:

42. Carl is 50 in. tall. John is $\frac{3}{4}$ as tall.
43. In a school of 432 pupils $\frac{4}{9}$ are boys.
44. A bridge is 100 ft. long and $\frac{3}{8}$ as wide.
45. Elmer weighs 98 lb. and Roy $\frac{7}{8}$ as much.
46. Marion had \$1.32. Anna had $\frac{5}{6}$ as much.

Oral Exercises

1. How many cents are $4 \times 6\text{¢}$? $\frac{1}{2}$ of 6¢ ? $4 \times 6\text{¢}$ and $\frac{1}{2}$ of 6¢ ? Then, how many cents are $4\frac{1}{2} \times 6\text{¢}$?
2. Find $2\frac{1}{2} \times 4$. 3. Find $3\frac{1}{3} \times 6$. 4. Find $1\frac{1}{4} \times 4$.
5. How many feet are there in $2\frac{1}{3}$ yd.?
6. How many quarts are there in $3\frac{1}{2}$ gal.?
7. Find the cost of $3\frac{1}{4}$ yd. of ribbon at 8¢ a yard.

Written Exercises

1. Multiply 26 by
- $7\frac{1}{4}$
- .

$$\begin{array}{r} 26 \\ 7\frac{1}{4} \\ \hline \end{array}$$

Product by $\frac{1}{4} = 6\frac{1}{2}$

Product by 7 = 182

Product by $7\frac{1}{4} = 188\frac{1}{2}$

2. Multiply \$1.24 by
- $7\frac{3}{4}$
- .

$$\begin{array}{r} \$1.24 \\ 7\frac{3}{4} \\ \hline \end{array}$$

93

8 68

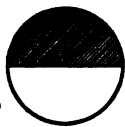
\$9.61

Multiply:

- | | | |
|------------------------|--------------------------|------------------------------|
| 3. 5 by $8\frac{1}{2}$ | 7. 12 by $6\frac{2}{3}$ | 11. \$1.40 by $3\frac{1}{2}$ |
| 4. 6 by $5\frac{1}{4}$ | 8. 34 by $4\frac{1}{5}$ | 12. \$1.77 by $5\frac{2}{3}$ |
| 5. 9 by $4\frac{2}{3}$ | 9. 50 by $5\frac{1}{5}$ | 13. \$2.60 by $2\frac{3}{4}$ |
| 6. 8 by $7\frac{3}{4}$ | 10. 42 by $4\frac{1}{8}$ | 14. \$2.75 by $4\frac{2}{5}$ |

Find the cost of:

15. $7\frac{1}{4}$ yards of cloth at \$.72 a yard.
16. $8\frac{2}{3}$ yards of carpet at \$1.08 a yard.
17. $5\frac{3}{8}$ gallons of molasses at \$.64 a gallon.
18. $4\frac{3}{4}$ dozen window pulleys at \$.40 a dozen.
19. $9\frac{5}{8}$ dozen ears of green corn at \$.18 a dozen.

Multiplication of a fraction by a fraction.

1. What part of a circle is $\frac{1}{2}$ of $\frac{1}{2}$ of it?
George had $\frac{1}{2}$ of a dollar, and he gave $\frac{1}{2}$ of his money to his sister. What part of a dollar did he give her?

2. What part of a rectangle is $\frac{1}{2}$ of $\frac{1}{4}$ of it? $\frac{1}{4}$ of $\frac{1}{2}$ of it? $\frac{3}{4}$ of $\frac{1}{2}$ of it?



3. How does $\frac{1}{2}$ of $\frac{1}{4}$ compare in value with $\frac{1 \times 1}{2 \times 4}$, or $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{2}$ with $\frac{1 \times 1}{4 \times 2}$? $\frac{3}{4}$ of $\frac{1}{2}$ with $\frac{3 \times 1}{4 \times 2}$?

4. Find the value of $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{1}{3}$ of $\frac{3}{4}$; $\frac{2}{3}$ of $\frac{3}{4}$.

To multiply a fraction by a fraction, *multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.*

Oral Exercises

Reduce mixed numbers to improper fractions and find:

1. $\frac{1}{2}$ of $\frac{2}{3}$ 4. $\frac{1}{2} \times \frac{3}{5}$ 7. $\frac{1}{2}$ of $\frac{1}{6}$ 10. $\frac{1}{6} \times \frac{3}{5}$ 13. $\frac{1}{2}$ of $1\frac{1}{3}$
2. $\frac{2}{3}$ of $\frac{1}{2}$ 5. $\frac{4}{5} \times \frac{1}{4}$ 8. $\frac{2}{3}$ of $\frac{3}{5}$ 11. $\frac{3}{5} \times \frac{5}{8}$ 14. $\frac{1}{3}$ of $2\frac{1}{4}$
3. $\frac{2}{3}$ of $\frac{1}{4}$ 6. $\frac{1}{5} \times \frac{5}{8}$ 9. $\frac{1}{5}$ of $\frac{5}{6}$ 12. $\frac{1}{2} \times 1\frac{1}{2}$ 15. $\frac{1}{4}$ of $1\frac{3}{5}$

16. What part of a pound of rice is $\frac{1}{2}$ of $\frac{1}{4}$ lb.?
17. At $\$ \frac{4}{5}$ a pound, find the cost of $\frac{1}{2}$ lb. of tea.
18. What part of a pound of raisins is $\frac{1}{2}$ of $\frac{3}{4}$ lb.?
19. How much will $\frac{1}{3}$ yd. of cloth cost at $\$ \frac{3}{5}$ a yard?
20. If James earned $\frac{1}{3}$ of $\$ \frac{3}{4}$, how much did he earn?
21. The champion runner of our school circled a $\frac{1}{8}$ -mile track in a minute. How far did he run in $\frac{2}{3}$ of a minute?

Written Exercises

1. Find $\frac{4}{5}$ of $\frac{15}{16}$; that is, multiply $\frac{15}{16}$ by $\frac{4}{5}$.

$$\frac{4}{5} \text{ of } \frac{15}{16} = \frac{60}{80} = \frac{3}{4}$$

Or,

$$\frac{4}{5} \text{ of } \frac{15}{16} = \frac{4 \times 15}{5 \times 16} = \frac{3}{4}$$

If we multiply the numerators for the numerator of the product and the denominators for the denominator of the product, the result is $\frac{60}{80}$, which may be reduced to $\frac{3}{4}$.

If we indicate the work and cancel before multiplying, the direct result is $\frac{3}{4}$.

Cancel whenever possible.

Find the value of:

- | | | | |
|-----------------------------------|--------------------------------------|-------------------------------------|--|
| 2. $\frac{2}{3}$ of $\frac{3}{4}$ | 8. $\frac{2}{5} \times \frac{5}{8}$ | 14. $\frac{2}{3}$ of $\frac{3}{10}$ | 20. $\frac{4}{15} \times \frac{5}{8}$ |
| 3. $\frac{3}{4}$ of $\frac{5}{6}$ | 9. $\frac{5}{9} \times \frac{3}{4}$ | 15. $\frac{4}{5}$ of $\frac{5}{12}$ | 21. $\frac{9}{16} \times \frac{2}{3}$ |
| 4. $\frac{3}{8}$ of $\frac{2}{3}$ | 10. $\frac{7}{8} \times \frac{4}{5}$ | 16. $\frac{5}{6}$ of $\frac{9}{10}$ | 22. $\frac{3}{20} \times \frac{5}{6}$ |
| 5. $\frac{2}{5}$ of $\frac{5}{6}$ | 11. $\frac{5}{6} \times \frac{3}{8}$ | 17. $\frac{6}{7}$ of $\frac{7}{12}$ | 23. $\frac{5}{24} \times \frac{4}{5}$ |
| 6. $\frac{2}{5}$ of $\frac{3}{8}$ | 12. $\frac{3}{4} \times \frac{8}{9}$ | 18. $\frac{3}{4}$ of $\frac{8}{15}$ | 24. $\frac{14}{15} \times \frac{3}{4}$ |
| 7. $\frac{3}{7}$ of $\frac{7}{9}$ | 13. $\frac{4}{5} \times \frac{5}{6}$ | 19. $\frac{4}{5}$ of $\frac{3}{16}$ | 25. $\frac{15}{16} \times \frac{8}{9}$ |

26. What part of a dollar is $\frac{2}{3}$ of $\$ \frac{3}{8}$?

27. What part of an acre is $\frac{4}{5}$ of $\frac{1}{6}$ of an acre?

28. Find the area of the cover of a book, $\frac{2}{3}$ ft. by $\frac{3}{4}$ ft.

Reduce mixed numbers to improper fractions and find:

- | | | | |
|-------------------------------------|---------------------------------------|-------------------------------------|--|
| 29. $\frac{2}{3}$ of $4\frac{1}{2}$ | 32. $\frac{1}{8} \times 6\frac{2}{3}$ | 35. $\frac{5}{6}$ of $9\frac{3}{4}$ | 38. $4\frac{1}{2} \times 1\frac{1}{3}$ |
| 30. $\frac{3}{4}$ of $3\frac{1}{3}$ | 33. $\frac{5}{6} \times 5\frac{3}{8}$ | 36. $\frac{5}{8}$ of $6\frac{2}{5}$ | 39. $5\frac{1}{3} \times 2\frac{1}{4}$ |
| 31. $\frac{4}{5}$ of $7\frac{1}{2}$ | 34. $\frac{3}{8} \times 9\frac{1}{3}$ | 37. $\frac{7}{8}$ of $9\frac{3}{8}$ | 40. $4\frac{1}{6} \times 2\frac{5}{6}$ |

41. At $\$ \frac{1}{5}$ a pound, find the cost of $22\frac{1}{2}$ lb. of nuts.

42. Frank bought a book for $\$ 1\frac{3}{4}$ and afterwards sold it for $\frac{4}{5}$ of the cost. How much did he receive for it?

Miscellaneous Exercises

Find the cost of :

1. $1\frac{1}{4}$ lb. soda @ 8¢.
2. $1\frac{1}{2}$ lb. rice @ 10¢.
3. $3\frac{1}{2}$ lb. beans @ 6¢.
4. $2\frac{1}{2}$ doz. rolls @ 10¢.
5. $\frac{1}{2}$ lb. cheese @ 22¢.
6. $\frac{3}{8}$ qt. olives @ 40¢.
7. $\frac{1}{2}$ gal. vinegar @ 30¢.
8. $1\frac{1}{2}$ lb. raisins @ 12¢.
9. 36 textbooks at $\$ \frac{3}{4}$ each.
10. 6 yd. silk at $\$ 1\frac{1}{4}$ a yard.
11. 4 stepladders at $\$ 1\frac{3}{4}$ each.
12. $5\frac{1}{4}$ yd. velvet at $\$ 3$ a yard.
13. $6\frac{1}{2}$ tons of hay at $\$ 20$ a ton.
14. $12\frac{3}{8}$ bu. apples at $\$.40$ a bushel.
15. $5\frac{1}{2}$ yd. broadcloth at $\$ 2\frac{1}{2}$ a yard.
16. Helen counted 75 roses in the garden $\frac{2}{5}$ of which were white. How many white roses were there?
17. Mr. Williams paid \$.48 a bushel for a bag of oats containing $2\frac{1}{8}$ bu. How much did the oats cost?
18. If it requires $1\frac{3}{4}$ bu. of seed wheat to sow 1 acre, how many bushels are needed to sow 8 acres?
19. A meadow of 10 acres produced on the average $1\frac{1}{5}$ tons of hay per acre. What was the total production?
20. A dressmaker bought a piece of cloth containing $8\frac{2}{3}$ yd. at $\$ 1.50$ a yard. Find the cost.
21. A chimney measuring 150 ft. from the bottom of its foundation is $\frac{7}{8}$ above ground. How high is it above ground?

22. A farmer had 6 stacks of hay containing $16\frac{1}{2}$ tons each. How many tons of hay were in the 6 stacks?

23. What is the product when the multiplicand is two hundred forty and the multiplier three eighths?

24. Mr. Harmon set out 224 tomato plants and Mr. Gage $\frac{5}{8}$ as many. How many plants did Mr. Gage set out?

25. I bought $1\frac{3}{4}$ lb. of 40-cent tea and handed the grocer a dollar. How much change should I receive?

26. If Mr. Smith runs his automobile at the rate of $25\frac{3}{4}$ miles an hour, how many miles does he go in 8 hours?

27. If an express train runs at the rate of 48 miles an hour, how far does it run in $2\frac{2}{3}$ hours?

28. Frank could dig potatoes $\frac{2}{3}$ as fast as John. How many bushels could he dig while John was digging $12\frac{3}{4}$ bu.?

29. A wheel in making one revolution travels $12\frac{2}{5}$ ft. How far will it travel in making 10 revolutions?

30. I bought a horse for \$174 and sold him for $\frac{5}{8}$ of the cost. How much did I get for him?

31. Mrs. Cook bought 4 yd. of linen at $\$ \frac{1}{2}$ a yard. How much change did she receive from \$5?

32. A man rented a house at the rate of \$240 a year. He lived in it $4\frac{1}{2}$ years. How much rent did he pay?

33. Mr. Bell's salary is \$984 a year, and his expenses are $\frac{3}{4}$ as much. How much are his expenses per year?

34. Mr. Joy has 4 bins in his granary, holding $96\frac{3}{4}$ bu. each. How many bushels of grain do they all hold?

35. Find the total cost of $1\frac{1}{2}$ doz. sheets at \$7.60 per dozen and $2\frac{1}{2}$ doz. pillowcases at \$2 per dozen.

DIVISION OF FRACTIONS

Division of a fraction by an integer.

1. Read, filling the blanks:

$$4 \text{ apples} \div 2 = \text{--- apples.}$$

$$4 \text{ fifths} \div 2 = \text{--- fifths.}$$

$$\frac{4}{5} \div 2 = \text{---.}$$

2. *Multiply the denominator of $\frac{4}{5}$ by 2 and reduce to lowest terms. Divide the numerator of $\frac{4}{5}$ by 2. Compare results. You have divided $\frac{4}{5}$ by 2 in two ways.*

3. Divide in two ways: $\frac{3}{4}$ by 3; $\frac{4}{7}$ by 2; $\frac{8}{9}$ by 4.

From the preceding and from p. 163, 2, observe that:

To divide a fraction by an integer, *divide the numerator or multiply the denominator by the integer.*

Always divide the numerator when it is divisible by the integer.

Oral Exercises

Divide as indicated, giving results in lowest terms:

- | | | | | |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 1. $\frac{1}{2} \div 2$ | 11. $\frac{4}{6} \div 4$ | 21. $\frac{4}{8} \div 4$ | 31. $\frac{3}{4} \div 2$ | 41. $\frac{6}{8} \div 3$ |
| 2. $\frac{2}{3} \div 2$ | 12. $\frac{1}{2} \div 5$ | 22. $\frac{2}{3} \div 3$ | 32. $\frac{6}{8} \div 6$ | 42. $\frac{6}{9} \div 2$ |
| 3. $\frac{3}{4} \div 3$ | 13. $\frac{6}{7} \div 3$ | 23. $\frac{6}{7} \div 2$ | 33. $\frac{8}{9} \div 2$ | 43. $\frac{2}{3} \div 4$ |
| 4. $\frac{1}{2} \div 4$ | 14. $\frac{2}{5} \div 2$ | 24. $\frac{1}{5} \div 2$ | 34. $\frac{3}{8} \div 2$ | 44. $\frac{3}{6} \div 2$ |
| 5. $\frac{4}{5} \div 2$ | 15. $\frac{1}{3} \div 3$ | 25. $\frac{5}{8} \div 5$ | 35. $\frac{3}{8} \div 3$ | 45. $\frac{2}{3} \div 6$ |
| 6. $\frac{1}{2} \div 3$ | 16. $\frac{4}{7} \div 2$ | 26. $\frac{2}{8} \div 2$ | 36. $\frac{4}{9} \div 2$ | 46. $\frac{2}{4} \div 3$ |
| 7. $\frac{2}{4} \div 2$ | 17. $\frac{1}{2} \div 6$ | 27. $\frac{1}{4} \div 3$ | 37. $\frac{5}{6} \div 2$ | 47. $\frac{2}{5} \div 4$ |
| 8. $\frac{4}{5} \div 4$ | 18. $\frac{3}{5} \div 3$ | 28. $\frac{6}{9} \div 3$ | 38. $\frac{8}{9} \div 4$ | 48. $\frac{2}{4} \div 4$ |
| 9. $\frac{1}{3} \div 2$ | 19. $\frac{1}{3} \div 4$ | 29. $\frac{1}{4} \div 2$ | 39. $\frac{4}{6} \div 2$ | 49. $\frac{2}{4} \div 5$ |
| 10. $\frac{2}{6} \div 2$ | 20. $\frac{3}{6} \div 3$ | 30. $\frac{6}{8} \div 2$ | 40. $\frac{4}{8} \div 2$ | 50. $\frac{2}{3} \div 8$ |

Written Exercises

1. Divide
- $\frac{15}{24}$
- by 10.

$$\frac{15}{24} \div 10 = \frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{24}} \times \underset{2}{\cancel{10}}} = \frac{1}{16}$$

Since the divisor is not exactly contained in the numerator, we *multiply* the denominator 24 by 10.

Indicating the operation and *canceling*, we have for the result $\frac{1}{16}$.

Cancel whenever possible.

Divide as indicated:

- | | | | |
|--------------------------|--------------------------|-----------------------------|----------------------------|
| 2. $\frac{2}{4} \div 6$ | 11. $\frac{2}{6} \div 4$ | 20. $\frac{6}{10} \div 4$ | 29. $\frac{15}{4} \div 10$ |
| 3. $\frac{3}{4} \div 4$ | 12. $\frac{6}{8} \div 4$ | 21. $\frac{9}{12} \div 6$ | 30. $\frac{16}{3} \div 12$ |
| 4. $\frac{3}{5} \div 6$ | 13. $\frac{3}{6} \div 6$ | 22. $\frac{8}{16} \div 6$ | 31. $\frac{12}{5} \div 24$ |
| 5. $\frac{2}{6} \div 4$ | 14. $\frac{4}{5} \div 6$ | 23. $\frac{12}{15} \div 8$ | 32. $\frac{21}{2} \div 14$ |
| 6. $\frac{2}{4} \div 8$ | 15. $\frac{4}{6} \div 8$ | 24. $\frac{15}{20} \div 9$ | 33. $\frac{25}{6} \div 50$ |
| 7. $\frac{2}{5} \div 6$ | 16. $\frac{2}{8} \div 6$ | 25. $\frac{16}{24} \div 6$ | 34. $\frac{32}{3} \div 16$ |
| 8. $\frac{4}{5} \div 8$ | 17. $\frac{4}{6} \div 6$ | 26. $\frac{15}{18} \div 10$ | 35. $\frac{15}{8} \div 30$ |
| 9. $\frac{6}{7} \div 4$ | 18. $\frac{6}{9} \div 4$ | 27. $\frac{12}{15} \div 16$ | 36. $\frac{54}{5} \div 18$ |
| 10. $\frac{3}{8} \div 6$ | 19. $\frac{4}{8} \div 8$ | 28. $\frac{16}{24} \div 12$ | 37. $\frac{15}{4} \div 45$ |

Reduce mixed numbers to improper fractions and divide:

- | | | | |
|-------------------------|-------------------------|-------------------------|--------------------------|
| 38. $1\frac{2}{3}$ by 5 | 41. $3\frac{3}{5}$ by 4 | 44. $5\frac{5}{6}$ by 7 | 47. $12\frac{1}{2}$ by 5 |
| 39. $1\frac{4}{5}$ by 3 | 42. $4\frac{1}{8}$ by 5 | 45. $6\frac{3}{4}$ by 6 | 48. $16\frac{2}{3}$ by 4 |
| 40. $2\frac{1}{4}$ by 6 | 43. $7\frac{1}{8}$ by 3 | 46. $9\frac{1}{3}$ by 8 | 49. $25\frac{1}{5}$ by 9 |

Find the cost of each when

- | | |
|--------------------------------------|---------------------------------------|
| 50. 4 pads cost \$ $\frac{2}{5}$. | 53. 2 lamps cost \$ $6\frac{1}{2}$. |
| 51. 3 caps cost \$ $1\frac{1}{2}$. | 54. 6 clocks cost \$ $7\frac{1}{2}$. |
| 52. 5 sleds cost \$ $3\frac{3}{4}$. | 55. 8 knives cost \$ $6\frac{2}{3}$. |

Division of a fraction by a fraction.

1. Read, filling the blanks:

$$4 \text{ books} \div 2 \text{ books} = \text{---}.$$

$$4 \text{ fifths} \div 2 \text{ fifths} = \text{---}.$$

$$\frac{4}{5} \div \frac{2}{5} = \text{---}.$$

2. $\frac{6}{7} \div \frac{3}{7} = ?$ 3. $\frac{4}{6} \div \frac{2}{6} = ?$ 4. $\frac{6}{8} \div \frac{2}{8} = ?$ 5. $\frac{6}{8} \div \frac{3}{8} = ?$

6. How may one fraction be divided by another when the fractions have a common denominator?

Oral Exercises*Divide the numerator of the dividend by the numerator of the divisor and thus find:*

1. $\frac{2}{3} \div \frac{1}{3}$

5. $\frac{2}{4} \div \frac{3}{4}$

9. $\frac{6}{7} \div \frac{2}{7}$

13. $\frac{6}{10} \div \frac{2}{10}$

2. $\frac{1}{3} \div \frac{2}{3}$

6. $\frac{3}{4} \div \frac{2}{4}$

10. $\frac{3}{6} \div \frac{4}{6}$

14. $\frac{2}{12} \div \frac{4}{12}$

3. $\frac{4}{5} \div \frac{2}{5}$

7. $\frac{2}{6} \div \frac{4}{6}$

11. $\frac{4}{8} \div \frac{6}{8}$

15. $\frac{8}{10} \div \frac{2}{10}$

4. $\frac{2}{5} \div \frac{4}{5}$

8. $\frac{2}{8} \div \frac{6}{8}$

12. $\frac{8}{9} \div \frac{2}{9}$

16. $\frac{10}{12} \div \frac{5}{12}$

Reduce to a common denominator and divide:

17. $\frac{1}{2} \div \frac{1}{4}$

21. $\frac{1}{4} \div \frac{3}{8}$

25. $\frac{1}{2} \div \frac{3}{4}$

29. $\frac{1}{3} \div \frac{1}{2}$

18. $\frac{1}{6} \div \frac{1}{3}$

22. $\frac{1}{2} \div \frac{5}{6}$

26. $\frac{2}{3} \div \frac{1}{9}$

30. $\frac{1}{2} \div \frac{2}{3}$

19. $\frac{2}{3} \div \frac{1}{6}$

23. $\frac{1}{3} \div \frac{1}{9}$

27. $\frac{1}{3} \div \frac{5}{6}$

31. $\frac{1}{5} \div \frac{1}{2}$

20. $\frac{1}{8} \div \frac{1}{2}$

24. $\frac{3}{8} \div \frac{1}{2}$

28. $\frac{5}{8} \div \frac{1}{4}$

32. $\frac{1}{4} \div \frac{1}{3}$

33. How does $\frac{6}{7} \div \frac{3}{7}$ compare with $\frac{6}{7} \times \frac{7}{3}$? $\frac{4}{6} \div \frac{2}{6}$ with $\frac{4}{6} \times \frac{6}{2}$? $\frac{1}{2} \div \frac{1}{4}$ with $\frac{1}{2} \times \frac{4}{1}$? $\frac{1}{4} \div \frac{3}{8}$ with $\frac{1}{4} \times \frac{8}{3}$, that is, with $\frac{1}{4}$ multiplied by $\frac{3}{8}$ *inverted* (turned upside down)?34-49. Perform the divisions in exercises 17-32 by *multiplying the dividend by the divisor inverted*.

To divide a fraction by a fraction, *multiply the dividend by the divisor inverted.*

Written Exercises

1. Divide $\frac{14}{15}$ by $\frac{7}{10}$.

$$\frac{14}{15} \div \frac{7}{10} = \frac{\cancel{14}^2}{\cancel{15}_3} \times \frac{\cancel{10}_7^2}{7} = \frac{4}{3} = 1\frac{1}{3}$$

To divide $\frac{14}{15}$ by $\frac{7}{10}$, we may multiply $\frac{14}{15}$ by $\frac{10}{7}$ inverted.

Indicating the work and *canceling*, we find the result to be $1\frac{1}{3}$.

Divide as indicated:

- | | | | |
|------------------------------------|------------------------------------|-------------------------------------|---------------------------------------|
| 2. $\frac{1}{4} \div \frac{1}{6}$ | 12. $\frac{2}{3} \div \frac{4}{5}$ | 22. $\frac{2}{5} \div \frac{3}{10}$ | 32. $\frac{3}{10} \div \frac{4}{5}$ |
| 3. $\frac{3}{5} \div \frac{3}{4}$ | 13. $\frac{3}{4} \div \frac{1}{6}$ | 23. $\frac{5}{6} \div \frac{5}{12}$ | 33. $\frac{5}{12} \div \frac{3}{4}$ |
| 4. $\frac{1}{8} \div \frac{1}{6}$ | 14. $\frac{4}{9} \div \frac{2}{3}$ | 24. $\frac{2}{3} \div \frac{4}{15}$ | 34. $\frac{8}{15} \div \frac{4}{5}$ |
| 5. $\frac{3}{4} \div \frac{3}{8}$ | 15. $\frac{3}{8} \div \frac{3}{5}$ | 25. $\frac{7}{8} \div \frac{7}{12}$ | 35. $\frac{9}{16} \div \frac{3}{8}$ |
| 6. $\frac{2}{3} \div \frac{6}{7}$ | 16. $\frac{2}{5} \div \frac{4}{9}$ | 26. $\frac{4}{5} \div \frac{8}{15}$ | 36. $\frac{9}{20} \div \frac{3}{4}$ |
| 7. $\frac{3}{8} \div \frac{1}{6}$ | 17. $\frac{5}{6} \div \frac{5}{8}$ | 27. $\frac{3}{4} \div \frac{3}{16}$ | 37. $\frac{7}{24} \div \frac{5}{8}$ |
| 8. $\frac{3}{4} \div \frac{2}{3}$ | 18. $\frac{3}{5} \div \frac{6}{7}$ | 28. $\frac{3}{5} \div \frac{9}{20}$ | 38. $\frac{14}{15} \div \frac{7}{12}$ |
| 9. $\frac{2}{3} \div \frac{5}{6}$ | 19. $\frac{3}{4} \div \frac{5}{6}$ | 29. $\frac{3}{8} \div \frac{5}{16}$ | 39. $\frac{15}{16} \div \frac{3}{10}$ |
| 10. $\frac{5}{8} \div \frac{3}{4}$ | 20. $\frac{7}{8} \div \frac{3}{4}$ | 30. $\frac{5}{6} \div \frac{5}{24}$ | 40. $\frac{12}{25} \div \frac{9}{10}$ |
| 11. $\frac{7}{9} \div \frac{7}{8}$ | 21. $\frac{8}{9} \div \frac{3}{5}$ | 31. $\frac{5}{8} \div \frac{3}{20}$ | 41. $\frac{16}{25} \div \frac{8}{15}$ |

Reduce mixed numbers to improper fractions and divide

- | | | | |
|-------------------------------------|-------------------------------------|--------------------------------------|--------------------------------------|
| 42. $2\frac{1}{2}$ by $\frac{5}{6}$ | 48. $3\frac{3}{4}$ by $\frac{3}{5}$ | 54. $2\frac{1}{4}$ by $\frac{3}{10}$ | 60. $2\frac{1}{2}$ by $3\frac{1}{3}$ |
| 43. $3\frac{1}{3}$ by $\frac{2}{3}$ | 49. $5\frac{5}{6}$ by $\frac{7}{8}$ | 55. $3\frac{1}{3}$ by $\frac{5}{12}$ | 61. $3\frac{3}{4}$ by $7\frac{1}{2}$ |
| 44. $2\frac{1}{4}$ by $\frac{3}{8}$ | 50. $8\frac{1}{3}$ by $\frac{5}{6}$ | 56. $5\frac{2}{5}$ by $\frac{9}{10}$ | 62. $4\frac{2}{3}$ by $5\frac{1}{4}$ |
| 45. $3\frac{3}{5}$ by $\frac{3}{4}$ | 51. $5\frac{5}{8}$ by $\frac{3}{4}$ | 57. $6\frac{1}{8}$ by $\frac{7}{16}$ | 63. $6\frac{1}{8}$ by $3\frac{1}{2}$ |
| 46. $4\frac{1}{6}$ by $\frac{5}{8}$ | 52. $4\frac{4}{9}$ by $\frac{2}{3}$ | 58. $9\frac{1}{8}$ by $1\frac{1}{2}$ | 64. $4\frac{1}{6}$ by $1\frac{7}{8}$ |
| 47. $6\frac{2}{3}$ by $\frac{4}{5}$ | 53. $9\frac{3}{8}$ by $\frac{5}{8}$ | 59. $8\frac{3}{4}$ by $1\frac{5}{6}$ | 65. $5\frac{5}{6}$ by $6\frac{1}{4}$ |

Division of an integer by a fraction.

1. Find results : $\frac{4}{4} \div \frac{1}{4}$; $1 \div \frac{1}{4}$; $\frac{4}{4} \div \frac{2}{4}$; $1 \div \frac{2}{4}$.

Compare $\frac{4}{4} \div \frac{1}{4}$ with $\frac{4}{4} \times \frac{4}{1}$; $1 \div \frac{1}{4}$ with $1 \times \frac{4}{1}$; $\frac{4}{4} \div \frac{2}{4}$ with $\frac{4}{4} \times \frac{4}{2}$; $1 \div \frac{2}{4}$ with $1 \times \frac{4}{2}$.

2. Give answers : $\frac{6}{3} \div \frac{1}{3}$; $2 \div \frac{1}{3}$; $\frac{6}{3} \div \frac{2}{3}$; $2 \div \frac{2}{3}$.

How does $\frac{6}{3} \div \frac{1}{3}$ compare with $\frac{6}{3} \times \frac{3}{1}$? $2 \div \frac{1}{3}$ with $2 \times \frac{3}{1}$? $\frac{6}{3} \div \frac{2}{3}$ with $\frac{6}{3} \times \frac{3}{2}$? $2 \div \frac{2}{3}$ with 2 multiplied by $\frac{3}{2}$ inverted?

3. Just as $\frac{8}{4} \div \frac{3}{4} = \frac{8}{4} \times \frac{4}{3} = 2\frac{2}{3}$, so $2 \div \frac{3}{4} = 2 \times \frac{4}{3} = 2\frac{2}{3}$.

To divide an integer by a fraction, *multiply the integer by the fraction inverted.*

Observe that the method of *dividing by a fraction* is the same whether the dividend is a fraction, a mixed number, or an integer.

Oral Exercises

Divide as indicated :

- | | | | | |
|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 1. $2 \div \frac{1}{2}$ | 6. $2 \div \frac{1}{3}$ | 11. $4 \div \frac{1}{8}$ | 16. $8 \div \frac{2}{3}$ | 21. $5 \div \frac{2}{3}$ |
| 2. $4 \div \frac{1}{3}$ | 7. $2 \div \frac{2}{5}$ | 12. $6 \div \frac{3}{4}$ | 17. $6 \div \frac{3}{8}$ | 22. $9 \div \frac{3}{4}$ |
| 3. $4 \div \frac{2}{3}$ | 8. $5 \div \frac{5}{6}$ | 13. $5 \div \frac{5}{8}$ | 18. $5 \div \frac{3}{4}$ | 23. $3 \div \frac{5}{8}$ |
| 4. $3 \div \frac{1}{4}$ | 9. $4 \div \frac{4}{5}$ | 14. $2 \div \frac{4}{5}$ | 19. $3 \div \frac{5}{6}$ | 24. $5 \div \frac{3}{8}$ |
| 5. $3 \div \frac{3}{4}$ | 10. $6 \div \frac{2}{3}$ | 15. $7 \div \frac{7}{8}$ | 20. $8 \div \frac{2}{5}$ | 25. $6 \div \frac{5}{6}$ |
26. At $\$ \frac{1}{2}$ a pound, how much tea can I buy for \$3?
27. How long will it take, at $\$ \frac{7}{10}$ a day, to earn \$7?
28. How much silk, at $\$ \frac{3}{4}$ a yard, can be bought for \$9?
29. Mr. Hay divided 5 lb. of candy among his children, giving each $\frac{5}{8}$ lb. How many children had he?
30. How many steps will a man take in walking 10 yd., if his step is $\frac{5}{6}$ yd. long?

Written Exercises

1. Divide 18 by
- $\frac{4}{5}$
- .

$$18 \div \frac{4}{5} = 18 \times \frac{5}{4} = \frac{45}{2} = 22\frac{1}{2}$$

To divide 18 by $\frac{4}{5}$, we may multiply 18 by $\frac{5}{4}$ inverted.

Indicating the work and *canceled*, we find the result to be $22\frac{1}{2}$.

Divide:

- | | | | |
|-----------------------|-------------------------|-------------------------|-------------------------|
| 2. 6 by $\frac{3}{8}$ | 8. 6 by $\frac{9}{10}$ | 14. 10 by $\frac{4}{5}$ | 20. 39 by $\frac{3}{8}$ |
| 3. 8 by $\frac{2}{7}$ | 9. 7 by $\frac{5}{12}$ | 15. 27 by $\frac{3}{4}$ | 21. 30 by $\frac{4}{5}$ |
| 4. 9 by $\frac{3}{5}$ | 10. 4 by $\frac{2}{15}$ | 16. 15 by $\frac{6}{7}$ | 22. 36 by $\frac{2}{3}$ |
| 5. 7 by $\frac{5}{6}$ | 11. 9 by $\frac{3}{16}$ | 17. 18 by $\frac{5}{6}$ | 23. 22 by $\frac{4}{9}$ |
| 6. 8 by $\frac{4}{9}$ | 12. 8 by $\frac{4}{15}$ | 18. 28 by $\frac{2}{3}$ | 24. 34 by $\frac{5}{6}$ |
| 7. 9 by $\frac{5}{8}$ | 13. 6 by $\frac{9}{20}$ | 19. 33 by $\frac{3}{4}$ | 25. 42 by $\frac{8}{9}$ |

Reduce mixed numbers to improper fractions and divide:

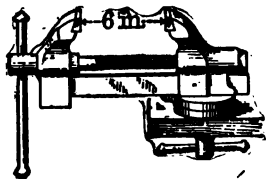
- | | | | |
|-------------------------|-------------------------|--------------------------|--------------------------|
| 26. 4 by $2\frac{2}{3}$ | 31. 8 by $4\frac{4}{5}$ | 36. 14 by $3\frac{1}{2}$ | 41. 40 by $5\frac{1}{3}$ |
| 27. 6 by $4\frac{1}{2}$ | 32. 9 by $6\frac{3}{4}$ | 37. 15 by $2\frac{1}{4}$ | 42. 45 by $6\frac{1}{4}$ |
| 28. 7 by $5\frac{2}{3}$ | 33. 7 by $6\frac{1}{8}$ | 38. 28 by $4\frac{2}{3}$ | 43. 55 by $6\frac{2}{3}$ |
| 29. 5 by $6\frac{1}{4}$ | 34. 8 by $6\frac{2}{3}$ | 39. 35 by $3\frac{3}{4}$ | 44. 39 by $7\frac{1}{2}$ |
| 30. 6 by $7\frac{1}{5}$ | 35. 9 by $5\frac{5}{8}$ | 40. 42 by $4\frac{1}{5}$ | 45. 49 by $8\frac{2}{5}$ |
46. How much ribbon at $\$ \frac{3}{8}$ a yard can I buy for \$6?
47. At $\$ \frac{3}{4}$ per day, how long must a boy work to earn \$9?
48. If it takes $4\frac{1}{2}$ yd. of silk to make a waist, how many waists can be made from a piece containing 18 yd.?
49. If $12\frac{1}{2}$ yd. of lace cost \$5, find the cost per yard.
50. If a Hudson River steamboat goes 36 miles in $2\frac{2}{3}$ hours, what is its average rate per hour?

Miscellaneous Exercises

1. What number multiplied by $\frac{3}{4}$ gives a product of 6?
2. How much will a yard of linen cost, if 2 yd. cost $\$ \frac{4}{5}$?
3. How many eggs, at $\$ \frac{2}{5}$ per dozen, can I buy for $\$ 1 \frac{1}{5}$?
4. How much candy, at $\$ \frac{3}{4}$ a pound, can be bought for $\$ 2 \frac{1}{4}$?

5. A dealer wishes to put $\frac{7}{10}$ of a ton of coal into bags holding $\frac{1}{20}$ of a ton each. How many bags does he need?

6. A merchant paid \$ 6 for rakes. How many rakes did he buy, if each rake cost $\$ \frac{2}{5}$?



7. If each turn of the screw brings the jaws of this vise $\frac{1}{8}$ in. nearer together, how many turns will it take to close the vise?

8. How much lace at $\$ \frac{2}{5}$ a yard can be bought for $\$ 1 \frac{3}{10}$?
9. A rider went around a track 17 times, thus riding a distance of $4 \frac{1}{4}$ miles. How far is it around the track?
10. If a train runs $9 \frac{3}{8}$ miles in 12 minutes, how far does it run in 1 minute?
11. A farmer received $\$ 8 \frac{3}{4}$ for some corn at $\$ \frac{5}{8}$ a bushel. How many bushels of corn did he sell?
12. A farmer put 75 bu. of apples into barrels holding $2 \frac{1}{2}$ bu. each. How many barrels did he use?
13. How many pieces of cord $\frac{7}{12}$ yd. long can be cut from $5 \frac{1}{4}$ yd. of cord?
14. If I walk at the rate of $3 \frac{1}{2}$ miles an hour, how long will it take me to walk $12 \frac{1}{4}$ miles?

FRACTIONAL RELATIONS

Finding what fractional part one number is of another.

1. How many squares does the rectangle contain? how many are shaded? What part of the rectangle is shaded?



Then, what part of 12 is 3?

How does this result compare with $3 \div 12$ or with $\frac{3}{12}$ reduced to its lowest terms?

2. What part of 12 is 9? 4? 8? 6?

Oral Exercises

What part of

1. 4 is 2? 6. 10 is 2? 11. 10 is 6? 16. 12 is 10?
2. 6 is 3? 7. 12 is 6? 12. 16 is 4? 17. 15 is 10?
3. 8 is 4? 8. 14 is 7? 13. 12 is 8? 18. 16 is 12?
4. 9 is 3? 9. 15 is 5? 14. 18 is 9? 19. 20 is 15?
5. 8 is 6? 10. 16 is 8? 15. 16 is 6? 20. 18 is 12?
21. What part of a yard is 1 ft.? 2 ft.?
22. What part of a gallon is 1 qt.? 2 qt.? 3 qt.?
23. Tell what part of a dozen 3 is; 4; 6.
24. What part of a foot is 6 in.? 8 in.? 9 in.?
25. What part of 20¢ is 5¢? 4¢? 2¢? 6¢? 10¢? 15¢? 12¢? 16¢? 18¢?
26. A boy who had 20 problems to work solved 15 of them. What part of the problems did he solve?
27. A farmer raised 16 bu. of onions and sold 14 bu. of them. What part of his crop did he sell?

Written Exercises

Tell what part of the first number the second is:

- | | | | |
|-----------|------------|------------|-----------------------------------|
| 1. 30, 20 | 6. 60, 48 | 11. 96, 48 | 16. $\frac{1}{2}$, $\frac{1}{3}$ |
| 2. 36, 18 | 7. 75, 50 | 12. 80, 64 | 17. $\frac{3}{4}$, $\frac{1}{8}$ |
| 3. 40, 25 | 8. 64, 16 | 13. 48, 42 | 18. $\frac{2}{3}$, $\frac{1}{6}$ |
| 4. 50, 30 | 9. 36, 27 | 14. 64, 48 | 19. $\frac{5}{8}$, $\frac{1}{4}$ |
| 5. 48, 36 | 10. 40, 35 | 15. 70, 28 | 20. $\frac{7}{8}$, $\frac{1}{2}$ |
21. What part of 100 is 10? 20? 25? 40? 50? 60?
22. What part of \$1 is 10¢? 20¢? 25¢? 50¢? 75¢?
23. A grocer bought 64 doz. eggs. When he had sold 40 doz., what part of them had he sold?
24. A dealer, wishing to dispose of 75 suits, sold 60 of them at a bargain sale. What part of his suits did he sell?
- Finding the whole when a fractional part of it is given.**

Oral Exercises

1. If 3 oranges cost 9¢, how much will 1 orange cost? 4 oranges?
2. If 3 fourths of a cake cost 9¢, how much will 1 fourth of it cost? 4 fourths, or the whole cake?
3. If $\frac{3}{4}$ of the cost of a top is 9¢, how much is $\frac{1}{4}$ of the cost? $\frac{4}{4}$, or the whole cost?
4. If $\frac{3}{4}$ of a number is 9, what is $\frac{1}{4}$ of it? the number?
5. If $\frac{2}{5}$ of a number is 10, what is the number?
6. If $\frac{2}{3}$ of a number is 12, what is the number?
7. If $\frac{3}{8}$ of a number is 15, what is the number?

Find the number of which

8. 10 is $\frac{2}{3}$ 11. 16 is $\frac{4}{5}$ 14. 21 is $\frac{3}{5}$ 17. 36 is $\frac{3}{10}$
 9. 12 is $\frac{2}{5}$ 12. 20 is $\frac{5}{6}$ 15. 25 is $\frac{5}{8}$ 18. 45 is $\frac{5}{12}$
 10. 18 is $\frac{3}{4}$ 13. 24 is $\frac{3}{8}$ 16. 28 is $\frac{7}{8}$ 19. 66 is $\frac{11}{12}$
 20. Find the cost of a pound of mustard, if $\frac{3}{4}$ lb. costs 24¢.
 21. If $\frac{5}{8}$ of the pupils in our grade are girls and there are 25 girls, how many pupils are there in the grade?
 22. Elmer weighs 55 lb., and his weight is $\frac{11}{12}$ that of Henry. How much does Henry weigh?

Written Exercises

1. If 256 is $\frac{4}{5}$ of a number, what is the number?

$$\begin{array}{r} 4)256 \\ \underline{64} \\ 16 \\ \underline{5} \\ 320 \end{array}$$

$\frac{4}{5}$ of the number = 256;
 $\frac{1}{5}$ of the number = $\frac{1}{4}$ of 256 = 64;
 $\frac{5}{5}$, or the whole, of the number = $5 \times 64 = 320$.

Find the number of which

2. 36 is $\frac{2}{3}$ 7. 70 is $\frac{5}{8}$ 12. 120 is $\frac{2}{5}$ 17. 264 is $\frac{2}{3}$
 3. 45 is $\frac{3}{8}$ 8. 84 is $\frac{3}{4}$ 13. 138 is $\frac{3}{4}$ 18. 504 is $\frac{3}{4}$
 4. 56 is $\frac{4}{5}$ 9. 85 is $\frac{5}{6}$ 14. 256 is $\frac{2}{3}$ 19. 792 is $\frac{4}{5}$
 5. 63 is $\frac{3}{4}$ 10. 72 is $\frac{3}{5}$ 15. 280 is $\frac{5}{6}$ 20. 645 is $\frac{5}{6}$
 6. 75 is $\frac{5}{6}$ 11. 91 is $\frac{7}{8}$ 16. 342 is $\frac{3}{8}$ 21. 847 is $\frac{7}{8}$
 22. Find the cost of a horse, if $\frac{3}{4}$ of the cost is \$135.
 23. Find the cost of a piano, if $\frac{3}{5}$ of the cost is \$315.
 24. Mr. Day has \$278 in the bank. If this is $\frac{2}{3}$ of all the money he has, how much money has he?

ANALYSIS

Oral and Written Exercises

1. If 6 roses cost 30 ¢, how much do 5 roses cost?

MODEL SOLUTION

6 roses cost 30 ¢.

1 rose costs $30 \text{ ¢} \div 6$, or 5 ¢.

5 roses cost 5 times 5 ¢, or 25 ¢.

Observe that we first find the cost of 1 rose from the cost of 6 roses, and then find the cost of 5 roses from the cost of 1 rose.

Give orally the *analysis* of each problem; then write the solution:

2. If 2 pencils cost 8 ¢, how much do 7 pencils cost?
3. Find the cost of 10 qt. of oil, when 4 qt. cost 12 ¢.
4. Find the cost of 5 qt. of molasses at 36 ¢ a gallon.
5. When 7 eggs cost 28 ¢, find the cost of 12 eggs.
6. What is the cost of 9 pt. of milk at 6 ¢ a quart?
7. If 4 telephone calls cost 20 ¢, find the cost of 7 calls.
8. When 6 peaches cost 12 ¢, how much must be paid for 10 peaches?
9. How many pounds of coffee do 11 coffee trees yield, if 6 trees yield 12 lb. of coffee?
10. When 5 bunches of mint cost 30 ¢, how much do 9 bunches cost?
11. If 4 railroad tickets cost \$1, how much do 6 such tickets cost?
12. A woman bought 10 pineapples for \$1 and sold 3 to a neighbor at cost. How much did she receive for them?

Oral and Written Exercises

1. $10\phi = 5$ times 2ϕ ; $2\phi = \frac{1}{5}$ of 10ϕ .
2. $\$12 = \text{---}$ times $\$4$; $\$4 = \text{---}$ of $\$12$.

In the same way compare:

- | | |
|--|--|
| 3. 12ϕ and 3ϕ . | 7. 5 hats and 10 hats. |
| 4. 15ϕ and 5ϕ . | 8. 1 foot and 2 inches. |
| 5. $\$20$ and $\$4$. | 9. 1 quart and 1 gallon. |
| 6. $\$24$ and $\$6$. | 10. 1 bushel and 2 pecks. |
| 11. If 2 lemons cost 5ϕ ,
find the cost of 6 lemons. | 12. If 12 oranges cost 30ϕ ,
find the cost of 4 oranges. |

MODEL SOLUTION

2 lemons cost 5ϕ .
6 lemons are 3×2 lemons.
6 lemons cost $3 \times 5\phi$, or 15ϕ .

MODEL SOLUTION

12 oranges cost 30ϕ .
4 oranges are $\frac{1}{3}$ of 12 oranges.
4 oranges cost $\frac{1}{3}$ of 30ϕ , or 10ϕ .

Give orally the analysis of each problem; then write the solution:

13. If 4 plums cost 3ϕ , how much do 12 plums cost?
14. If 12 bananas cost 20ϕ , find the cost of 3 bananas.
15. If you can walk 5 miles in 2 hours, how far can you walk in 4 hours?
16. If a boat sails 48 miles in 4 hours, how far does it sail in 2 hours?
17. If 12 eggs cost 32ϕ , how much do 3 eggs cost?
18. How many valentines can you buy for 40ϕ , if 3 valentines cost 10ϕ ?
19. How long will it take a boy to work 42 problems at the rate of 6 problems in 10 minutes?

20. Eliza bought 6 small flags for 5¢. How many could she buy for 25¢?

21. A man earns \$3 in 8 hours. How much does he earn in 72 hours?

22-25. At the store Eva saw piles of oranges marked :

3 for 10¢

2 for 5¢

3 for 5¢

4 for 5¢

She had 40¢. How many oranges of the first kind could she buy? of each of the other kinds?

26-29. Find the cost of a dozen oranges of each kind.

Find the cost of:

30. 4 buns at 18¢ a dozen.

31. 8 rolls at 12¢ a dozen.

32. 6 bananas at 22¢ a dozen.

33. 18 pears at 6 pears for 10¢.

34. 6 lb. grapes at 2 lb. for 15¢.

35. 5 lb. beans at 10 lb. for 60¢.

36. 16 apples at 4 apples for 12¢.

37. 2 cans of corn at 10 cans for \$1.

38. 12 pairs of shoes at 2 pairs for \$7.

39. 8 boxes of berries at 4 boxes for 25¢.

40. 3 cans of tomatoes at 12 cans for \$1.

41. 1 doz. grapefruits at 3 grapefruits for 25¢.

42. 2 packages of oatmeal at 8 packages for \$1.

43. 18 cakes of soap at 6 cakes for a quarter dollar.

PROBLEM INTERPRETATION

Oral and Written Exercises

Without solving, tell *how to solve* each of these problems ; then write the solution :

1. At 50¢ each, how many books can I buy for \$ 5 ?
2. How many bushels are $8\frac{1}{2}$ bu., $10\frac{3}{4}$ bu., and $12\frac{3}{8}$ bu. ?
3. If 8 yd. silk cost \$ 10, how much do 4 yd. cost ?
4. Find the cost of 1 doz. melons, if 2 melons cost \$.25.
5. A barrel of flour weighs 196 lb. How much does a quarter of a barrel of flour weigh ?
6. How far does Edward walk in 4 hours, at the rate of $2\frac{1}{2}$ miles per hour ?
7. Henry bought 3 pencils at 5¢ each and had 14¢ left. How much money had he at first ?
8. If 2 plum trees bear 22 pk. of plums, how many pecks of plums do 3 such trees bear ?
9. Helen spent $\frac{4}{5}$ of the money she had for a fan. If the fan cost 60¢, how much money had she at first ?
10. If a motor car ran 2 miles while a train ran 3 miles, how far did the car go while the train ran 24 miles ?
11. The product of three numbers is 432. If two of the numbers are 8 and 9, what is the third number ?
12. If $\frac{2}{5}$ of the value of a house is \$ 800, how much is the house worth ?
13. A train averages 7 miles every 8 minutes. How long would it take to make a trip of 49 miles ?

PROBLEM MAKING

Oral and Written Exercises

Write as many problems as you can from the following conditions ; then solve your problems :

1. A boy had \$ $1\frac{1}{4}$ and afterward spent \$ $\frac{1}{2}$.
2. Alfred had 42 walnuts. He cracked $\frac{1}{6}$ of them.
3. Mary's pansy bed is 8 ft. long and 4 ft. wide.
4. Marbles cost 1¢ for 6, 1¢ for 3, 2¢ for 5, 3¢ for 5.

The following are prices at a fruit stand :

- | | |
|------------------------|-----------------------------|
| 5. Figs, 20¢ a pound. | 9. Nuts, 18¢ a pound. |
| 6. Dates, 12¢ a pound. | 10. Lemons, 2¢ apiece. |
| 7. Bananas, 2 for 3¢. | 11. Grapes, 15¢ a pound. |
| 8. Oranges, 2 for 5¢. | 12. Cantaloupes, 3 for 25¢. |

Here are some prices at a stationer's store :

- 13 Pencils, 1¢, 2¢, 3¢, 5¢ each ; 10¢ a doz. ; 25¢ a doz.
14. Pens, 6 for 5¢.
15. Books, from 25¢ to \$1 each.
16. Blotters, 8 for 5¢.
17. Ink, 5¢ a bottle, small size ; 25¢ a bottle, large size.
18. Edward spent $\frac{1}{2}$ of his money for a goat and $\frac{1}{6}$ of it for a wagon.
19. Frank had 60¢. The price of oranges was 3 for 10¢. The price of bananas was 20¢ a dozen.
20. Some berry pickers picked $8\frac{1}{4}$ crates of strawberries in the forenoon and $4\frac{1}{2}$ crates in the afternoon.

Camp Fire Girl Problems

1. At a union council fire there were 15 Camp Fires of 12 girls each. How many Camp Fire Girls attended?

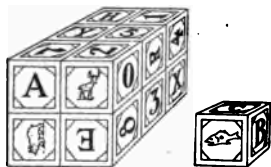


A Camp Fire Girl won an honor by:

2. Riding horseback $8\frac{1}{4}$ miles per day, on the average, for 5 days. How many miles did she ride?
3. Hemming the ends of 2 towels each 18 in. wide. How many yards of hemming did she do?
4. Filling a regular position for 18 weeks. Her wages were \$6.25 per week. How much did she earn?
5. Walking $42\frac{1}{2}$ miles in 10 days. How many miles did she average per day?
6. Raising 82 qt. of strawberries that sold at $12\frac{1}{2}\text{¢}$ per quart. How much did she receive for them?
7. Make out and receipt a bill for: 150 honor beads @ 1¢ ; 3 Torch Bearer's pins @ 50¢ ; 12 hat pins @ 25¢ ; 8 pairs of moccasins @ \$1.25.

VOLUME MEASURES

1. The blocks shown in the picture are **cubes**.



How many corners has a cube?
how many edges? how many flat
sides, or *faces*?

2. How do the edges compare in
length? the faces in shape? in
area? Describe the shape of each face.

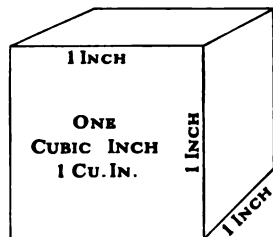
3. How long is the front face of this cube? how wide?
What is its area?

Each face is a *square inch*.

A cube whose faces are each a
square inch is a **cubic inch**.

4. What name is given to a cube,
if each of its faces is a *square foot*?

How long, how wide, and how
high is a cubic inch? a cubic
foot?



5. Let us see if we can find the number of blocks in
the pile at the top of the page, without counting them.

There are 2 layers of blocks, and 2 rows in each layer.

Now, in 1 row there are 3 blocks.

In 1 layer (2 rows) there are 2×3 blocks.

In 2 layers there are 2 times 2×3 blocks.

That is, the pile contains $2 \times 2 \times 3$ blocks, or 12 blocks.

6. On the next page there is a picture of a block of
wood 4 in. long, 2 in. wide, and 3 in. high, or "4 in. by 2
in. by 3 in.," marked to show cubic inches.

Find how many cubic inches there are in this block.

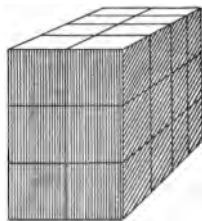
There are 3 layers of cubic inches with
2 rows in each.

In 1 row there are 4 cu. in.

In 1 layer there are 2×4 cu. in.

In 3 layers there are $3 \times 2 \times 4$ cu. in.

That is, the block contains $3 \times 2 \times 4$ cu.
in., or 24 cu. in.



This is the **volume** of the block.

7. A block is 5 in. by 4 in. by 3 in. Find its volume.

Think of rows and layers of cubic inches.

8. What is the volume of a cube whose edge is 4 in.?

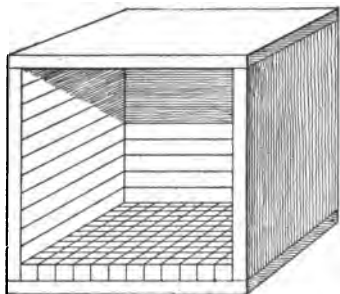
Such a cube is called a *4-inch cube*. What is a 6-inch cube?

Oral Exercises

Find the volume of:

1. A 3-foot cube.
2. A tank, 5 ft. by 4 ft. by 4 ft.
3. A brick, 8 in. by 4 in. by 2 in.
4. A coal bin, 6 ft. by 5 ft. by 3 ft.
5. A cake of ice, 3 ft. by 2 ft. by 1 ft.
6. A stone block, 4 ft. by 4 ft. by 3 ft.
7. A cement wall, 20 ft. by 3 ft. by 1 ft.
8. How many cubic feet of dirt can be put into the box of a cart that is 5 ft. by 4 ft. by 2 ft. on the inside?
9. A packing case is 4 ft. long, 3 ft. wide, and 3 ft. high. How many cubic feet of space does it occupy?

1. The inside of this box is 1 ft. long, 1 ft. wide, and 1 ft. deep. How many cubic feet will the box contain?

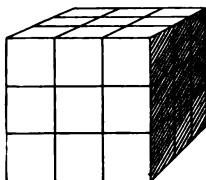


2. To find how many cubic *inches* it will hold, put a layer of inch cubes in the bottom of the box, as in the picture.

Since the bottom of the box is 1 ft. square, how many cubic inches are there in this layer?

3. Since the box is 1 ft. high, how many such layers are required to fill the box? Find by multiplication the number of cubic inches in a cubic foot.

4. Each edge of this block is 1 yd. long. What is the volume of the block?



5. Since the top face of the block is 1 yd. square, how many cubic feet are there in the top layer of foot cubes?
6. Since the block is 1 yd. high, how many such layers are there? how many foot cubes?
7. Learn this table of **volume measures** :

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

8. How many cubic feet are there in 3 cu. yd.? in $\frac{1}{3}$ cu. yd.? cubic inches in 2 cu. ft.? in $\frac{1}{2}$ cu. ft.?

Written Exercises

Find the volume of:

1. A bin, 7 ft. by 3 ft. by 4 ft.
2. A trough, 16 ft. by 2 ft. by 2 ft.
3. A room, 18 ft. by 12 ft. by 10 ft.
4. A suitcase, 24 in. by 14 in. by 6 in.
5. A poultry crate, 39 in. by 39 in. by 13 in.
6. A medicine cabinet, 24 in. by 19 in. by 7 in.
7. How many cubic feet of air does a room contain that is 20 ft. long, 18 ft. wide, and 12 ft. high?
8. Leo's wagon box is 20 in. by 16 in. by 5 in., on the inside. How much less than a cubic foot will it hold?
9. When snow is 1 ft. deep, how many cubic yards of it will be shoveled in making a path 27 ft. long and 2 ft. wide?
10. In digging a cellar 24 ft. by 18 ft. by 6 ft., how many cubic yards of dirt were removed?

Roy estimated and measured as follows. Find the true volume and the error in each case.

We often write (') instead of "ft." and (") instead of "in."

THING MEASURED	ESTIMATED DIMENSIONS	MEASURED DIMENSIONS
11. Bin	15' by 6' by 8'	14' by 5' by 8'
12. Room	17' by 14' by 9'	18' by 12' by 9'
13. Box	16" by 12" by 9"	15" by 12" by 10"
14. Trunk	35" by 20" by 25"	34" by 21" by 24"
15. Stone step	96" by 12" by 12"	90" by 10" by 10"
16. Estimate and measure the dimensions and volume of various things, as boxes, rooms, cabinets, etc.		

REVIEW

Oral Exercises

1. Give five fractions.

What are the terms of a fraction called? What does the denominator show? the numerator?

2. Read the following fractions and name the numerator and the denominator of each: $\frac{5}{6}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{8}$; $\frac{7}{16}$.

3. Reduce $\frac{8}{12}$ to its lowest terms; to twenty-fourths.

When is a fraction expressed in its lowest terms? How may a fraction be reduced to higher terms?

4. Reduce to lowest terms: $\frac{4}{8}$; $\frac{3}{6}$; $\frac{6}{9}$; $\frac{4}{6}$; $\frac{6}{8}$; $\frac{8}{12}$; $\frac{6}{10}$.
5. Change to twenty-fourths: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{5}{6}$; $\frac{5}{12}$; $\frac{11}{12}$.
6. Name the integers, mixed numbers, proper and improper fractions: $2\frac{1}{2}$; 1; $\frac{7}{8}$; $\frac{8}{9}$; 7; $\frac{2}{3}$; $3\frac{4}{5}$; 9; $\frac{3}{8}$.
7. Reduce 4 to eighths; $7\frac{3}{4}$ to fourths.

How may integers and mixed numbers be reduced to fractions?

8. Change 8 to halves; to fifths; to sixths; to eighths.
9. Reduce to a fraction: $8\frac{1}{3}$; $6\frac{1}{4}$; $4\frac{5}{8}$; $9\frac{5}{8}$; $6\frac{4}{5}$; $5\frac{7}{12}$.
10. Change $2\frac{4}{8}$ to an integer; $1\frac{6}{5}$ to a mixed number.

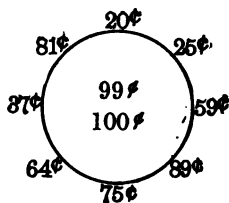
What is the value of a fraction?

11. Reduce to a mixed number: $\frac{8}{5}$; $\frac{7}{3}$; $\frac{9}{4}$; $\frac{15}{8}$; $\frac{25}{6}$; $\frac{49}{5}$.
12. Reduce to fractions having the least common denominator: $\frac{3}{8}$ and $\frac{1}{4}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{5}{6}$ and $\frac{5}{8}$; $\frac{11}{12}$ and $\frac{7}{8}$.
13. Add $\frac{3}{8}$ and $\frac{7}{8}$. Subtract $\frac{2}{3}$ from $\frac{3}{4}$. Explain.
14. Find the g. c. d. of 15 and 20; of 12, 18, and 24.

Read the following:

15. XL 17. CD 19. XLIX 21. DCCC 23. XLVII
16. XC 18. CM 20. LXIV 22. XCIV 24. LXXXI

25. Add each number of cents outside the ring to 100¢, or \$1, and then to 99¢. Give answers in dollars and cents. Add rapidly in either direction.



26. Subtract each number of cents outside the ring from 99¢ and then from \$1, or 100¢. Subtract rapidly in either direction.

In the following, give the parts indicated:

- | $\frac{2}{3}$ of | $\frac{3}{4}$ of | $\frac{3}{8}$ of | $\frac{5}{6}$ of | $\frac{4}{5}$ of | $\frac{5}{8}$ of |
|------------------|------------------|------------------|------------------|------------------|------------------|
| 27. 24 | 30. 20 | 33. 32 | 36. 42 | 39. 25 | 42. 80 |
| 28. 27 | 31. 40 | 34. 40 | 37. 54 | 40. 45 | 43. 72 |
| 29. 36 | 32. 48 | 35. 64 | 38. 72 | 41. 60 | 44. 96 |

Read the following:

45. \$.08 47. 580,058 49. \$13.79 51. 101,650,048
46. \$.62 48. 709,045 50. \$37.05 52. 310,000,969

Give answers promptly:

- | | | | |
|--------------------------------------|------------------------------------|---------------------------------------|----------------------------------|
| 53. $\frac{1}{4} + \frac{3}{8}$ | 60. $8 \div \frac{2}{3}$ | 67. $\frac{5}{6} + \frac{11}{12}$ | 74. $1\frac{1}{2} + \frac{3}{4}$ |
| 54. $\frac{5}{6} - \frac{1}{2}$ | 61. $\frac{3}{4} \div \frac{2}{5}$ | 68. $\frac{3}{8} \times 16$ | 75. $2\frac{2}{3} - \frac{1}{6}$ |
| 55. $\frac{2}{3} \times 9$ | 62. $\frac{3}{5} + \frac{1}{2}$ | 69. $\frac{4}{5} - \frac{7}{10}$ | 76. $3\frac{1}{3} \div 2$ |
| 56. $5 \times \frac{3}{8}$ | 63. $\frac{3}{4} \div 6$ | 70. $\frac{5}{6} \div \frac{5}{12}$ | 77. $4\frac{3}{4} + \frac{1}{2}$ |
| 57. $\frac{4}{5} \times 6$ | 64. $\frac{5}{6} - \frac{2}{3}$ | 71. $\frac{2}{3} \times \frac{3}{10}$ | 78. $5\frac{1}{2} - \frac{7}{8}$ |
| 58. $\frac{1}{3} \div 8$ | 65. $8 \times \frac{3}{4}$ | 72. $9 \div \frac{9}{10}$ | 79. $6\frac{1}{4} + \frac{5}{6}$ |
| 59. $\frac{3}{4} \times \frac{5}{6}$ | 66. $\frac{3}{4} \times 6$ | 73. $8 \times \frac{1}{12}$ | 80. $4\frac{1}{2} + \frac{2}{3}$ |

Complete :

- | | |
|-------------------|---------------------------------------|
| 81. 1 pk. = — qt. | 86. 1 sq. yd. = — sq. ft. |
| 82. 1 ft. = — in. | 87. 1 sq. ft. = — sq. in. |
| 83. 1 ft. = — yd. | 88. 1 cu. yd. = — cu. ft. |
| 84. 1 pt. = — qt. | 89. 1 cu. ft. = — cu. in. |
| 85. 1 pk. = — bu. | 90. $\frac{1}{3}$ sq. yd. = — sq. ft. |

91. Find the volume of a 4-foot cube.

92. When 4 yd. of satin cost \$8, find the cost of 12 yd.

93. Percy spent 5¢, or $\frac{1}{6}$ of his money, for peanuts. How much money had he at first?

Frank, Clara, Mabel, and Alfred made pop corn balls and candy according to the following receipts:

POP CORN BALLS	PEANUT CANDY	WALNUT CANDY
$\frac{1}{2}$ pt. molasses	$\frac{3}{4}$ pt. molasses	1 pt. molasses
$\frac{1}{2}$ lb. butter	$\frac{1}{4}$ lb. butter	$\frac{1}{8}$ lb. butter
$\frac{1}{4}$ lb. red sugar	5 qt. peanuts	$\frac{3}{4}$ lb. brown sugar
2 $\frac{1}{2}$ lb. pop corn	Salt	2 $\frac{1}{2}$ lb. English walnuts
Salt		1 tablespoon vinegar

94. Frank bought the molasses. How many pints did he buy? How much did it cost at \$.16 a quart?

95. Clara bought the butter at \$.32 a pound and the pop corn at \$.10 a pound. How much did she spend?

96. Mabel bought the sugar. She paid 4¢ a pound for brown sugar and 28¢ a pound for red sugar. How much did she spend for sugar?

97. Alfred bought the peanuts at 5¢ a quart and the walnuts at 20¢ a pound. How much did both cost?

Written Exercises

Reduce to lowest terms :

1. $\frac{12}{20}$ 3. $\frac{24}{36}$ 5. $\frac{35}{60}$ 7. $\frac{24}{64}$ 9. $\frac{48}{72}$ 11. $\frac{25}{100}$
 2. $\frac{20}{25}$ 4. $\frac{25}{40}$ 6. $\frac{14}{56}$ 8. $\frac{27}{90}$ 10. $\frac{56}{96}$ 12. $\frac{75}{100}$

Write the following, using Roman numerals :

13. 12 15. 29 17. 56 19. 75 21. 92 23. 400
 14. 18 16. 42 18. 62 20. 80 22. 99 24. 900

Reduce to an integer or a mixed number :

25. $\frac{75}{5}$ 26. $\frac{91}{4}$ 27. $\frac{55}{3}$ 28. $\frac{88}{6}$ 29. $\frac{89}{5}$ 30. $\frac{96}{3}$

Subtract from \$100.00 :

31. \$4.95 33. \$38.50 35. \$57.64 37. \$87.16
 32. \$6.24 34. \$42.89 36. \$49.21 38. \$72.04
 39. Add each of the above numbers to \$85.98.
 40. Change to gallons and quarts: 37 qt.; 58 qt.;
 85 qt.; 74 qt.; 93 qt.; 69 qt.

Reduce to a fraction :

41. $8\frac{3}{8}$ 42. $9\frac{5}{6}$ 43. $18\frac{3}{4}$ 44. $25\frac{2}{3}$ 45. $32\frac{3}{5}$ 46. $44\frac{1}{8}$

Factor as on page 98 :

47. 45 51. 132 55. 486 59. 1320 63. 4620
 48. 64 52. 228 56. 650 60. 1008 64. 5760
 49. 56 53. 189 57. 936 61. 2250 65. 7000
 50. 96 54. 375 58. 840 62. 3024 66. 5544

67. Find the sum of 85 dollars and 32 cents, 76 dollars and 9 cents, 20 dollars and 10 cents, and 96 dollars.

68. Add $18\frac{3}{4}$ to $25\frac{3}{4}$. Explain your work.

69. Multiply $\frac{5}{8}$ by 48; 84 by $\frac{7}{8}$; $\frac{15}{8}$ by $\frac{4}{5}$; $3\frac{3}{4}$ by $3\frac{1}{2}$.

How do you multiply a fraction by an integer? any number by a fraction?

70. Divide $\frac{2}{3}$ by 24; 36 by $\frac{3}{8}$; $\frac{24}{5}$ by $\frac{4}{5}$. Explain.

Find the answers quickly:

- | | | | |
|--------------------------------------|--|--|--|
| 71. $\frac{7}{12} \times 36$ | 75. $\frac{25}{4} + 10$ | 79. $4\frac{3}{8} + 5\frac{5}{8}$ | 83. $4\frac{1}{6} \times 4\frac{4}{5}$ |
| 72. $\frac{9}{20} \div \frac{3}{10}$ | 76. $\frac{5}{12} \times \frac{3}{10}$ | 80. $\frac{9}{10} \times 5\frac{1}{3}$ | 84. $\frac{24}{5} \div \frac{8}{15}$ |
| 73. $21 \div \frac{7}{12}$ | 77. $32 \div 3\frac{1}{2}$ | 81. $\frac{20}{4} + 10$ | 85. $40 \times \frac{5}{16}$ |
| 74. $15 \times \frac{9}{10}$ | 78. $2\frac{2}{5} \times 3\frac{1}{8}$ | 82. $\frac{8}{15} \times \frac{5}{12}$ | 86. $5\frac{1}{4} + 1\frac{1}{8}$ |

Point off into periods and then write in words:

- | | | | |
|-----------|------------|-------------|---------------|
| 87. 56004 | 89. 150245 | 91. 6001245 | 93. 683120041 |
| 88. 30467 | 90. 870017 | 92. 4890063 | 94. 860054003 |

What part of

- | | | | |
|---------------|---------------|----------------|----------------|
| 95. 30 is 20? | 97. 36 is 24? | 99. 72 is 48? | 101. 90 is 54? |
| 96. 40 is 15? | 98. 64 is 32? | 100. 80 is 32? | 102. 96 is 72? |

Find the number of which

- | | | | |
|--------------------------|--------------------------|---------------------------|---------------------------|
| 103. 36 is $\frac{4}{5}$ | 105. 72 is $\frac{2}{3}$ | 107. 125 is $\frac{5}{8}$ | 109. 320 is $\frac{2}{3}$ |
| 104. 48 is $\frac{3}{4}$ | 106. 84 is $\frac{7}{8}$ | 108. 280 is $\frac{6}{7}$ | 110. 516 is $\frac{3}{5}$ |

Find correct answers, timing yourself:

- | | | |
|-----------------------|--------------------------|-------------------------|
| 111. 1205×43 | 116. $25 \times \$32.25$ | 121. $\$156.60 \div 18$ |
| 112. $3564 \div 27$ | 117. $36 \times \$28.40$ | 122. $\$289.34 \div 23$ |
| 113. $4828 \div 71$ | 118. $52 \times \$40.56$ | 123. $\$732.48 \div 24$ |
| 114. 3460×35 | 119. $60 \times \$59.75$ | 124. $\$665.28 \div 32$ |
| 115. $5332 \div 62$ | 120. $42 \times \$72.08$ | 125. $\$895.40 \div 44$ |

Divide, using cancellation :

126. $8 \times 24 \times 6$ by 18×12 130. $24 \times 32 \times 15$ by $20 \times 8 \times 6$
 127. $7 \times 18 \times 9$ by 15×28 131. $19 \times 45 \times 12$ by $38 \times 9 \times 4$
 128. $5 \times 36 \times 8$ by 24×10 132. $48 \times 16 \times 22$ by $44 \times 5 \times 8$
 129. $6 \times 40 \times 7$ by 12×42 133. $17 \times 64 \times 18$ by $34 \times 6 \times 9$
 134. Change to forty-eighths: $\frac{1}{4}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{11}{12}$; $\frac{15}{16}$.
 135. Express in feet and inches: 225 in.; 343 in.; 405 in.

Reduce to fractions having the l. c. d. :

136. $\frac{2}{3}$ and $\frac{3}{5}$ 139. $\frac{5}{6}$ and $\frac{7}{8}$ 142. $\frac{3}{4}$ and $\frac{7}{10}$ 145. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$
 137. $\frac{1}{4}$ and $\frac{5}{6}$ 140. $\frac{4}{5}$ and $\frac{1}{6}$ 143. $\frac{1}{2}$ and $\frac{8}{15}$ 146. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{12}$
 138. $\frac{2}{5}$ and $\frac{3}{4}$ 141. $\frac{3}{8}$ and $\frac{1}{5}$ 144. $\frac{5}{8}$ and $\frac{7}{12}$ 147. $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{32}$

Perform the operations indicated :

148. $84 - 26 + 37 - 18$ 152. $5\frac{3}{4} + 6\frac{1}{2} - 4\frac{7}{8}$
 149. $46 + 39 - 58 + 24$ 153. $6\frac{2}{3} + 7\frac{1}{2} - 7\frac{5}{8}$
 150. $65 - 28 - 19 + 76$ 154. $8\frac{1}{2} - 3\frac{5}{8} + 8\frac{3}{4}$
 151. $54 + 62 + 75 - 98$ 155. $5\frac{4}{5} + 6\frac{1}{2} - 3\frac{7}{10}$

Reduce :

156. 18 bu. to pecks. 161. 5 ft. 7 in. to inches.
 157. 15 ft. to inches. 162. 3 bu. 5 pk. to pecks.
 158. 8 sq. ft. to sq. in. 163. 32 gal. 3 qt. to quarts.
 159. 192 qt. to gallons. 164. 25 sq. yd. 8 sq. ft. to sq. ft.
 160. 208 pk. to bushels. 165. 8 sq. ft. 96 sq. in. to sq. in.

166. W. Ryan bought of Harder & Co. 7 yd. of linoleum @ \$ 2.48 and 14 yd. of carpet @ \$ 1.25. Supplying necessary data, make out, foot, and receipt the bill.

167. How many times will $4\frac{3}{4}$ pk. of peanuts fill a quart measure?

168. How many cubic feet of water will a cistern 6 ft. square and 5 ft. deep hold?

169. The product of three numbers is 560. If two of the numbers are 10 and 7, find the third number.

170. A merchant bought some goods for \$637.45 and sold them for \$964.94. How much did he gain?

171. A cubic foot of water weighs $62\frac{1}{2}$ lb. Find the weight of 9 cu. ft. of water.

172. Floy's watch cost her father \$32.50 and the chain \$12.35. How much change did he receive out of \$50.00?

173. How far is it from Chicago to Detroit, if $\frac{3}{8}$ of the distance is 171 miles?

174. A farmer bought 560 fence posts for \$100.80. Find the cost of each post.

175. Each sheep of a flock of 20 yielded, on the average, $31\frac{5}{8}$ lb. of wool. Find the total yield.

176. One year Mr. West raised 364 bu. of grain on $6\frac{1}{2}$ acres of land. Find the average yield per acre.

177. I paid \$632 for a lot and sold it for $\frac{7}{8}$ of the cost. For how much did I sell it?

178. A grocer bought 96 doz. eggs. When he had sold 72 doz., what part of them had he sold?

179. How many times can $5\frac{1}{4}$ gal. of vinegar be drawn from a barrel that contains $31\frac{1}{2}$ gal.?

180. A grocer bought $96\frac{1}{2}$ lb. of mixed nuts at \$.12 a pound and sold them at \$.18 a pound. Find his gain.

FIFTH YEAR — FIRST HALF

Before beginning the work of this year review pages 216–222.

DECIMAL FRACTIONS

NOTATION AND NUMERATION OF DECIMALS

When anything is divided into 10 equal parts, each part is *one tenth*, which may be written either $\frac{1}{10}$ or .1.

The period before the figure 1 is the **decimal point**.

1. Read: $\frac{3}{10}$; .3; $\frac{5}{10}$; .5; .6; .9; .8.

2. Write in two ways: 2 tenths; 4 tenths; 7 tenths.

If each tenth of anything is divided into 10 equal parts, the whole thing is divided into 100 equal parts. Each part is a **hundredth**, which may be written either $\frac{1}{100}$ or .01.

3. Read: $\frac{3}{100}$; .03; $\frac{25}{100}$; .25; $\frac{68}{100}$; .68; .37; .06; .49; .09; .75.

4. Write in two ways: 8 hundredths; 35 hundredths.

If each hundredth of anything is divided into 10 equal parts, the whole thing is divided into 1000 equal parts. Each part is a **thousandth**, which may be written either $\frac{1}{1000}$ or .001.

5. Read: $\frac{3}{1000}$; .003; $\frac{19}{1000}$; .019; $\frac{225}{1000}$; .225; .007; .045; .372; .608; .256; .009; .075.

6. Write in two ways: 5 thousandths; 25 thousandths; 452 thousandths; 8 thousandths; 63 thousandths; 999 thousandths.

One *and* 1 tenth may be written 1.1; 1 *and* 1 hundredth, 1.01; 1 *and* 1 thousandth, 1.001.

7. Write: 5 and 7 tenths; 4 and 56 hundredths; 47 and 139 thousandths; 23 and 9 hundredths.

8. Read, using the word *and* between the integer and the decimal: 3.4; 5.9; 8.25; 19.07; 6.146; 7.042; 5.009.

Fractions that express tenths, hundredths, thousandths, etc., are **decimal fractions**, or simply **decimals**.

Other fractions are **common fractions**, or simply **fractions**.

The term "decimal" is commonly applied to a fraction that is written without its denominator by using the decimal point, as .3 and .25; while the term "fraction" refers to one that requires the denominator to be written in expressing it, as $\frac{3}{10}$ and $\frac{25}{100}$.

A number expressed by an integer and a decimal is a **mixed decimal**.

Oral Exercises

Read these decimals:

- | | | | |
|-------|--------|---------|---------|
| 1. .1 | 3. .01 | 5. .001 | 7. .111 |
| 2. .3 | 4. .03 | 6. .003 | 8. .333 |

9. How many figures are needed to express tenths? hundredths? thousandths?

10. What does 1 mean when it stands in the first place at the right of the decimal point? in the second place? in the third place? What does 3 mean in each of these places?

Read, then tell what each figure means:

- | | | | | | |
|--------|---------|---------|----------|----------|-----------|
| 11. .4 | 13. 5.3 | 15. .37 | 17. 2.41 | 19. .094 | 21. 3.246 |
| 12. .9 | 14. 3.8 | 16. .05 | 18. 5.02 | 20. .406 | 22. 6.008 |

How many tens are there in 1 hundred? units in 1 ten? tenths in 1 unit? hundredths in 1 tenth? thousandths in 1 hundredth?

Ten units of any order, integral or decimal, make one unit of the next higher order.

Orders of decimals are shown below :

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths
7	9	6	.	5	8	4	6
INTEGRAL				DECIMAL			

This number is read, "796 and 5846 ten-thousandths."

Lower orders are *hundred-thousandths, millionths, ten-millionths, etc.*

A decimal should be read as an integer with the name of the right-hand order added.

The word "and" is used between the integral and decimal parts of a mixed decimal, and not elsewhere.

Oral Exercises

Read :

- | | | | | |
|--------|----------|----------|-----------|-----------|
| 1. 3.1 | 8. 26.3 | 15. 3.41 | 22. 28.52 | 29. 8.006 |
| 2. .04 | 9. .314 | 16. .612 | 23. 1.125 | 30. .2008 |
| 3. 2.5 | 10. .281 | 17. .035 | 24. 90.02 | 31. .0126 |
| 4. .48 | 11. 4.08 | 18. 50.7 | 25. 1.005 | 32. 80.05 |
| 5. 6.6 | 12. .301 | 19. .048 | 26. 679.4 | 33. .0037 |
| 6. .08 | 13. .074 | 20. .080 | 27. 5.031 | 34. 3.062 |
| 7. 9.2 | 14. .800 | 21. .009 | 28. .4503 | 35. .0007 |

The denominator of a decimal is not usually written, but it is 1 with as many 0's annexed as there are decimal orders in the decimal. It is *always* named when the decimal is read.

In writing a decimal, the numerator is written and ciphers are prefixed, if necessary, to indicate the denominator.

The decimal point is placed before the tenths.

Written Exercises

Express as a decimal or as a mixed decimal :

1. $\frac{23}{100}$

4. $24\frac{9}{10}$

7. $335\frac{7}{10}$

10. $71\frac{9}{1000}$

2. $6\frac{3}{10}$

5. $\frac{341}{1000}$

8. $56\frac{3}{100}$

11. $4\frac{225}{10000}$

3. $\frac{8}{100}$

6. $8\frac{21}{100}$

9. $3\frac{13}{1000}$

12. $8\frac{31}{10000}$

13. 5 tenths; 9 tenths; 12 hundredths.

14. 8 tenths; 5 hundredths; 162 thousandths.

15. Six tenths; 28 thousandths; one hundredth.

16. Seven and 3 tenths; 141 and 15 hundredths.

17. Six thousandths; 2 hundred and 35 hundredths.

18. Nine hundredths; one hundred and 4 thousandths.

19. Fifty and ten hundredths; 98 and two thousandths.

20. Seventy and one thousandth; 1273 ten-thousandths.

21. 212 ten-thousandths; thirteen and 85 thousandths.

22. 3127 and 8 ten-thousandths; seventeen thousandths.

Write in words:

23. 401

25. .401

27. .0366

29. 2000.002

24. 400.001

26. 401,000

28. 300.0066

30. 2002.002

REDUCTION OF DECIMALS

Decimals that have the same number of decimal places are **similar decimals**.

.14 and .65 are *similar* decimals, but .8 and .92 are *dissimilar*.

Reduction of dissimilar decimals to similar decimals.

How do the fractions $\frac{6}{10}$, $\frac{60}{100}$, and $\frac{600}{1000}$ compare in value? the decimals .6, .60, .600 in form?

Annexing zeros to a decimal does not change its value.

Written Exercises

1. Reduce .9, .34, and .0750 to similar decimals.

.9 = .900 It is best to give the similar decimals the least possible number of decimal places.

.34 = .340 Since .0750 = .075, we must reduce each of the other given decimals to thousandths, that

.0750 = .075 is, to decimals having *three* places. This may be done by annexing zeros as shown in the process.

Give all the decimals the same number of decimal places by annexing or cutting off zeros.

Reduce to similar decimals:

- | | | |
|-------------|---------------|-------------------|
| 2. .4, .12 | 9. 8.4, .64 | 16. 13.1, 1.08 |
| 3. .03, .8 | 10. .4, .250 | 17. .03, .1200 |
| 4. .5, .10 | 11. 3.46, .9 | 18. .127, .2104 |
| 5. .45, .3 | 12. .570, .7 | 19. .281, .1450 |
| 6. .7, .50 | 13. .07, .200 | 20. .8, .17, .195 |
| 7. .04, .2 | 14. .8, .0125 | 21. .200, .20, .2 |
| 8. .6, .175 | 15. .0270, .5 | 22. .125, .7, .04 |

Reduction of decimals to common fractions.

1. How many halves are there in $\frac{5}{10}$? in .5?
2. To how many fifths is $\frac{2}{10}$ equal? .2? $\frac{4}{10}$? .4? .6? .8?

Written Exercises

1. Reduce .75 to a common fraction; also .075.

$$.75 = \frac{75}{100} = \frac{3}{4}$$

$$.075 = \frac{75}{1000} = \frac{3}{40}$$

Omit the decimal point, supply the proper denominator and reduce the resulting fraction to its lowest terms.

Reduce to a common fraction or to a mixed number:

- | | | | | |
|--------|----------|----------|-----------|-----------|
| 2. .25 | 9. .600 | 16. 10.5 | 23. 2.125 | 30. .1875 |
| 3. .20 | 10. .125 | 17. 4.10 | 24. 8.500 | 31. .0625 |
| 4. .50 | 11. .250 | 18. 3.25 | 25. 18.20 | 32. 7.075 |
| 5. .05 | 12. .375 | 19. 12.6 | 26. 6.375 | 33. .4375 |
| 6. .40 | 13. .750 | 20. 5.75 | 27. 22.80 | 34. .3125 |
| 7. .04 | 14. .625 | 21. 14.4 | 28. 9.625 | 35. 8.875 |
| 8. .65 | 15. .875 | 22. 16.8 | 29. 15.08 | 36. .5625 |

37. Reduce $.16\frac{2}{3}$ to a common fraction.

$$.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{\frac{50}{3}}{100} = \frac{50}{300} = \frac{1}{6}$$

$.16\frac{2}{3}$ written as a common fraction becomes $\frac{50}{300}$, which means $\frac{50}{3} \div 100$, and equals $\frac{50}{300}$, or $\frac{1}{6}$.

Reduce to a common fraction or to a mixed number:

- | | | | | |
|----------------------|----------------------|----------------------|-----------------------|-----------------------|
| 38. $.12\frac{1}{2}$ | 41. $.08\frac{1}{3}$ | 44. $.62\frac{1}{2}$ | 47. $3.12\frac{1}{2}$ | 50. $6.62\frac{1}{2}$ |
| 39. $.33\frac{1}{3}$ | 42. $.06\frac{1}{4}$ | 45. $.83\frac{1}{3}$ | 48. $2.16\frac{2}{3}$ | 51. $8.66\frac{2}{3}$ |
| 40. $.37\frac{1}{2}$ | 43. $.66\frac{2}{3}$ | 46. $.87\frac{1}{2}$ | 49. $5.33\frac{1}{3}$ | 52. $9.87\frac{1}{2}$ |

Reduction of common fractions to decimals.

1. How many tenths are there in $\frac{1}{2}$? in $\frac{1}{5}$?
2. Then, express as a decimal: $\frac{1}{2}$ or $\frac{5}{10}$; $\frac{1}{5}$ or $\frac{2}{10}$.

Written Exercises

1. Reduce $\frac{3}{4}$ to a decimal; also $\frac{18}{60}$.

$$\frac{3}{4} = \frac{75}{100} = .75$$

$$\frac{18}{60} = \frac{3}{10} = .3$$

Reduce to *hundredths*, as on page 174, and write as a decimal.

Reduce to *tenths*, as on page 174, and write as a decimal.

Reduce to *tenths* and write as a decimal:

- | | | | | |
|------------------|------------------|-------------------|--------------------|---------------------|
| 2. $\frac{1}{5}$ | 4. $\frac{2}{5}$ | 6. $\frac{4}{5}$ | 8. $\frac{12}{30}$ | 10. $\frac{35}{50}$ |
| 3. $\frac{1}{2}$ | 5. $\frac{3}{5}$ | 7. $\frac{6}{20}$ | 9. $\frac{28}{40}$ | 11. $\frac{45}{50}$ |

Reduce to *hundredths* and write as a decimal:

- | | | | | |
|-------------------|--------------------|--------------------|---------------------|-----------------------|
| 12. $\frac{1}{4}$ | 14. $\frac{5}{10}$ | 16. $\frac{7}{20}$ | 18. $\frac{12}{25}$ | 20. $\frac{32}{200}$ |
| 13. $\frac{3}{4}$ | 15. $\frac{1}{20}$ | 17. $\frac{3}{25}$ | 19. $\frac{4}{50}$ | 21. $\frac{160}{500}$ |

Reduce to *thousandths* and write as a decimal:

- | | | | | |
|-------------------|--------------------|---------------------|---------------------|------------------------|
| 22. $\frac{1}{8}$ | 24. $\frac{9}{20}$ | 26. $\frac{5}{8}$ | 28. $\frac{27}{40}$ | 30. $\frac{150}{2000}$ |
| 23. $\frac{3}{8}$ | 25. $\frac{4}{25}$ | 27. $\frac{19}{40}$ | 29. $\frac{8}{125}$ | 31. $\frac{48}{3000}$ |

This topic is continued on page 244 after *division of decimals*.

Since cents are written as decimals of \$1, in decimals, addition, subtraction, multiplication, and division are performed just as in United States money. Also since decimals are really fractions in another form, the principles of fractions are employed in explaining work in decimals.

ADDITION OF DECIMALS

1. How many tenths are $\frac{3}{10} + \frac{1}{10}$? $.3 + .1$?
2. How many hundredths are $\frac{7}{100} + \frac{23}{100}$? $.07 + .23$?
how many tenths?

Written Exercises

Add:

- | | | | | |
|-------------|-------------|-------------|-------------|--------------|
| 1. \$4.36 | 2. \$2.38 | 3. \$4.22 | 4. \$16.47 | 5. \$71.16 |
| <u>7.92</u> | <u>8.45</u> | <u>9.06</u> | <u>8.33</u> | <u>24.62</u> |

6. Find the sum of 1.125, 4.27, and 6.075.

Write the units in one column, the tenths in another, and so on. When this is done, *the decimal points are in a column.*

1.125	
4.27	
6.075	
<u>11.47</u>	

Add and place the decimal point in the sum directly under the other decimal points. The sum is 11.470, but since (p. 227) $.470 = .47$, we write 11.47.

Find the sum of:

- | | |
|------------------------|----------------------------|
| 7. 4.7 and 3.8 | 12. 45, 3.5, 10.05 |
| 8. 6.75 and 2.63 | 13. 3.6, 4.82, 5.756 |
| 9. 4.235, 6.41, 2.567 | 14. 34.704, .436, 2.47 |
| 10. 3.625, 1.05, 3.385 | 15. 17.205, .495, 3.7509 |
| 11. 24.63, .75, 61.006 | 16. 3.3, 4.03, 6.003, .667 |

Add and test:

- | | | | | | |
|------------|-------------|-------------|-------------|--------------|--------------|
| 17. | 18. | 19. | 20. | 21. | 22. |
| 3.4 | 38.9 | 4.28 | .462 | 819.2 | 93.84 |
| 6.8 | 26.2 | 7.62 | .817 | 274.7 | 34.02 |
| 7.5 | 64.7 | 9.36 | .325 | 696.3 | 84.63 |
| <u>6.3</u> | <u>46.4</u> | <u>5.27</u> | <u>.459</u> | <u>841.5</u> | <u>25.97</u> |

Add and test :

23.	24.	25.	26.	27.	28.
.38	34.2	4.56	.648	7.627	.8287
.47	69.8	3.78	.463	6.35	.493
.55	73.5	2.4	.025	7.142	.6472
<u>.91</u>	<u>12.7</u>	<u>6.27</u>	<u>.138</u>	<u>4.869</u>	<u>.8078</u>

29.	30.	31.	32.	33.	34.
3.4 $\frac{1}{4}$.38 $\frac{1}{6}$	4.25 $\frac{1}{8}$.462 $\frac{1}{3}$	19.2 $\frac{1}{4}$	3.84 $\frac{2}{5}$
6.8 $\frac{1}{2}$.26 $\frac{2}{3}$	7 62 $\frac{5}{8}$.817 $\frac{1}{2}$	74.7 $\frac{1}{2}$	4.02 $\frac{1}{2}$
<u>7.5$\frac{1}{4}$</u>	<u>.64$\frac{1}{6}$</u>	<u>9.36$\frac{1}{4}$</u>	<u>.325$\frac{1}{6}$</u>	<u>96.3$\frac{1}{8}$</u>	<u>4.63$\frac{3}{8}$</u>

Add columns ; rows ; the sums of columns ; of rows :

	35.	36.	37.	38.	39.	40.					
41.	3.6	+	.38	+	9.24	+	58.9	+	.675	+	7.975
42.	6.2	+	.84	+	6.49	+	37.4	+	.925	+	9.346
43.	7.4	+	.60	+	2.83	+	56.7	+	.828	+	5.305
44.	4.8	+	.49	+	3.62	+	98.5	+	.956	+	9.999
45.	7.9	+	.75	+	7.34	+	33.3	+	.666	+	8.625

Add and test, timing yourself :

46.	47.	48.	49.	50.
32.5	87.89	46.25	79.82	68.8255
63.9	43.21	65.92	22.389	10.253
17.6	37.34	15.13	75.693	32.9847
42.2	39.46	.978	24.64	49.229
34.2	71.68	37.92	13.37	86.167
15.4	29.35	7.47	83.189	5.4528
<u>68.5</u>	<u>27.58</u>	<u>64.63</u>	<u>42.598</u>	<u>8.7163</u>

51. There are 5.875 cords of wood in one pile and 12.25 cords in another. How many cords are there in both piles?

52. A horse was fed daily 13 lb. of oats, 6.5 lb. of hay, and 8.25 lb. of straw. Find the weight of his daily food.

53. Edward rode 439.4 miles on the train in going from New York to Buffalo, and 535.9 miles in going from Buffalo to Chicago. How many miles did he ride?

54. At one time Minnesota produced .4 of the iron ore of this country, Michigan .25, and Alabama .125. What part of the country's output was produced by these states?

55. Add: seventy and six tenths; forty-five and nine hundredths; six hundred and sixteen hundredths.

56. A chimney stands 220.25 ft. above ground and extends 18.5 ft. below, where it rests on a foundation 7.75 ft. deep. Find the total height of chimney and foundation.

57. A ball was thrown from the home plate to second base, 127.28 ft.; thence to first base, 90 ft.; then from first base to third base, 127.28 ft. How far did the ball travel?

58. Add: four hundred fifty-six; four hundred and fifty-six thousandths; nine thousand and ninety-five hundredths; eighty-six and eighty-six thousandths.

59. For six months one year, the rainfall in a certain place was: 4.09 in., 2.26 in., 5.49 in., .62 in., 5.03 in., and 1.86 in. Find the total rainfall for those months.

60. The Times Building extends 143 ft. along Broadway, 58.33 $\frac{1}{3}$ ft. along 42d Street, 137.83 $\frac{1}{3}$ ft. along 7th Avenue, and 20 ft. along 43d Street. Find the distance around it.

SUBTRACTION OF DECIMALS

1. How many tenths are $\frac{7}{10} - \frac{3}{10}$? $.7 - .3$? $.9 - .4$?
 2. How many hundredths are $\frac{34}{100} - \frac{4}{100}$? $.34 - .04$?
 how many tenths?

Written Exercises

Subtract :

1. $\begin{array}{r} \$8.35 \\ 2.13 \\ \hline \end{array}$ 2. $\begin{array}{r} \$6.43 \\ 4.36 \\ \hline \end{array}$ 3. $\begin{array}{r} \$8.44 \\ 2.69 \\ \hline \end{array}$ 4. $\begin{array}{r} \$38.00 \\ 14.75 \\ \hline \end{array}$ 5. $\begin{array}{r} \$68.14 \\ 39.37 \\ \hline \end{array}$

6. Subtract 2.25 from 6.82.

 $\begin{array}{r} 6.82 \\ 2.25 \\ \hline \end{array}$

Write the numbers so that *the decimal points are in a column.*

 $\begin{array}{r} 2.25 \\ 4.57 \\ \hline \end{array}$

Subtract and place the decimal point in the difference directly under the other decimal points.

Subtract as indicated :

7. $45.6 - 27.4$ 9. $3.48 - 2.57$ 11. $6.325 - 4.146$

8. $53.2 - 13.8$ 10. $8.06 - 5.28$ 12. $9.218 - 5.326$

13. From 100 subtract 24.58.

 $\begin{array}{r} 100 \\ 24.58 \\ \hline \end{array}$ $\begin{array}{r} 24.58 \\ 75.42 \\ \hline \end{array}$

The decimal point is assumed to follow all integers.

Without writing them we may consider 0's to occupy tenths' and hundredths' places of the minuend.

Subtract as indicated :

14. $10 - 7.5$ 19. $500 - 4.8$ 24. $345.7 - 26.45$

15. $10 - 6.67$ 20. $1.24 - .854$ 25. $8.006 - 3.647$

16. $68 - 9.53$ 21. $800 - 4.47$ 26. $92.04 - 7.264$

17. $6.5 - 2.75$ 22. $51.2 - 3.48$ 27. $23.22 - 4.546$

18. $1.8 - .963$ 23. $46.8 - .875$ 28. $620.1 - .9875$

Subtract :

$$\begin{array}{r r r r r}
 29. & 3.7\frac{1}{2} & 30. & 7.4\frac{7}{8} & 31. & 9.4\frac{3}{4} & 32. & 6.37\frac{1}{2} & 33. & 8.76\frac{2}{3} \\
 & \underline{1.2\frac{1}{4}} & & \underline{4.6\frac{1}{2}} & & \underline{5.8\frac{5}{8}} & & \underline{2.56\frac{1}{6}} & & \underline{3.91\frac{3}{4}}
 \end{array}$$

34-40. The increase in the price of cotton during one season lasted eight months. The average price per pound for each month was :

JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.
8.95¢	9.65¢	10.08¢	10.44¢	11.46¢	12.40¢	12.74¢	12.75¢

Find the advance in price from each month to the next.

41. The sum of two numbers is 25. If one of the numbers is 8.125, what is the other number?

42. If 25.75 barrels of water were pumped from a cistern containing 30.5 barrels, how much water remained in the cistern?

43. A gallon of water weighs 8.355 lb., and a gallon of milk, 8.622 lb. Which is the heavier and how much?

44. From thirty-four and two hundred forty-five thousandths subtract eighteen and nine hundredths.

45. How much larger is the English bushel of 2218.192 cu. in. than the American bushel of 2150.42 cu. in.?

46. Syracuse is situated on a railroad 439.44 miles long between New York and Buffalo, and is 148.72 miles from Buffalo. How far is Syracuse from New York?

47. If the average weight of a boy is 61.28 lb. at 10 years of age, 64.89 lb. at 11 years, 72.55 lb. at 12 years, 78.32 lb. at 13 years, 87.41 lb. at 14 years, and 103.29 lb. at 15 years, find the gain in average weight for each year.

MULTIPLICATION OF DECIMALS

Multiplication of decimals by integers.

1. How many tenths are $2 \times \frac{3}{10}$? $2 \times .3$? $3 \times .3$?
 2. How many hundredths are $5 \times \frac{7}{100}$? $5 \times .07$?
 $2 \times .15$?

Written Exercises

Multiply:

- | | | | | |
|---|---|---|---|--|
| 1. $\begin{array}{r} \$3.12 \\ 3 \\ \hline \end{array}$ | 2. $\begin{array}{r} \$8.74 \\ 5 \\ \hline \end{array}$ | 3. $\begin{array}{r} \$5.24 \\ 4 \\ \hline \end{array}$ | 4. $\begin{array}{r} \$7.33 \\ 7 \\ \hline \end{array}$ | 5. $\begin{array}{r} \$4.375 \\ 9 \\ \hline \end{array}$ |
| 6. $\begin{array}{r} 4.18 \\ 4 \\ \hline 16.72 \end{array}$ | 7. $\begin{array}{r} 3.69 \\ 9 \\ \hline 33.21 \end{array}$ | 8. $\begin{array}{r} 5.45 \\ 8 \\ \hline 43.60 \end{array}$ | 9. $\begin{array}{r} 8.75 \\ 4 \\ \hline 35.00 \end{array}$ | 10. $\begin{array}{r} 9.785 \\ 6 \\ \hline 58.710 \end{array}$ |

Place the decimal point in the product directly under the decimal point in the multiplicand.

Zeros at the right of a decimal may be *omitted*, except that in United States money two decimal places are retained for cents.

- | | | |
|---|---|---|
| 11. $\begin{array}{r} 34.68 \\ 24 \\ \hline 138\ 72 \\ 693\ 6 \\ \hline 832.32 \end{array}$ | 12. $\begin{array}{r} \$\ 5.16 \\ 8\frac{2}{3} \\ \hline 3\ 44 \\ 41\ 28 \\ \hline \$44.72 \end{array}$ | 13. $\begin{array}{r} 5.16 \\ 8\frac{2}{3} \\ \hline 3\ 44 \\ 41\ 28 \\ \hline 44.72 \end{array}$ |
|---|---|---|

We find $8\frac{2}{3} \times 5.16$ just as we find $8\frac{2}{3} \times \$5.16$. Dollar signs and decimal points are omitted from the partial products.

- | | | |
|---------------|----------------|----------------------------|
| 14. 24.3 by 5 | 19. 1.74 by 12 | 24. 37.4 by $5\frac{1}{2}$ |
| 15. .458 by 7 | 20. 36.8 by 25 | 25. 6.78 by $4\frac{2}{3}$ |
| 16. 6.19 by 6 | 21. .429 by 47 | 26. 54.5 by $6\frac{2}{3}$ |
| 17. .732 by 8 | 22. 68.4 by 39 | 27. 7.04 by $8\frac{3}{4}$ |
| 18. 58.6 by 9 | 23. 8.05 by 68 | 28. .882 by $9\frac{5}{6}$ |

Multiply :

- | | | |
|----------------|-----------------|-----------------|
| 29. 12.03 by 4 | 33. 251.4 by 15 | 37. 4.65 by 132 |
| 30. 451.6 by 6 | 34. 5.625 by 32 | 38. 60.8 by 274 |
| 31. .3752 by 7 | 35. 60.78 by 67 | 39. .837 by 409 |
| 32. 5.867 by 9 | 36. .8139 by 84 | 40. 9.21 by 658 |

41. Find the cost of 4 boxes of apples at \$ 1.25 per box.
42. At \$ 2.75 per day, how much will I earn in 8 days?
43. Find the cost of 5 tons of straw at \$ 15.75 per ton.
44. A farmer had 6 acres of corn which averaged 85.4 bu. per acre. Find the total yield of corn.
45. If a steamer burns 457.6 tons of coal a day, how many tons will it burn on a voyage of 7 days?
46. How far will a train go in 24 hours at the rate of 48.25 miles per hour?
47. How much was received for 8 geese that weighed 11 lb. each, if they were sold at \$.23 a pound?
48. An apple tree yielded $12\frac{3}{4}$ bu. of apples worth \$.72 a bushel. Find the total value of the apples.
49. A record cow gave an average of 46.7 qt. of milk daily for 182 days. Find the total yield in that time.
50. If the average yield from an $8\frac{3}{4}$ -acre field of oats was 45.75 bu. per acre, what was the total yield?
51. A cubic foot of water weighs 62.5 lb. Find the weight of water that a tank 8' by 4' by 3' will hold.
52. The average yield from 32 acres of potatoes was 235.75 bu. per acre. Find the total yield.

Multiplication of decimals by multiples of 10.

1. What is the value of $10 \times .1$? of $10 \times .2$?

Multiplying tenths by 10 gives units.

2. What is the value of $10 \times .01$? of $10 \times .03$?

Multiplying hundredths by 10 gives tenths.

3. What is the value of $10 \times .001$? of $10 \times .005$?

Multiplying thousandths by 10 gives hundredths.

4. Since $10 \times .2 = 2$, $10 \times .03 = .3$, and $10 \times .005 = .05$, when .235 is multiplied by 10, each figure of .235 is advanced into the next higher place. .235

A short way to do this is to *move the decimal point one place toward the right.* $\frac{10}{2.350}$

5. Multiply .235 by 10 and the product by 10, and so obtain $100 \times .235$; multiply again by 10 and obtain $1000 \times .235$.

A decimal may be multiplied by 10, 100, 1000, etc., by moving the decimal point toward the right as many places as there are zeros in the multiplier.

If necessary, annex 0's to give the multiplicand enough places; thus, $1000 \times 6.4 = 6400$.

Oral Exercises

Multiply by 10; by 100; by 1000:

- | | | | | |
|--------|-----------|-----------|------------|-----------|
| 1. .05 | 6. 4.07 | 11. .006 | 16. \$1.50 | 21. .0048 |
| 2. 1.6 | 7. 69.1 | 12. \$.09 | 17. \$4.06 | 22. 3.257 |
| 3. .27 | 8. .586 | 13. 5.24 | 18. .0273 | 23. 50.69 |
| 4. .08 | 9. \$.20 | 14. 70.8 | 19. 4.958 | 24. 7.284 |
| 5. 4.9 | 10. \$.38 | 15. 9.37 | 20. 60.73 | 25. 800.3 |

Change to cents:

26. \$4.25	28. \$5.63	30. \$20.36	32. \$56.78
27. \$3.54	29. \$7.02	31. \$38.05	33. \$42.24

Written Exercises

1. Multiply 7.628 by 200.

$$\begin{array}{r} 7.628 \\ \times 200 \\ \hline 1525.6 \end{array}$$

We first multiply by 2 and then by 100; that is, we multiply by 2 and move the decimal point *two* places toward the right.

Find products:

- | | | |
|---------------------|-----------------------|-------------------------|
| 2. 40×13.5 | 9. $300 \times .025$ | 16. 70×14.03 |
| 3. $20 \times .266$ | 10. 400×1.36 | 17. 500×3.125 |
| 4. 30×5.74 | 11. 200×47.2 | 18. 900×278.4 |
| 5. $60 \times .382$ | 12. 500×3.82 | 19. $3000 \times .0653$ |
| 6. 50×70.9 | 13. $700 \times .569$ | 20. 4000×62.48 |
| 7. 80×4.56 | 14. 600×74.8 | 21. 6000×819.7 |
| 8. $90 \times .678$ | 15. 800×9.06 | 22. 7000×9.356 |

23. A French franc is worth \$.193. Find the value of 200 francs.

24. Which is the greater and how much, the product or the sum of .625 and 500?

25. Find the weight of a slate roof 40 ft. long and 20 ft. wide, if every square foot weighs 9.25 lb.

26. If it takes 40 poles for each mile of a telegraph line, find the cost of the poles for 75 miles at \$2.89 apiece.

Multiplication of decimals or integers by decimals.

1. How many hundredths are $\frac{3}{10} \times \frac{2}{10}$? $.3 \times .2$?
2. How many thousandths are $\frac{2}{100} \times \frac{3}{10}$? $.02 \times .3$?
3. How many ten-thousandths are $\frac{7}{100} \times \frac{3}{100}$? $.07 \times .03$?
4. How does the number of 0's in the denominator of the product compare with the number in both factors?
5. Then, how does the number of decimal places in the product compare with the number in both multiplicand and multiplier?

The number of decimal places in the product is equal to the number in both multiplicand and multiplier.

Written Exercises

1. Multiply .63 by .48.

$$\begin{array}{r}
 .63 \\
 .48 \\
 \hline
 504 \\
 252 \\
 \hline
 .3024
 \end{array}$$

Since the multiplicand .63 has two decimal places and the multiplier .48 two decimal places, the product must contain $2 + 2$, or *four*, decimal places.

Therefore, the product is .3024.

Multiply as if the numbers were integers, and from the right of the product point off as many decimal places as there are decimal places in both multiplicand and multiplier.

Find products :

- | | | | |
|--------------------|---------------------|----------------------|-----------------------|
| 2. $.4 \times .28$ | 6. $.37 \times .48$ | 10. $.4 \times 265$ | 14. $.27 \times 416$ |
| 3. $.6 \times .34$ | 7. $.49 \times .56$ | 11. $.5 \times .307$ | 15. $.45 \times .503$ |
| 4. $.7 \times .45$ | 8. $.62 \times .67$ | 12. $.8 \times .682$ | 16. $.36 \times .842$ |
| 5. $.9 \times .67$ | 9. $.84 \times .34$ | 13. $.9 \times .745$ | 17. $.78 \times .971$ |

Multiply :

$$\begin{array}{r} 18. \quad .32 \\ \quad .26 \\ \hline 192 \\ 64 \\ \hline .0832 \end{array}$$

$$\begin{array}{r} 19. \quad .34 \\ \quad .45 \\ \hline 170 \\ 136 \\ \hline .1530 \end{array}$$

$$\begin{array}{r} 20. \quad 60.29 \\ \quad .095 \\ \hline 30145 \\ 5 \ 4261 \\ \hline 5.72755 \end{array}$$

Decimal ciphers are *prefixed*, when necessary, to give the product the proper number of decimal places.

- | | | |
|----------------|-----------------|-------------------------------|
| 21. .16 by .4 | 39. 1.16 by .07 | 57. .1345 by .08 |
| 22. 1.8 by .6 | 40. .209 by .04 | 58. 1.426 by .07 |
| 23. .27 by .3 | 41. 27.4 by .05 | 59. .1578 by .05 |
| 24. .26 by .5 | 42. .312 by .03 | 60. 185.4 by .12 |
| 25. 1.9 by .7 | 43. 1.24 by .09 | 61. 16.09 by 2.8 |
| 26. 24 by .9 | 44. 2.06 by .16 | 62. 2.736 by 3.5 |
| 27. 33 by .8 | 45. 43.1 by .18 | 63. 2.425 by .44 |
| 28. 35 by .06 | 46. 309 by 4.6 | 64. .3562 by 3.7 |
| 29. 27 by .05 | 47. 245 by .37 | 65. 4050 by .52 $\frac{1}{3}$ |
| 30. 53 by .08 | 48. 503 by 2.9 | 66. 3845 by 4.6 $\frac{2}{5}$ |
| 31. 3.6 by 1.2 | 49. 273 by .64 | 67. 5972 by .65 $\frac{3}{4}$ |
| 32. .42 by .13 | 50. 408 by 3.6 | 68. 6016 by .58 $\frac{5}{8}$ |
| 33. 2.6 by 4.1 | 51. 387 by 5.3 | 69. .3125 by .016 |
| 34. 3.8 by .15 | 52. 61.4 by .45 | 70. 47.56 by .038 |
| 35. .63 by 3.1 | 53. .537 by .68 | 71. 2.895 by .054 |
| 36. .46 by 2.2 | 54. 4.89 by 5.5 | 72. 31.62 by 25.8 |
| 37. 5.7 by .48 | 55. 70.6 by .39 | 73. .2685 by 5.26 |
| 38. .69 by .56 | 56. .825 by .46 | 74. 520.8 by .315 |

Miscellaneous Exercises

Multiply:

- | | | |
|---------------|-----------------|-------------------|
| 1. 325 by .8 | 7. 405 by 3.4 | 13. 3450 by .252 |
| 2. 6.09 by 3 | 8. .286 by 15 | 14. .2025 by 128 |
| 3. .872 by 4 | 9. 7.05 by 32 | 15. 15.36 by 4.25 |
| 4. 5.86 by 6 | 10. .582 by 50 | 16. 3.964 by .025 |
| 5. .479 by .2 | 11. .125 by .06 | 17. .1206 by 17.5 |
| 6. .095 by .7 | 12. 2.84 by .03 | 18. .2375 by .132 |
- 19-22. Multiply 42.75 by .08; by 20; by $6\frac{3}{4}$; by .002.
- 23-26. Multiply 2.108 by .16; by 700; by $2\frac{3}{4}$; by 27.5.
- 27-30. Multiply .1215 by .48; by 400; by $4\frac{2}{5}$; by 1.04.
31. Find the area of a floor 9.25 ft. by 8 ft.
32. At \$ 1.20 a yard, how much will 7.5 yd. of silk cost?
33. Find the cost of $5\frac{3}{4}$ doz. oranges at \$.40 per dozen.
34. A house and lot cost \$ 2100. Find the annual rent charged for it, if the rent is .15 of its cost.
35. A farmer had 74 sheep from which the average clip per sheep was 9.75 lb. Find the total yield of wool.
36. If it costs \$ 5.94 to irrigate an acre of land in Colorado, how much does it cost to irrigate 87.5 acres?
37. If 4.5 lb. of onion seed is required to sow 1 acre, find the cost of the seed used on 4.8 acres at \$ 2.25 a pound.
38. A California farmer raised 27.5 acres of sugar beets that averaged 9.81 tons per acre. Find the total yield.
39. Find the cost at \$.12 per bushel of enough cotton seed to plant 245 acres of land, if each acre requires 1.75 bu.

DIVISION OF DECIMALS

Division of decimals by integers.

1. How many tenths are $\frac{6}{10} \div 2$? $.6 \div 2$? $.9 \div 3$?
2. How many hundredths are $\frac{35}{100} \div 5$? $.35 \div 5$? $.15 \div 3$?

Written Exercises

Find quotients :

1.	2.	3.	4.	5.
3) $\$9.36$	5) $\$7.35$	2) 573.6	3) 76.74	5) 6.725
$\$3.12$	$\$1.47$	286.8	25.58	1.345

Place the decimal point in the quotient directly under the decimal point in the dividend, in short division ; directly over it, in long division.

6. Divide 11.4 by 8.

$$\begin{array}{r} 8 \overline{)11.400} \\ 1.425 \end{array}$$

7. Divide .34 by 6.

$$\begin{array}{r} 6 \overline{).340} \\ .056\frac{2}{3} \end{array}$$

Zeros may be annexed to the dividend and the quotient may be carried to any desired number of decimal places. In this case by annexing two 0's the division is found to be exact in the third decimal place.

Here one 0 is annexed, but it is seen that the *remainder* in each place will be 4, however far the division is carried. When we desire to stop, we indicate the division $\frac{4}{3}$, reduce to lowest terms, and write $\frac{2}{3}$ as a part of the quotient.

Give results in best form (not over three decimal places):

8. $73.5 \div 2$ 12. $97.1 \div 4$ 16. $1.642 \div 3$ 20. $4.674 \div 4$
 9. $4.37 \div 5$ 13. $.614 \div 6$ 17. $21.57 \div 2$ 21. $63.84 \div 5$
 10. $34.1 \div 4$ 14. $49.7 \div 5$ 18. $18.34 \div 7$ 22. $357.1 \div 8$
 11. $.625 \div 3$ 15. $.546 \div 8$ 19. $3.752 \div 6$ 23. $6.525 \div 9$

Divide, carrying the quotient to *three decimal places* in exercise **24**, and to the *nearest second decimal place* in exercises **25** and **26** :

$$\begin{array}{r} .157 \\ 24. \quad 22 \overline{)3.460} \\ \underline{22} \\ 126 \\ \underline{110} \\ 160 \\ \underline{154} \end{array}$$

$$\begin{array}{r} .842, \text{ or } .84^+ \\ 25. \quad 42 \overline{)35.390} \\ \underline{336} \\ 179 \\ \underline{168} \\ 110 \\ \underline{84} \end{array}$$

$$\begin{array}{r} .537, \text{ or } .54^- \\ 26. \quad 37 \overline{)19.900} \\ \underline{185} \\ 140 \\ \underline{111} \\ 290 \\ \underline{259} \end{array}$$

When the quotient to three decimal places is required, we stop upon finding the figure for the third place, whatever the remainder may be.

Since .002 is less than $\frac{1}{2}$ of .01, the quotient, to the nearest second decimal place, is .84⁺.

The sign ⁺ shows that the quotient is a little more than .84.

Since .007 is greater than $\frac{1}{2}$ of .01, the quotient, to the nearest second decimal place, is .54⁻.

The sign ⁻ indicates that the quotient is a little less than .54.

Divide, carrying inexact quotients to three decimal places:

$$27. \quad 29.4 \text{ by } 35 \qquad 32. \quad 17.92 \text{ by } 32 \qquad 37. \quad 15.435 \text{ by } 63$$

$$28. \quad 5.63 \text{ by } 24 \qquad 33. \quad 2.128 \text{ by } 28 \qquad 38. \quad 145.32 \text{ by } 42$$

$$29. \quad 828 \text{ by } 21 \qquad 34. \quad 57.23 \text{ by } 47 \qquad 39. \quad 2360.9 \text{ by } 46$$

$$30. \quad .765 \text{ by } 43 \qquad 35. \quad 6.344 \text{ by } 52 \qquad 40. \quad 284.97 \text{ by } 58$$

$$31. \quad 3.64 \text{ by } 56 \qquad 36. \quad 56.42 \text{ by } 69 \qquad 41. \quad 35.898 \text{ by } 62$$

42. A flock of ducks at an experiment station gained 108.75 lb. in 5 weeks. Find the average gain per week.

43. One year a farmer cut 67.5 tons of hay from a field of 18 acres. Find the average yield per acre.

Find quotients:

To nearest tenth	To nearest hundredth	To nearest thousandth
44. $32.9 \div 23$	49. $396.3 \div 28$	54. $24.319 \div 124$
45. $732 \div 45$	50. $53.29 \div 35$	55. $452.96 \div 138$
46. $45.6 \div 27$	51. $683.6 \div 44$	56. $306.75 \div 283$
47. $386 \div 46$	52. $7.651 \div 57$	57. $6.9182 \div 314$
48. $52.9 \div 37$	53. $891.4 \div 39$	58. $8876.5 \div 452$

A common fraction is an indicated division.

Hence, to reduce a **common fraction** to a **decimal**:

Place a decimal point after the numerator, annex zeros, and divide by the denominator.

Another method was given on page 229.

Written Exercises

Reduce to a decimal:

1. $\frac{3}{8}$

2. $\frac{2}{3}$

3. $\frac{5}{6}$

$$\begin{array}{r} 8 \overline{)3.000} \\ \underline{.375} \end{array}$$

$$\begin{array}{r} 3 \overline{)2.000} \\ \underline{.667-} \end{array}$$

$$\begin{array}{r} 6 \overline{)5.00} \\ \underline{.83\frac{1}{3}} \end{array}$$

Reduce to an exact decimal equivalent:

4. $\frac{2}{5}$

6. $\frac{5}{8}$

8. $\frac{9}{20}$

10. $\frac{3}{20}$

12. $\frac{7}{16}$

14. $\frac{3}{16}$

5. $\frac{3}{4}$

7. $\frac{7}{8}$

9. $\frac{17}{20}$

11. $\frac{19}{20}$

13. $\frac{9}{16}$

15. $\frac{15}{16}$

Reduce to a decimal, expressing the result:

In hundredths, as in Ex. 3.

In thousandths, as in Ex. 2.

16. $\frac{1}{3}$

19. $\frac{5}{12}$

22. $\frac{14}{15}$

25. $\frac{23}{24}$

28. $\frac{49}{60}$

31. $\frac{59}{60}$

17. $\frac{1}{6}$

20. $\frac{7}{15}$

23. $\frac{17}{30}$

26. $\frac{5}{48}$

29. $\frac{43}{48}$

32. $\frac{71}{72}$

18. $\frac{7}{9}$

21. $\frac{17}{18}$

24. $\frac{25}{36}$

27. $\frac{35}{36}$

30. $\frac{53}{60}$

33. $\frac{89}{96}$

Division of decimals or integers by multiples of 10.

1. Since $1 \div 10 = \frac{1}{10}$, or .1, what is the value of $2 \div 10$?
Dividing units by 10 gives tenths.

2. Since $.1 \div 10 = .01$, what is the value of $.5 \div 10$?
Dividing tenths by 10 gives hundredths.

3. Since $.01 \div 10 = .001$, what is the value of $.06 \div 10$?
Dividing hundredths by 10 gives thousandths.

4. Since $2 \div 10 = .2$, $.5 \div 10 = .05$, and $.06 \div 10 = .006$,
 when 2.56 is divided by 10, each figure of
 2.56 is moved into the next lower place.

$$\begin{array}{r} 10 \overline{) 2.56} \\ .256 \end{array}$$

A short way to do this is to *move the decimal point one place toward the left.*

5. Divide 2.56 by 10 and the quotient by 10 and so obtain $2.56 \div 100$; divide again by 10 and obtain the quotient $2.56 \div 1000$.

A number may be divided by 10, 100, 1000, etc., by moving the decimal point toward the left as many places as there are zeros in the divisor.

If necessary, prefix 0's to give the quotient enough places; thus,
 $23.5 \div 1000 = .0235$.

Oral Exercises

Divide by 10; by 100; by 1000:

- | | | | | |
|--------|---------|----------|----------|-----------|
| 1. 3.5 | 6. 74 | 11. 124 | 16. 32.4 | 21. 3244 |
| 2. 72 | 7. 5.2 | 12. 235 | 17. 4.16 | 22. 67.12 |
| 3. 25 | 8. .63 | 13. 14.3 | 18. 3.57 | 23. 346.3 |
| 4. 36 | 9. 7.5 | 14. 16.7 | 19. .130 | 24. .6200 |
| 5. 4.8 | 10. .81 | 15. 2.51 | 20. 5.29 | 25. 5.173 |

Reduce to dollars or to dollars and cents :

26. 300¢ 28. 700¢ 30. 674¢ 32. 1200¢ 34. 6504¢
 27. 500¢ 29. 320¢ 31. 805¢ 33. 5060¢ 35. 7165¢

Since division may be indicated by a fraction whose value is the quotient of the numerator divided by the denominator, it follows from principle 3, page 163, that :

Dividing both dividend and divisor by the same number does not change the quotient.

Then, when the divisor is an integer and has zeros at the right, it is easier and shorter first to divide both dividend and divisor by such of the numbers 10, 100, 1000, etc., as will cut off the 0's at the right of the divisor.

Thus, in $48.6 \div 200$, first divide both dividend and divisor by 100 by cutting off *two* 0's from 200 and moving the decimal point in 48.6 *two* places toward the left, for $48.6 \div 200$ is equal to $.486 \div 2$.

Written Exercises

1. Divide 56.7 by 3000.

$$56.7 \div 3000$$

$$\begin{array}{r} 3 \overline{) .0567} \\ \underline{.0189} \end{array}$$

The divisor is 3000. Prepare for division by cutting off the *three* 0's from the right of 3000 and moving the decimal point in 56.7 *three* places toward the left. $56.7 \div 3000$ equals $.0567 \div 3$.

Divide .0567 by 3; the quotient is .0189.

Divide :

- | | | |
|---------------|----------------|-------------------|
| 2. 48.4 by 20 | 6. 4.28 by 200 | 10. 846.8 by 4000 |
| 3. 6.28 by 40 | 7. 64.5 by 300 | 11. 710.4 by 6000 |
| 4. 87.5 by 50 | 8. 31.5 by 700 | 12. 63.75 by 5000 |
| 5. 7.44 by 80 | 9. 8.46 by 600 | 13. 581.6 by 8000 |

Division of decimals or integers by decimals.

Since division may be indicated by a fraction whose value is the quotient of the numerator divided by the denominator, it follows from principle 3, page 163, that:

Multiplying both dividend and divisor by the same number does not change the quotient.

1. When the divisor is a decimal, we make the division easier by multiplying both dividend and divisor by a number that will change the *divisor* to an *integer*.

2. Change $.3)\underline{.6}$ to $3)\underline{6}$ and divide.

3. Change $.4)\underline{.08}$ to $4)\underline{.8}$ and divide.

4. Change $.2)\underline{1.2}$ to $2)\underline{12}$ and divide.

5. Change $.03)\underline{.6}$ to $3)\underline{60}$ and divide.

6. Change $.04)\underline{.08}$ to $4)\underline{8}$ and divide.

7. Change $.02)\underline{1.2}$ to $2)\underline{120}$ and divide.

8. Change $.003)\underline{.6}$ to $3)\underline{600}$ and divide.

9. Change $.004)\underline{.08}$ to $4)\underline{80}$ and divide.

10. How should the dividend and the divisor be prepared for division when the divisor is a decimal?

Change the divisor to an integer by moving the decimal point, in both dividend and divisor, toward the right as many places as there are decimal figures in the divisor. Proceed as in division of a decimal by an integer.

This is known as the **Austrian** method of division in decimals.

Written Exercises

1. Divide .1296 by .08.

8) $\overline{12.96}$ Since the divisor expresses hundredths, to change it to an integer *without changing the quotient*, both dividend and divisor are multiplied by 100 by moving the decimal point in each two places toward the right. The dividend becomes 12.96 and the divisor, 8.

Divide as indicated :

2. $12.3 \div .3$

7. $23.42 \div .02$

12. $3.269 \div .007$

3. $2.45 \div .5$

8. $4.168 \div .04$

13. $.6204 \div .004$

4. $.328 \div .4$

9. $.5605 \div .05$

14. $7.005 \div .005$

5. $8.04 \div .6$

10. $6.928 \div .08$

15. $.8136 \div .009$

6. $5.08 \div .2$

11. $73.02 \div .06$

16. $5.768 \div .008$

17.

$6.448 \div 1.6$

$\begin{array}{r} 4\ 03 \\ 16 \overline{)64.48} \\ \underline{64} \\ 48 \\ \underline{48} \\ 48 \\ \underline{48} \end{array}$

18.

$.011 \div .44$

$\begin{array}{r} .025 \\ 44 \overline{)1.100} \\ \underline{88} \\ 220 \\ \underline{220} \end{array}$

19.

$9 \div .375$

$\begin{array}{r} 24 \\ 375 \overline{)9000} \\ \underline{750} \\ 1500 \\ \underline{1500} \end{array}$

When the dividend contains fewer decimal figures than the divisor, annex decimal ciphers before moving the decimal point. Thus,

$9 \div .375 = 9.000 \div .375 = 9000 \div 375.$

20. $2.87 \div .35$

25. $5.628 \div 1.4$

30. $5 \div .125$

21. $46.2 \div 2.4$

26. $.6345 \div .75$

31. $18 \div .225$

22. $.155 \div 6.2$

27. $.0403 \div .65$

32. $50 \div .0625$

23. $.022 \div 5.5$

28. $.0198 \div 3.6$

33. $63 \div .0875$

24. $.014 \div .56$

29. $2.002 \div .22$

34. $45 \div .9375$

Divide :

35. 15.4 by .2	52. 13.2 by 4.4	69. 4.325 by .005
36. 34.5 by .5	53. 2.79 by 3.1	70. .7624 by .008
37. 2.55 by .3	54. .984 by .24	71. 3.458 by .013
38. .098 by .7	55. 6.02 by 1.4	72. .1232 by .022
39. 7.11 by .9	56. .572 by 2.2	73. 48.72 by 1.45
40. 4.08 by .6	57. .144 by 4.8	74. 8.906 by 3.65
41. 2.24 by .8	58. .371 by 5.3	75. .6275 by 1.25
42. .136 by .4	59. .119 by .34	76. .0216 by .225
43. .305 by .5	60. .011 by .55	77. 17.28 by 1.44
44. .296 by .04	61. 122.5 by 3.5	78. 42.602 by 8.95
45. .894 by .06	62. 3.612 by .43	79. .20732 by 1.46
46. .093 by .05	63. 45.72 by 3.6	80. 1.0897 by .425
47. .007 by .04	64. .5424 by .24	81. .02275 by .625
48. 4326 by .06	65. 2.842 by 1.4	82. 49.245 by 23.45
49. 8109 by .03	66. 6.237 by .55	83. 793.35 by 32.25
50. 2544 by .08	67. .2346 by .68	84. .77221 by 8.215
51. 1692 by .09	68. .0546 by .65	85. .02328 by .2425

Find quotients :

To nearest tenth	To nearest hundredth	To nearest thousandth
86. $.438 \div .07$	91. $.625 \div 2.64$	96. $2.165 \div .375$
87. $.071 \div .14$	92. $8.45 \div 2.21$	97. $3.564 \div .095$
88. $.226 \div .15$	93. $7.24 \div .046$	98. $.8008 \div 25.4$
89. $3.54 \div .46$	94. $5.326 \div .085$	99. $.07468 \div 3.284$
90. $2.05 \div .98$	95. $.9006 \div 32.3$	100. $5.2064 \div .9244$

101. How many bats, at \$.75 each, can I buy for \$3.75?
102. At \$.40 a yard, how much lace can I buy for \$2.50?
103. How many books, at \$1.75 each, can I buy for \$12.25?
104. At \$3.50 each, how many sweaters can I buy for \$31.50?
105. If a railroad train travels .9 of a mile in one minute, how many minutes does it take to travel 29.16 miles?
106. A chest of tea was sold for \$28.31 at \$.38 per pound. How many pounds did the chest contain?
107. A planter pays \$519.65 per year for the rent of a farm at \$4.75 per acre. How many acres does he rent?
108. There are 5.5 yd. in a rod. How many rods are there in 37.125 yd.?
109. When 4.8 bu. of wheat make a barrel of flour, how many barrels of flour will 648 bu. of wheat make?
110. If a farmer sells potatoes for \$.65 per bushel, and receives \$1145.30 for them, how many bushels does he sell?
111. The strawberries grown on 4.5 acres of land were sold for \$2377.35. What was the income per acre?
112. How many pounds are there in a sack of coffee that sells for \$17.82 at \$.135 a pound?
113. The cost of drilling a well at \$.875 per foot was \$1333.50. How deep was the well?
114. A cow gave 5738.35 lb. of milk in a year. How many quarts of milk did she give, if 1 qt. weighs 2.15 lb.?
115. The cost of building a dirt road 21.7 miles long was \$9804.06. What was the cost per mile?

Miscellaneous Exercises

Divide as indicated :

- | | | |
|--------------------|-----------------------|------------------------|
| 1. $2.32 \div 4$ | 7. $85.36 \div .08$ | 13. $26 \div .1625$ |
| 2. $82.2 \div 6$ | 8. $207.6 \div 16$ | 14. $88 \div .0125$ |
| 3. $.027 \div .8$ | 9. $1.458 \div 200$ | 15. $25.9 \div 20.72$ |
| 4. $48.9 \div .05$ | 10. $.4065 \div .006$ | 16. $1.311 \div 874$ |
| 5. $.936 \div .24$ | 11. $5.402 \div .037$ | 17. $1.6412 \div .044$ |
| 6. $7.65 \div 1.2$ | 12. $864.9 \div 12.4$ | 18. $48.783 \div 805$ |

19. A blacksmith bought an anvil for \$9, paying \$.10 per pound for it. How much did the anvil weigh?

20. If 1 lb. of grapes makes .3 lb. of raisins, how many pounds of grapes are required for 10.5 lb. of raisins?

21. If Henry spends \$.05 a day for car fare, how long will \$6.45 last him?

22. A train went a distance of 61 miles in 1.25 hours. At what rate per hour did it run?

23. A police dog after being trained was worth \$72, or 3.6 times as much as he cost. How much did he cost?

24. Find the quotient when the dividend is twenty-four and the divisor is one hundred twenty-five ten-thousandths.

25. At \$2.85 a box, how many boxes of oranges can a wholesale dealer buy for \$2485.20?

26. A train of 105 cars carried 5544 tons of coal. Find the average amount of coal in each car.

27. If it requires a flow of 3.77 gal. of water per minute to irrigate 1 acre of land, how many acres can be irrigated by a flow of 207.35 gal. per minute?

Review Exercises in Decimals

Multiply:

1. 673.6 by .1; by .01
2. .0415 by 10; by 100

Divide:

3. .0543 by .1; by .01
4. 562.8 by 10; by 100

Find answers quickly:

5. $.2 \times 3$ 8. $.08 + .8$ 11. $.25 \times .04$ 14. $.325 \times 10$
6. $6 - .8$ 9. $.25 \div .5$ 12. $.24 \div .12$ 15. $12.8 + .09$
7. $.2 \times .3$ 10. $.02 \times .3$ 13. $.85 - .67$ 16. $5.36 - .009$

Reduce to a common fraction in its lowest terms:

17. .75 18. .125 19. .875 20. .1875 21. .5625

Subtract from 25.3:

Subtract from 100:

22. 12.5 24. 1.437 26. 24.8 28. 48.32
23. 9.64 25. .8786 27. 9.75 29. 8.375

Reduce to a decimal to the nearest thousandth:

30. $\frac{5}{8}$ 31. $\frac{11}{12}$ 32. $\frac{7}{15}$ 33. $\frac{17}{24}$ 34. $\frac{23}{30}$

Find the sum of:

35. .025 and 1.75 38. 84, .075, 21.96
36. 4.38, .625, .0328 39. .0824, 96.24, 1.25
37. 67.5, 2.84, 1.009 40. 28.3, .65, 7.008, .475

Find the answers:

41. 5.75×17 45. $.018 \times 300$ 49. 72.38×1.25
42. $336 \div 9.6$ 46. $245 \times .014$ 50. $.0042 \div .056$
43. $48.8 \times 6\frac{3}{4}$ 47. $76.5 \div 600$ 51. $2.045 \times .046$
44. $.674 \div .08$ 48. $8.04 \div .024$ 52. $6.848 \div 1.28$

Add and test, timing yourself:

53. 1.606	54. 98.046	55. 3.8649	56. 3.1416	57. 5.2064
5.21	48.792	8.0095	2.75	6.7187
.043	25.4	.8008	4.853	4.6562
2.531	3.96	32.84	.9244	3.4687
4.937	8.265	1.7854	4.422	29.726
<u>6.006</u>	<u>23.061</u>	<u>6.4139</u>	<u>2.4627</u>	<u>4.8495</u>

58. At \$ 1.75 a day, how much will I earn in 25 days?
59. If 50 lb. of sugar cost \$ 2.25, find the cost per pound.
60. A rod is 16.5 ft. How many rods are there in 132 ft.?
61. Find the perimeter of a rectangle 12.5 ft. by 6.75 ft.
62. Dividend = 6.8; divisor = 200. Find the quotient.
63. How much will 30 stoves cost at \$ 24.25 each?
64. An iron rod weighed 6.5 lb. I cut 1.75 lb. from it. Find the weight of the piece that remained.
65. Find the total snowfall for two winters in New York, if it was 52.4 in. one winter and 32.2 in. the next.
66. If 1 gal. of paint will cover 500 sq. ft. of surface on wood, how much surface will 5.75 gal. of paint cover?
67. In a month 4 cows produced 1922.5 lb. of milk. Find the average number of pounds produced per cow.
68. A nail 5 in. long is driven through a board so that it projects 2.419 in. on one side and 1.706 in. on the other. How thick is the board?
69. A factory employed 2400 persons. Their wages amounted to \$3360 per day. Find the average daily wages of each.

Find the cost of :

70. $5\frac{1}{2}$ yd. voile at \$1.10 per yard.
71. 7.5 yd. ribbon at \$.88 per yard.
72. 14.5 yd. carpet at \$1.50 per yard.
73. $2\frac{3}{4}$ doz. towels at \$5.60 per dozen.
74. $12\frac{1}{3}$ doz. buttons at \$.75 per dozen.
75. It cost a farmer \$3.51 per load to draw his produce to market, 6.75 miles. Find the cost per mile.
76. How much will it cost to drill a gas well to a depth of 1125 ft. at \$1.05 per foot?
77. How long will it take a gasoline launch that has a speed of 6.75 miles per hour to travel 20.25 miles?
78. If 123.25 qt. of milk make 17 lb. of butter, how many quarts of milk are required to make 1 lb. of butter?
79. A champion cow produced 34.32 lb. of butter in 1 week. At that rate, find the yield in 8 weeks.
80. Find the value of the oil from 750 coconuts, if 100 coconuts yield 2 gal. of oil worth 50¢ a gallon.
81. Find the perimeter of a triangular field whose sides are 60.5 rods, 48.25 rods, and 56.375 rods, respectively.
82. One day's product of a flour mill sold for \$8002.80, at \$3.42 per barrel. How much flour was produced?
83. A farmer sold his wheat for \$248.25, rye for \$176.45, and hay for \$369.50. Find the total income.
84. Mr. Green had a farm of 342.75 acres. He sold from it at different times 37.5 acres, 126.25 acres, and 88.5 acres. How many acres did he have left?

Find the cost of each when

85. 7 mufflers cost \$6.86.
86. 8 suitcases cost \$52.
87. 7.75 tons of coal cost \$51.15.
88. 6.25 tons of hay cost \$112.50.
89. 37.5 bu. of apples cost \$41.25.
90. 5.9 acres of land cost \$737.50.
91. Find the area of a lot 60.4 ft. by 40.5 ft.
92. One season 20 trees produced 712 boxes of apples that sold at \$1.50 per box. Find the average return per tree.
93. The yield of grapes from a 35-acre vineyard was sold for \$4014.50. Find the average income per acre.
94. Our country one year produced 50 million brooms worth 10 million dollars. Find the average value per broom.
95. A tree produced 910 lb. of nuts one season, worth \$.125 per pound. Find the value of the yield.
96. How long will it take an automobile that has a speed of 41.7 miles per hour to go 150.12 miles?
97. An automobile used 1.6 pt. of oil per 100 miles. How many gallons of oil were used on a trip of 2280 miles?
98. How much more will it cost to send 2575 tons of coal by rail at \$2.70 per ton than by boat at \$.43 per ton?
99. When wheat costs \$.625 per bushel, how many bushels can be bought for \$1000?
100. A California road 25 miles long was sprinkled with petroleum, 175.5 barrels to the mile. How much did the petroleum cost at \$.70 per barrel?

ALIUOT PARTS

1. What part of 100 is 10? 50? 20? 25? $12\frac{1}{2}$? $33\frac{1}{3}$?
2. What part of \$1 is \$.10? \$.50? \$.20? \$.12 $\frac{1}{2}$?
3. How many times does \$1 contain \$.10? \$.15? \$.50? \$.12 $\frac{1}{2}$? Which of these numbers *exactly* divide \$1?

The parts of a number that exactly divide it are **aliquot parts** of the number.

Thus, 25 is an aliquot part of 100; $12\frac{1}{2}$ ¢, of \$1.

Oral Exercises

1. Memorize this table of the aliquot parts of \$1 and their multiples.
2. What part of \$1 is \$.12 $\frac{1}{2}$?
3. Beginning with \$.12 $\frac{1}{2}$, add \$.12 $\frac{1}{2}$ successively until you reach \$1.
4. Which of the parts of \$1 given in the table are aliquot parts? Which are multiples of aliquot parts?
5. What is the cost of 24 articles at \$1 each? at \$.50, or $\frac{1}{2}$, each? at \$.25, or $\frac{1}{4}$, each?
6. From the parts of \$1 given in the table make a similar table of the corresponding parts of 100: thus, 50 is $\frac{1}{2}$ of 100; 25 is $\frac{1}{4}$ of 100; etc.
7. Memorize the table you have made.

50¢ =	\$ $\frac{1}{2}$ =	\$.50
25¢ =	\$ $\frac{1}{4}$ =	\$.25
75¢ =	\$ $\frac{3}{4}$ =	\$.75
20¢ =	\$ $\frac{1}{5}$ =	\$.20
40¢ =	\$ $\frac{2}{5}$ =	\$.40
60¢ =	\$ $\frac{3}{5}$ =	\$.60
80¢ =	\$ $\frac{4}{5}$ =	\$.80
10¢ =	\$ $\frac{1}{10}$ =	\$.10
30¢ =	\$ $\frac{3}{10}$ =	\$.30
70¢ =	\$ $\frac{7}{10}$ =	\$.70
90¢ =	\$ $\frac{9}{10}$ =	\$.90
5¢ =	\$ $\frac{1}{20}$ =	\$.05
12 $\frac{1}{2}$ ¢ =	\$ $\frac{1}{8}$ =	\$.12 $\frac{1}{2}$
37 $\frac{1}{2}$ ¢ =	\$ $\frac{3}{8}$ =	\$.37 $\frac{1}{2}$
62 $\frac{1}{2}$ ¢ =	\$ $\frac{5}{8}$ =	\$.62 $\frac{1}{2}$
87 $\frac{1}{2}$ ¢ =	\$ $\frac{7}{8}$ =	\$.87 $\frac{1}{2}$
6 $\frac{1}{4}$ ¢ =	\$ $\frac{1}{16}$ =	\$.06 $\frac{1}{4}$
33 $\frac{1}{3}$ ¢ =	\$ $\frac{1}{3}$ =	\$.33 $\frac{1}{3}$
66 $\frac{2}{3}$ ¢ =	\$ $\frac{2}{3}$ =	\$.66 $\frac{2}{3}$
16 $\frac{2}{3}$ ¢ =	\$ $\frac{1}{6}$ =	\$.16 $\frac{2}{3}$
83 $\frac{1}{3}$ ¢ =	\$ $\frac{5}{6}$ =	\$.83 $\frac{1}{3}$
8 $\frac{1}{3}$ ¢ =	\$ $\frac{1}{12}$ =	\$.08 $\frac{1}{3}$

Written Exercises

1. Find the cost of 44 pairs of tennis shoes @ \$.66 $\frac{2}{3}$.

SOLUTION. — At \$1 each, 44 pairs tennis shoes would cost \$44.
Then, at \$.66 $\frac{2}{3}$, or $\frac{2}{3}$, each, 44 pairs cost $\frac{2}{3}$ of \$44, or \$29.33.

NOTE. — Give results to the nearest cent, discarding less than $\frac{1}{2}$ cent and counting $\frac{1}{2}$ cent or a greater fraction of a cent as a whole cent.

Find the cost of 120 articles @ :

- | | | | | |
|--------|--------|-----------------------|------------------------|------------------------|
| 2. 20¢ | 6. 60¢ | 10. 90¢ | 14. 33 $\frac{1}{3}$ ¢ | 18. 66 $\frac{2}{3}$ ¢ |
| 3. 50¢ | 7. 25¢ | 11. 75¢ | 15. 12 $\frac{1}{2}$ ¢ | 19. 62 $\frac{1}{2}$ ¢ |
| 4. 40¢ | 8. 80¢ | 12. 8 $\frac{1}{3}$ ¢ | 16. 16 $\frac{2}{3}$ ¢ | 20. 83 $\frac{1}{3}$ ¢ |
| 5. 30¢ | 9. 70¢ | 13. 6 $\frac{1}{4}$ ¢ | 17. 37 $\frac{1}{2}$ ¢ | 21. 87 $\frac{1}{2}$ ¢ |

Find the cost of each lot of cloth :

- | | |
|---------------------|----------------------|
| 22. 25 yd. @ \$.20. | 26. 100 yd. @ \$.05. |
| 23. 45 yd. @ \$.10. | 27. 125 yd. @ \$.70. |
| 24. 48 yd. @ \$.25. | 28. 150 yd. @ \$.80. |
| 25. 75 yd. @ \$.30. | 29. 144 yd. @ \$.75. |
30. Find the cost of 72 plates @ 10¢.
31. Find the cost of 84 knives @ 50¢.
32. Find the cost of 95 lb. tea @ 40¢.
33. Find the cost of 50 towels @ 25¢.

Find the cost at wholesale of :

- | | |
|---------------------------------------|--|
| 34. 25 balls @ \$.50. | 39. 30 brushes @ \$.37 $\frac{1}{2}$. |
| 35. 18 books @ \$.12 $\frac{1}{2}$. | 40. 50 thimbles @ \$.33 $\frac{1}{3}$. |
| 36. 20 forks @ \$.37 $\frac{1}{2}$. | 41. 60 yd. serge @ \$.87 $\frac{1}{2}$. |
| 37. 24 clocks @ \$.62 $\frac{1}{2}$. | 42. 70 yd. damask @ \$.66 $\frac{2}{3}$. |
| 38. 48 purses @ \$.83 $\frac{1}{3}$. | 43. 100 lb. pecans @ \$.16 $\frac{2}{3}$. |

44. Find the cost of 15 whips at \$1.25 each.

\$15 The business man writes \$15, the cost at \$1 each;
 3.75 below \$15 he writes \$3.75, the cost at \$.25, or $\frac{1}{4}$,
 found by dividing \$15 by 4.
 \$18.75 Adding, he obtains \$18.75, the cost at \$1.25.

45. Find the cost of 22 yd. of carpet at \$.87 $\frac{1}{2}$ per yard.

\$22 What is the cost at \$1 per yard? at \$.12 $\frac{1}{2}$, or $\frac{1}{8}$,
 2.75 per yard? at \$1-\$.12 $\frac{1}{2}$, or \$.87 $\frac{1}{2}$, per yard?

 \$19.25

Find the cost of 240 articles @ :

- | | | | |
|-------------------------|--------------------------|--------------------------|--------------------------|
| 46. \$.83 $\frac{1}{8}$ | 49. \$1.20 | 52. \$1.12 $\frac{1}{2}$ | 55. \$1.62 $\frac{1}{2}$ |
| 47. \$1.10 | 50. \$1.60 | 53. \$1.37 $\frac{1}{2}$ | 56. \$1.06 $\frac{1}{4}$ |
| 48. \$.66 $\frac{2}{3}$ | 51. \$1.33 $\frac{1}{3}$ | 54. \$1.66 $\frac{2}{3}$ | 57. \$1.87 $\frac{1}{2}$ |

Find the cost at wholesale of:

58. 44 yd. silk @ \$1.25. 61. 56 croquet sets @ \$1.50.
 59. 48 yd. satin @ \$1.33 $\frac{1}{3}$. 62. 33 fishing rods @ \$2.25.
 60. 32 yd. linen @ \$1.12 $\frac{1}{2}$. 63. 30 bathing suits @ \$1.75.

64. Find the cost of 27 buggies at \$75 each.

675 At \$100 each, 27 buggies would cost
 $\frac{3}{4}$ of \$2700 = \$2025 \$2700. At \$75 each, then, they cost $\frac{3}{4}$
 of \$2700, or \$2025.

Find the cost of:

- | | |
|---------------------------------------|--|
| 65. 24 cows @ \$50. | 69. 12 ponies @ \$137 $\frac{1}{2}$. |
| 66. 18 buffets @ \$25. | 70. 18 horses @ \$162 $\frac{1}{2}$. |
| 67. 30 stoves @ \$37 $\frac{1}{2}$. | 71. 16 pianos @ \$187 $\frac{1}{2}$. |
| 68. 25 surreys @ \$87 $\frac{1}{2}$. | 72. 20 fur coats @ \$133 $\frac{1}{3}$. |

73. Find the cost of 400 lb. of butter at 25¢ per pound.
74. Mr. Hopkins bought 236 bu. of oats at \$.50 a bushel. How much did they cost him?
75. How much will a boy earn in 28 days at 75¢ a day?
76. At a fair 3248 50-cent tickets were sold in one day. Find the gate receipts for that day.
77. An orchard yielded 840 bu. of peaches. How much were they worth at \$1.50 per bushel?
78. A car contained 175 barrels of apples. How much were they worth at \$1.40 per barrel?
79. How much must be paid for 3548 bu. of wheat at 90¢ per bushel?
80. A cargo of lemons consisting of 25,000 boxes was bought for \$1.60 per box. Find the cost of the cargo.
81. A dealer bought 50 cameras at \$12½ each. Find the total cost of the cameras.
82. How much did a farmer receive for 2456 bu. of corn which he sold at 62½¢ per bushel?
83. Thirty olive pickers and an overseer received \$50 a day. If each picker received \$1.50 per day, how much did the overseer receive?
84. Find the cost of sending 30,000 lb. of oranges from California to New York at \$1.25 per hundredweight.
85. Find the cost, at \$.80 per square foot, of a lot 135 ft. deep and 48 ft. wide.
86. A grocer bought 650 lb. of coffee at 22½¢ per pound, and sold it at 35¢ per pound. How much did he gain?

Oral Exercises

How many 50-cent articles can be bought for

1. \$1? 2. \$2? 3. \$5? 4. \$12? 5. \$1.50? 6. \$3.50?
7. How many 25-cent articles will these amounts buy?

How many $33\frac{1}{3}$ -cent articles can be bought for

8. \$1? 9. \$2? 10. \$4? 11. \$10? 12. \$15? 13. \$33?
14. How many $16\frac{2}{3}$ -cent articles will these amounts buy?

Written Exercises

1. How many articles, at $37\frac{1}{2}\phi$ each, can be bought for \$84?

28

$$84 \times \frac{8}{\cancel{3}} = 224$$

Since $37\frac{1}{2}\phi = \$\frac{3}{8}$, the number of articles is equal to the number of times \$84 contains $\frac{3}{8}$, which may be found by dividing 84 by $\frac{3}{8}$, or by multiplying 84 by $\frac{8}{3}$.

Hence, 224 articles can be bought.

Find the number of articles that can be bought for:

2. \$25 @ 25ϕ 5. \$98 @ $16\frac{2}{3}\phi$ 8. \$1.50 @ $61\frac{1}{4}\phi$
3. \$42 @ 75ϕ 6. \$87 @ $37\frac{1}{2}\phi$ 9. \$495 @ $62\frac{1}{2}\phi$
4. \$56 @ 80ϕ 7. \$96 @ $66\frac{2}{3}\phi$ 10. \$560 @ $83\frac{1}{3}\phi$

11. How many articles, at $\$1.33\frac{1}{3}$ each, will \$112 buy?

SUGGESTION.—Since $\$1.33\frac{1}{3} = \$1\frac{1}{3}$, or $\frac{4}{3}$, the number of articles is equal to the number of times 112 contains $\frac{4}{3}$.

Find the number of articles that can be bought for:

12. \$65 @ \$1.25 15. \$54 @ \$2.25 18. \$660 @ $\$1.37\frac{1}{2}$
13. \$85 @ \$2.50 16. \$36 @ $\$1.33\frac{1}{3}$ 19. \$735 @ $\$1.66\frac{2}{3}$
14. \$28 @ \$3.50 17. \$63 @ $\$1.16\frac{2}{3}$ 20. \$825 @ $\$1.87\frac{1}{2}$

21. How many articles, @ $\$87\frac{1}{2}$, can I buy for $\$1400$?

SUGGESTION. — Since $\$87\frac{1}{2} = \frac{7}{8}$ of $\$100$, we may divide $\$1400$ by $\$87\frac{1}{2}$ by dividing first by $\$100$ and then by $\frac{7}{8}$.

Find the number of articles that can be bought for:

22. $\$650$ @ $\$12\frac{1}{2}$ 24. $\$1500$ @ $\$125$ 26. $\$1200$ @ $\$66\frac{2}{3}$

23. $\$900$ @ $\$16\frac{2}{3}$ 25. $\$2625$ @ $\$175$ 27. $\$2750$ @ $\$83\frac{1}{3}$

28. At 75¢ a day, how long will it take Roy to earn $\$6.75$?

29. A grocer paid $\$50$ for eggs at 25¢ a dozen. How many dozen eggs did he buy?

30. How many pails, at $37\frac{1}{2}\text{¢}$ each, can you buy for $\$54$?

31. Eleanor paid $\$10$ for some cloth at $\$1.25$ a yard. How many yards did she buy?

32. At $\$.66\frac{2}{3}$ a yard, how many yards of linen can a merchant buy for $\$75$?

33. Mr. Clark paid $\$103$ for some maple sugar at $\$.12\frac{1}{2}$ a pound. How many pounds did he buy?

34. An upholsterer paid $\$112.50$ for some mohair plush at $\$1.50$ per yard. How many yards did he buy?

35. A dealer bought some refrigerators for $\$800$, at $\$16\frac{2}{3}$ each. How many refrigerators did he buy?

36. It cost $\$206.25$ to dig a reservoir at $62\frac{1}{2}\text{¢}$ a cubic yard. How many cubic yards of earth were removed?

37. At $\$66\frac{2}{3}$ apiece, how many cows will $\$3000$ buy?

38. A jobber bought oranges at $\$2.25$ per box, paying $\$1629$ for them. How many boxes of oranges did he buy?

39. A wholesale dealer paid $\$2277$ for peaches at $\$.12\frac{1}{2}$ per basket. How many baskets of peaches did he buy?

Any desired part of 1, 10, 100, 1000 may be constructed as illustrated here :

Parts	Of 1	Of 10	Of 100	Of 1000
$\frac{1}{8}$.125	1.25	12.5	125
$\frac{1}{4}$.25	2.5	25	250
$\frac{3}{8}$.375	3.75	37.5	375

Written Exercises

1. Multiply 72 by 375.

SOLUTION. — 375 is $\frac{3}{8}$ of 1000; then, $72 \times 1000 \times \frac{3}{8} = 27,000$.

Find products :

- | | | |
|---------------------|---------------------|-----------------------|
| 2. $27 \times .125$ | 5. 250×67 | 8. 125×3.9 |
| 3. 71×750 | 6. 875×48 | 9. 87.5×48 |
| 4. 62×37.5 | 7. $625 \times .81$ | 10. 440×3.75 |

11. Divide 720 by 37.5.

SUGGESTION. — Since 37.5 is $\frac{3}{8}$ of 100, we divide 720 by 100 and the result by $\frac{3}{8}$.

Find quotients :

- | | | |
|-------------------|---------------------|----------------------|
| 12. $47 \div .25$ | 15. $428 \div 250$ | 18. $648 \div 1.25$ |
| 13. $64 \div 2.5$ | 16. $240 \div 12.5$ | 19. $34.3 \div 87.5$ |
| 14. $48 \div 7.5$ | 17. $111 \div 37.5$ | 20. $.425 \div .125$ |

Multiply by $.66\frac{2}{3}$; by 3.75; by 12.5; by 625 :

- | | | | | |
|--------|---------|----------|----------|-----------|
| 21. 48 | 23. 288 | 25. 6.72 | 27. 52.8 | 29. 12.96 |
| 22. 72 | 24. 456 | 26. .192 | 28. .264 | 30. .5616 |

Divide by .25; by 7.5; by 87.5; by $333\frac{1}{3}$:

- | | | | | |
|--------|---------|---------|----------|-----------|
| 31. 63 | 33. 252 | 35. 756 | 37. 1071 | 39. 724.5 |
| 32. 84 | 34. 504 | 36. 987 | 38. 2352 | 40. 49.14 |

MEASURES AND MEASUREMENTS

LENGTH MEASURES

Memorize the table :

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

1 rod = $16\frac{1}{2}$ feet; 1 mile = 5280 feet.

A *hand*, used in measuring the height of horses, is 4 inches; a *furlong* is $\frac{1}{8}$ of a mile.

A *fathom*, used in measuring the depth of water, is 6 feet; a *nautical mile (knot)*, used in measuring distances at sea, is 6080.27 feet, or about 1.15 miles.

Written Exercises

1. How many yards are there in 8 rd. 2 yd. ? 2. Express 278 in. as feet and inches.

SOLUTION

1 rd. = $5\frac{1}{2}$ yd.
 8 rd. = $8 \times 5\frac{1}{2}$ yd. = 44 yd.
 8 rd. 2 yd. = 44 yd. + 2 yd.
 = 46 yd.

SOLUTION

12 in. = 1 ft.
 278 in. \div 12 in. = 23 (ft.), 2 (in.).
 Then, 278 in. = 23 ft. 2 in.

Reduce to the next lower unit :

3. 6 rd. 8. 16 yd. 13. 25 ft. 18. $2\frac{1}{3}$ yd. 23. 110 ft.
 4. 3 mi. 9. 14 ft. 14. 16 mi. 19. $8\frac{3}{4}$ ft. 24. 225 yd.
 5. 9 rd. 10. 12 rd. 15. 24 rd. 20. $9\frac{2}{3}$ yd. 25. 244 ft.
 6. 5 mi. 11. 19 yd. 16. 38 yd. 21. $3\frac{1}{3}$ rd. 26. 350 rd.
 7. 8 rd. 12. 20 rd. 17. 27 mi. 22. $4\frac{1}{2}$ mi. 27. 230 mi.

Change to feet:

Change to rods:

28. 18 yd.	33. 16 rd.	38. 6 mi.	43. 99 ft.
29. 10 rd.	34. 84 yd.	39. 33 yd.	44. 77 yd.
30. 32 yd.	35. 24 rd.	40. 66 ft.	45. 18 mi.
31. 12 rd.	36. 168 in.	41. 44 yd.	46. 88 yd.
32. 45 yd.	37. 216 in.	42. 12 mi.	47. 22 mi.

Reduce to the next higher unit:

48. 48 ft.	53. 114 ft.	58. 440 yd.	63. 720 rd.
49. 22 yd.	54. 132 yd.	59. 375 ft.	64. 432 ft.
50. 72 ft.	55. 640 rd.	60. 400 rd.	65. 660 yd.
51. 55 yd.	56. 264 ft.	61. 561 yd.	66. 880 rd.
52. 96 ft.	57. 480 rd.	62. 960 rd.	67. 814 yd.

68-87. Reduce exercises 48-67 to the next lower unit.

Reduce to the lower of the given units:

88. 7 ft. 4 in.	93. 18 ft. 9 in.	98. 45 ft. 7 in.
89. 6 yd. 2 ft.	94. 17 yd. 2 ft.	99. 34 yd. 1 ft.
90. 8 rd. 5 ft.	95. 24 ft. 6 in.	100. 18 rd. 4 yd.
91. 4 mi. 9 rd.	96. 36 rd. 5 yd.	101. 44 rd. 9 ft.
92. 6 rd. 3 yd.	97. 10 mi. 8 rd.	102. 29 mi. 8 rd.

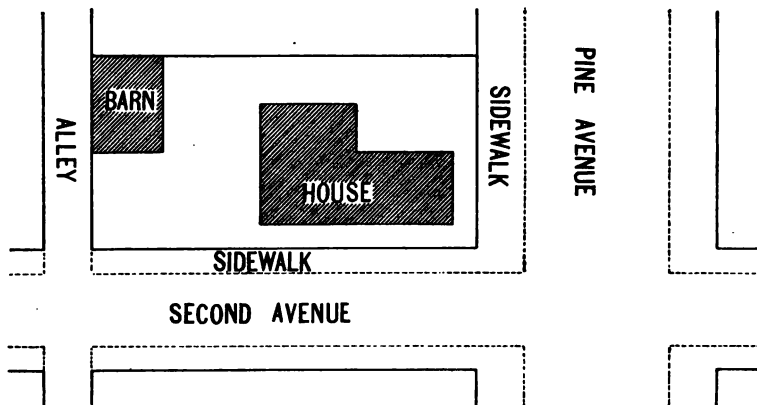
Express in the next higher and the given unit:

103. 32 ft.	108. 82 in.	113. 150 in.	118. 260 in.
104. 28 in.	109. 64 ft.	114. 188 ft.	119. 443 ft.
105. 40 ft.	110. 90 in.	115. 650 rd.	120. 358 rd.
106. 75 in.	111. 59 ft.	116. 227 ft.	121. 683 ft.
107. 50 ft.	112. 98 in.	117. 189 in.	122. 970 rd.

MEASUREMENTS FROM SCALE DRAWINGS

Written Exercises

1. In this map, or plan, 1 inch represents 64 feet. If you measure the plan of the house, you will find that it is 1 inch long. Then, the house is 64 feet long.



If 1 in. represents 64 ft., what distance is represented by

2. $\frac{1}{8}$ in.? 3. $\frac{1}{4}$ in.? 4. $\frac{3}{8}$ in.? 5. $\frac{5}{8}$ in.? 6. 2 in.?

Using a rule divided to eighths of an inch, find:

- | | |
|--------------------|-----------------------------|
| 7. Width of barn. | 10. Width of alley. |
| 8. Length of barn. | 11. Width of each avenue. |
| 9. Width of house. | 12. Width of each sidewalk. |

When $\frac{1}{4}$ in. represents 1 ft., how many inches represent

13. 10 ft. 6 in.? 14. 12 ft. 3 in.? 15. 24 ft. 9 in.?

16. Draw (scale $\frac{1}{4}$ in. to 1 ft.) the plan of a cellar bottom, 19 ft. by 23 ft., with a coal bin, 6 ft. by 8 ft., in one corner.

AREA MEASURES

Memorize this table:

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

A square mile in the form of a square is a *section*.

Written Exercises

Change to square feet:

- | | | | |
|---------------|----------------|---------|-----------------|
| 1. 15 sq. yd. | 4. 288 sq. in. | 7. 5 A. | 10. 121 sq. yd. |
| 2. 27 sq. yd. | 5. 432 sq. in. | 8. 7 A. | 11. 242 sq. yd. |
| 3. 32 sq. yd. | 6. 576 sq. in. | 9. 8 A. | 12. 363 sq. yd. |

Change to square rods:

- Reduce to the next lower unit:
- | | | | |
|-----------------|-----------------|-----------------|-------------|
| 13. 117 sq. ft. | 17. 605 sq. yd. | 21. 800 sq. rd. | 25. 1280 A. |
| 14. 320 sq. rd. | 18. 207 sq. ft. | 22. 847 sq. yd. | 26. 1920 A. |
| 15. 484 sq. yd. | 19. 640 sq. rd. | 23. 315 sq. ft. | 27. 2560 A. |
| 16. 162 sq. ft. | 20. 726 sq. yd. | 24. 968 sq. yd. | 28. 4480 A. |

29-44. Reduce exercises 13-28 to the next higher unit.

Reduce to the lower of the given units:

- | | |
|-------------------------|--------------------------|
| 45. 9 sq. yd. 7 sq. ft. | 47. 15 A. 10 sq. rd. |
| 46. 8 sq. ft. 4 sq. in. | 48. 12 sq. rd. 5 sq. yd. |

Express in the next higher and the given unit:

- | | | | |
|-----------------|-----------------|-----------------|-------------|
| 49. 110 sq. ft. | 51. 230 sq. rd. | 53. 200 sq. ft. | 55. 1300 A. |
| 50. 290 sq. in. | 52. 350 sq. rd. | 54. 440 sq. in. | 56. 1950 A. |

MEASUREMENT OF RECTANGLES

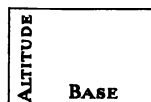
The difference in the direction of two lines that meet is an **angle**.



A figure that has four straight sides and four *equal* angles is a **rectangle**; the equal angles are **right angles**.

An equal-sided rectangle is a **square**.

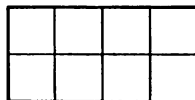
The two lines that form a right angle are **perpendicular** to each other.



The side of a figure on which it is assumed to stand is the **base**, and the height of the figure measured on a line perpendicular to the base is the **altitude**.

The number of square units in a surface is its **area**.

Thus, the area of a rectangle 4 in. long and 2 in. wide is 8 sq. in., for it may be divided into 2 rows, each containing 4 sq. in.



If the base and the altitude of a rectangle are expressed in *inches*, the area is found by multiplying the *number* of inches in the base by the *number* of inches in the altitude and calling the result *square inches*. If both dimensions are in *feet*, the area is found by multiplying one dimension by the other and calling the result *square feet*.

This is what is meant by the briefer statement:

The area of a rectangle is equal to the product of its base and altitude, expressed in like units.

It follows from the above that:

Either dimension is the quotient of the area and the other dimension, expressed in corresponding units.

Written Exercises

1. Find the area of a rectangle 12 ft. by 2 ft. 6 in.

SOLUTION. — Express the dimensions in *like units*, thus:

$$\text{Base} = 12 \text{ ft.}; \text{altitude} = 2 \text{ ft. } 6 \text{ in.} = 2\frac{1}{2} \text{ ft.}$$

$$\text{Then, area} = (2\frac{1}{2} \times 12) \text{ sq. ft.} = 30 \text{ sq. ft.}$$

2. A 24-acre rectangle is 60 rd. long. Find its width.

SOLUTION. — Express the length and area in *corresponding* units:

$$\text{Length} = 60 \text{ rd.}; \text{area} = 24 \text{ acres} = 24 \times 160 \text{ sq. rd.}$$

$$\text{Then, width} = \frac{24 \times 160}{60} \text{ rd.} = 64 \text{ rd.}$$

Find the area of a square whose side is:

3. 25 yd. 4. 34 ft. 5. 26.4 rd. 6. 12 ft. 6 in.

Find the area of a rectangle whose dimensions are:

7. 18 ft., 16 ft. 9. 24 rd., 20 rd. 11. 12 ft., $11\frac{1}{2}$ ft.
8. 17 yd., 13 yd. 10. 10 yd., 27 ft. 12. 21 ft., $16\frac{1}{3}$ ft.

Find the other dimension:

13. Area, 270 sq. ft.; length, 18 ft. 15. 660 sq. ft., 30 ft.
14. Area, 336 sq. yd.; width, 16 yd. 16. 896 sq. rd., 32 rd.

17. A baseball back stop is 12 ft. by 24 ft. Find the number of square feet of wire netting required to cover it.

18. The area of a concrete cellar floor is 1710 sq. ft. and its width is 38 ft. How long is it?

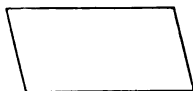
19. How many square yards of surface are there in the side wall of a room 16 ft. long and 12 ft. high?

20. A football field is 160 ft. wide, and its area is 48,000 sq. ft. Find its length.

MEASUREMENT OF PARALLELOGRAMS

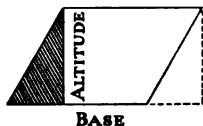
Lines that cannot meet, however far they are extended, are **parallel lines**.

A four-sided figure whose opposite sides are parallel is a **parallelogram**.



A rectangle is a right-angled parallelogram.

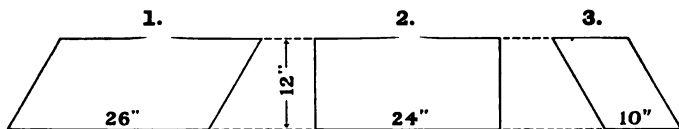
If the shaded part of this parallelogram is cut off and placed in the position indicated by dotted lines, the resulting figure will be a rectangle of the same area, base, and altitude as the parallelogram. Hence,



The area of a parallelogram is equal to the product of its base and altitude, expressed in like units.

Written Exercises

Find the area of each of these parallelograms:

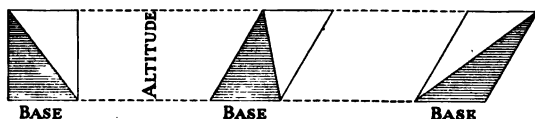


Find the missing area or part for each parallelogram:

	BASE	ALTITUDE	AREA
4.	22 ft.	9 ft. 6 in.	— sq. ft.
5.	16 ft.	42 in.	— sq. ft.
6.	12 rd.	99 ft.	— sq. rd.
7.	— rd.	26 rd.	988 sq. rd.
8.	84.3 ft.	20 yd.	— sq. yd.
9.	160 rd.	— rd.	120 acres

MEASUREMENT OF TRIANGLES

A plane figure bounded by three straight lines is a triangle.



Each shaded triangle is one half of a parallelogram of the same base and altitude as the triangle. Hence,

The area of a triangle is equal to half the product of its base and altitude, expressed in like units.

Written Exercises

Find the area of each of the following triangles :

BASE	ALTITUDE	BASE	ALTITUDE
1. 25 in.	18 in.	7. 56 rd.	42 rd.
2. 42 in.	16 in.	8. 42 yd	84 ft.
3. 38 ft.	22 ft.	9. 36 ft.	26½ ft.
4. 32 yd.	28 yd.	10. 72 ft.	56 ft. 9 in.
5. 54 rd.	40 rd.	11. 68 ft.	37 ft. 6 in.
6. 65 yd.	28 yd.	12. 84 yd.	68 yd. 1 ft.

13. Find the area of a triangular felt banner whose base is 10 in. and altitude 22 in.

14. The roof of this tower is composed of 5 triangular parts, each having a base of 7 ft. and an altitude of 9 ft. Find the area of the roof.



15. The gable end of a house is a triangle whose base is 32 ft. and altitude 8 ft. Find the area.

Miscellaneous Exercises

Find the area of each of the following:

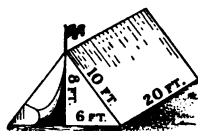
FIGURE	BASE	ALTITUDE	AREA
1. Square	36 ft.	36 ft.	—— sq. ft.
2. Rectangle	28 ft.	24 ft. 3 in.	—— sq. ft.
3. Rectangle	54 yd.	32 yd. 2 ft.	—— sq. yd.
4. Triangle	24 ft.	10 ft. 4 in.	—— sq. ft.
5. Triangle	42 ft.	24 yd. 1 ft.	—— sq. yd.
6. Parallelogram	24 rd.	330 ft.	—— acres.

7. The main passage of the New York post office is 280 ft. long and 29 ft. wide. Find its area.

8. A triangular grass plot has a base of 42 ft. and an altitude of 28 ft. 6 in. Find its area.

Find the number of square yards of canvas needed for:

9. Each side of the tent.
10. Each end of the tent.
11. Find the area of the floor.



12. Find the area of a belt 72 ft. long and $3\frac{1}{2}$ ft. wide.
13. How wide is a 4-acre strip of land that is 80 rd. long?
14. How many acres are there in a farm in the form of a parallelogram whose base is 128 rd. and altitude 120 rd.?
15. Find the cost of a hardwood floor for your school-room at 8¢ per square foot.
16. The area of a lot in the form of a parallelogram is 1 acre and its altitude is 12 rd. How long is it?

LATHING AND PLASTERING

Plastering is usually computed by the **square yard**.

Estimates for plastering, which often include the lathing, are figured to the *nearest* square yard in final results.

Allowances are sometimes made for the whole or a part of the area of openings, but in this book the exact allowances to be made (if any) are given in the exercises.

Laths are 4 ft. long and are sold in **bundles** of 50 or 100.

In this book, 1 bundle means 100 laths. Part of a bundle cannot be bought.

When lathing is done under a separate contract, the lather is usually paid by the *bundle* or by the *square yard*. A bundle is estimated to cover 5 sq. yd. of surface.

Written Exercises

1. Find the cost of lathing 44 sq. yd. at 58¢ a bundle for material and labor.
2. What is the cost, at 35¢ per square yard, of lathing and plastering the walls and ceiling of a room $16' \times 14' \times 9'$, deducting 2 sq. yd. for each of 2 doors and 2 windows?

SOLUTION

The distance around the room $= 2 \times (16' + 14') = 60'$.

The area of the walls $= (9 \times 60)$ sq. ft. $= 540$ sq. ft.

The area of the ceiling $= (16 \times 14)$ sq. ft. $= 224$ sq. ft.

The total area of walls and ceiling $= 764$ sq. ft.

The total area in square yards $= (764 \div 9)$ sq. yd. $= 84\frac{2}{3}$ sq. yd.

The deductions are 4×2 sq. yd. $= 8$ sq. yd.

Then, the area to be plastered $= 84\frac{2}{3}$ sq. yd. $- 8$ sq. yd. $= 76\frac{2}{3}$ sq. yd., or to the nearest square yard, 77 sq. yd.

Hence, the cost at 35¢ per square yard $= 77 \times \$.35 = \26.95 .

3. How many bundles of lath are needed for the walls and ceiling of a room $15' \times 12' \times 9'$?

Find the number of square yards of plastering there are in a room 9' high whose ceiling dimensions are:

4. $12' \times 6'$ 7. $12' \times 12'$ 10. $18' \times 18'$ 13. $21' \times 15'$

5. $15' \times 9'$ 8. $18' \times 16'$ 11. $21' \times 18'$ 14. $18' \times 13'$

6. $11' \times 9'$ 9. $15' \times 15'$ 12. $18' \times 15'$ 15. $20' \times 18'$

16. At 34¢ per square yard, find the cost of lathing and plastering a room $19' \times 16' \times 10'$, deducting $2\frac{1}{2}$ sq. yd. for each of 4 doors and 2 sq. yd. for each of 3 windows.

17. Find the cost, at 45¢ per square yard, of plastering the brick side walls of a room $21' \times 15' \times 12'$, deducting 7 sq. yd. for openings.

Find the cost of plastering each room, height 10', at 23¢ per square yard, deducting 2 sq. yd. for each opening:

18. Hall, $20' \times 6'$, 8 doors.

19. Hall, $15' \times 6'$, 3 doors, 1 window.

20. Parlor, $18' \times 16'$, 3 doors, 4 windows.

21. Bedroom, $13' \times 10'$, 2 doors, 2 windows.

22. Bedroom, $12' \times 12' 3''$, 2 doors, 1 window.

23. Bathroom, $8' 6'' \times 6'$, 1 door, 1 window.

24. Kitchen, $12' \times 11' 9''$, 4 doors, 2 windows.

25. Dining room, $16' \times 15' 3''$, 4 doors, 3 windows.

26. A mason plastered a schoolroom $42' \times 30' \times 12'$ for \$140. If, after deducting 12 sq. yd. for openings, the work cost him 34¢ per square yard, what was his profit?

PAINTING AND CALCIMINING

Painting and calcimining are usually estimated by the square yard.

In this book, results are given to the nearest square yard.

Written Exercises

Making no deductions, find the cost, at 20¢ per square yard, of painting the front of a house:

- | | | | |
|--------------|--------------|--------------|---------------|
| 1. 18' × 26' | 4. 23' × 27' | 7. 29' × 36' | 10. 31' × 45' |
| 2. 22' × 27' | 5. 24' × 30' | 8. 30' × 39' | 11. 36' × 42' |
| 3. 21' × 24' | 6. 28' × 36' | 9. 27' × 34' | 12. 33' × 45' |

Compute the cost, at 21¢ per square yard, of filling and varnishing a floor:

- | | | | |
|--------------|---------------|---------------|------------------|
| 13. 12' × 6' | 15. 15' × 12' | 17. 15' × 15' | 19. 16' × 15' 9" |
| 14. 13' × 9' | 16. 18' × 15' | 18. 18' × 17' | 20. 18' × 16' 6" |

Find the cost of calcimining each of these rooms at 6¢ per square yard, deducting 2 sq. yd. for each opening:

21. Bathroom, 9' × 7' × 8' 9", 1 window, 1 door.
22. Bedroom, 12' × 12' × 8' 9", 2 windows, 1 door.
23. Bedroom, 14' × 12' × 8' 9", 2 windows, 2 doors.
24. Kitchen, 15' × 13' × 8' 9", 2 windows, 3 doors.
25. Two boys painted both sides of a tight board fence 180' long and 6' high. How many square yards did they paint?
26. At 14¢ per square yard, find the cost of painting a galvanized iron roof 63' 9" by 24'.
27. Find the cost of painting the ceiling of your school-room, at 11¢ per square yard.

CARPETING

Carpeting is sold by the linear yard. It is usually 1 yard or $\frac{3}{4}$ of a yard in width.

Matting, oilcloth, linoleum, etc., are of various widths.

In matching the pattern there is usually some waste, but no allowance is necessary for the first strip because the other strips are matched to it.

Sometimes it is necessary to make a carpet a little too wide and *turn it under*, because the width of a strip may not be exactly contained in the width of the room, and parts of strips cannot be purchased; consequently, in computing the cost of carpets, any part of a strip is considered a whole strip.

Unless stated to the contrary, the exercises in this book require the strips to run lengthwise of the room.

Written Exercises

1. Find the cost, at \$.99 per yard, of carpet 27" wide for a floor $20' \times 15'$, allowing $\frac{2}{3}$ yd. for matching.

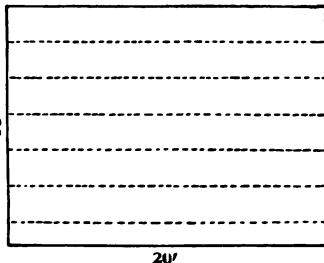
SOLUTION

Since the carpet is 27", or $2\frac{1}{4}'$, wide and the strips run lengthwise of the room, the number of strips required is $15' \div 2\frac{1}{4}'$, or $6\frac{2}{5}$, but 7 strips are necessary.

Then, the length of carpet needed is $7 \times 20'$, or 140', which equals $46\frac{2}{3}$ yd.

Since matching the pattern takes $\frac{2}{3}$ yd. more, $46\frac{2}{3}$ yd. + $\frac{2}{3}$ yd., or $47\frac{1}{3}$ yd., must be bought.

Hence, the cost of the carpet = $47\frac{1}{3} \times \$.99$, or \$46.86.



2. Find the cost, at \$ 1.10 per yard, of enough carpet 27" wide for a floor $18' \times 17'$, allowing $\frac{1}{2}$ yd. for matching.

Allowing an 18-inch margin on all sides, find the dimensions of a rug for a room :

3. $12' \times 9'$ 5. $15' \times 13'$ 7. $18' \times 15'$ 9. $12' \times 10'6''$
 4. $15' \times 9'$ 6. $18' \times 18'$ 8. $21' \times 18'$ 10. $15' \times 13'6''$

Find the number of yards of unfigured carpet 1 yard wide required to cover a floor :

11. $12' \times 9'$ 14. $13' \times 9'$ 17. $14' \times 11'$ 20. $13'6'' \times 12'$
 12. $15' \times 12'$ 15. $19' \times 18'$ 18. $15' \times 14'$ 21. $18'8'' \times 17'$
 13. $18' \times 15'$ 16. $24' \times 21'$ 19. $17' \times 15'$ 22. $19'6'' \times 18'$

Find the cost of carpet 27'' wide for each room :

ROOM	LENGTH	WIDTH	ALLOWANCE	PRICE PER YARD
23. Parlor	20'	18'	1 yd.	\$1.35
24. Library	21'	20'	$1\frac{1}{3}$ yd.	\$1.20
25. Bedroom	12'	10'9'	$\frac{2}{3}$ yd.	\$.90
26. Bedroom	15'	14'6''	$\frac{3}{4}$ yd.	\$1.00
27. Dining room	18'	17'9''	1 yd.	\$1.05

28. Find the cost of carpeting the New York senate chamber, $60' \times 55'$, with Victoria Wilton carpet 27'' wide at \$2.35 per yard, making no allowance for matching.

29. A room in which I wish to lay matting 1 yd. wide and costing 45¢ per yard is $18' \times 14'$. How much cheaper is it to lay the matting the short way of the room ?

30. How many yards of carpet 27'' wide will be required for a room, $18' \times 15'$, if there is a waste of $\frac{1}{6}$ yd. in each strip (except the first) in matching the pattern ?

VOLUME MEASURES

Memorize this table :

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

A *cubic yard* of earth is considered a *load*.

Written Exercises

Reduce to the next lower unit :

- | | | |
|--------------|---------------|----------------------------|
| 1. 3 cu. ft. | 5. 12 cu. yd. | 9. $2\frac{1}{2}$ cu. ft. |
| 2. 5 cu. yd. | 6. 16 cu. ft. | 10. $5\frac{1}{3}$ cu. yd. |
| 3. 7 cu. ft. | 7. 25 cu. yd. | 11. $6\frac{3}{4}$ cu. ft. |
| 4. 8 cu. yd. | 8. 36 cu. ft. | 12. $8\frac{2}{3}$ cu. yd. |

Reduce to the next higher unit :

- | | | |
|-----------------|------------------|--------------------|
| 13. 162 cu. ft. | 17. 1080 cu. ft. | 21. 10,368 cu. in. |
| 14. 378 cu. ft. | 18. 3456 cu. in. | 22. 13,824 cu. in. |
| 15. 486 cu. ft. | 19. 1350 cu. ft. | 23. 20,736 cu. in. |
| 16. 783 cu. ft. | 20. 6912 cu. in. | 24. 25,920 cu. in. |

Express in the lower of the given units :

- | | |
|-------------------------|-------------------------|
| 25. 5 cu. ft. 7 cu. in. | 27. 6 cu. ft. 8 cu. in. |
| 26. 8 cu. yd. 6 cu. ft. | 28. 9 cu. yd. 9 cu. ft. |

Express in the next higher and the given unit :

- | | | |
|-----------------|------------------|------------------|
| 29. 150 cu. ft. | 31. 1800 cu. in. | 33. 1000 cu. ft. |
| 30. 200 cu. ft. | 32. 3460 cu. in. | 34. 5200 cu. in. |

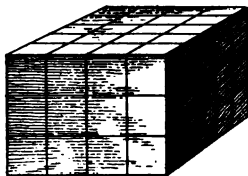
35. In digging a cellar, 6480 cu. ft. of earth were removed. Find the cost at $33\frac{1}{3}$ ¢ per cubic yard.

MEASUREMENT OF RECTANGULAR SOLIDS

A solid having six rectangular faces is a **rectangular solid**.

A rectangular solid whose faces are equal squares is a **cube**.

The number of cubic units that any solid contains is its **volume**.



Thus, if each cube in this rectangular solid represents some cubic unit, as a *cubic inch*, each row represents 5 cubic inches, each layer 4×5 cubic inches, and since there are 3 layers, the rectangular solid contains $3 \times 4 \times 5$ cubic inches; that is, its *volume* is 60 cubic inches.

If the length, the width, and the thickness of a rectangular solid are expressed in *inches*, the volume is found by multiplying the *number* of inches in the length by the *number* of inches in the width and that product by the *number* of inches in the thickness, calling the result *cubic inches*. If all the dimensions are in *feet*, the volume is obtained by finding the product of the *numbers* that measure them and calling the result *cubic feet*.

This is what is meant by the briefer statement :

The volume of a rectangular solid is equal to the product of its length, width, and thickness, all expressed in like units.

Thus, to find the volume of a rectangular solid 12 ft. by 8 ft. 6 in. by 4 ft. 4 in., since the dimensions must be expressed in *like units*, we may express them in *feet*, as follows, 12 ft. by $8\frac{1}{2}$ ft. by $4\frac{1}{3}$ ft. Then, the *number* of *cubic feet* in the volume is obtained by finding the product of 12, $8\frac{1}{2}$, and $4\frac{1}{3}$.

That is, the volume = $(12 \times 8\frac{1}{2} \times 4\frac{1}{3})$ cu. ft. = 442 cu. ft.

Written Exercises

Find the volume of each rectangular solid:

- | | |
|----------------------------|-------------------------------------|
| 1. 6 ft. by 5 ft. by 4 ft. | 7. $10' \times 5' \times 30''$ |
| 2. 9 ft. by 6 ft. by 2 ft. | 8. $12' \times 6' \times 40''$ |
| 3. 8 in. by 4 in. by 3 in. | 9. $15' \times 4'8'' \times 3'$ |
| 4. 9 in. by 7 in. by 5 in. | 10. $16' \times 6'9'' \times 5'$ |
| 5. 7 yd. by 4 yd. by 3 yd. | 11. $24' \times 6'3'' \times 5'6''$ |
| 6. 8 yd. by 7 yd. by 5 yd. | 12. $18' \times 5'6'' \times 4'8''$ |

13. How many cubic inches does a brick contain, if it is 8" long, 4" wide, and $2\frac{1}{4}$ " thick?

14. Find the number of cubic feet in a pile of stone 24' long, 4' wide, and 6' high.

15. How many cubic feet are there in a mow of hay 42' long, 30' wide, and 16' deep?

16. One of the largest bluestone flags ever used is 22' long, 12' wide, and 10' thick. Find its volume.

17. Find the volume of a cake of ice 3' long, 1'8" wide, and 14" thick.

18. A stone door sill is 6' long, 16" wide, and 9" thick. Find the number of cubic feet of stone it contains.

19. How much more than 2 cu. ft. of space will a crate 26" long, 12" wide, and 12" deep occupy?

20. A bale of cotton when compressed for shipment is about 4'6" by 2'3" by 16". Find its volume in cubic feet.

21. How many cubic feet of air does your schoolroom contain?

22. Find the number of cubic yards in a concrete pier 25' high, 6' wide, and 4'6" thick.

WOOD MEASURE

A pile of 4-foot wood, 8 feet long and 4 feet high is a cord.

$$128 \text{ cubic feet} = 1 \text{ cord (cd.)}$$



Wood is often cut into short lengths for use in stoves. Usually a pile of such stove wood is considered a cord, if its surface measure on one side is 32 square feet, regardless of the length of the sticks.

Written Exercises

Change to cubic feet :

Change to cords :

- | | | | |
|-----------|-----------|----------------|----------------|
| 1. 12 cd. | 3. 16 cd. | 5. 512 cu. ft. | 7. 640 cu. ft. |
| 2. 14 cd. | 4. 15 cd. | 6. 768 cu. ft. | 8. 896 cu. ft. |

Find the number of cords of 4-foot wood in a pile :

- | | | | |
|---------------------|---------------------|---------------------|---------------------|
| 9. $16' \times 4'$ | 12. $48' \times 4'$ | 15. $20' \times 8'$ | 18. $32' \times 7'$ |
| 10. $24' \times 8'$ | 13. $32' \times 6'$ | 16. $56' \times 4'$ | 19. $36' \times 8'$ |
| 11. $16' \times 6'$ | 14. $40' \times 4'$ | 17. $64' \times 6'$ | 20. $64' \times 5'$ |

21. Find the number of cords of stove wood in a pile 48' long and 6' high.

CAPACITY EQUIVALENTS

1. Water in large quantities is measured by the barrel of $31\frac{1}{2}$ gallons, by the cubic foot, and by the 1000 gallons.

A **gallon** is equivalent to **231 cu. in.**; hence, 1 cu. ft. (1728 cu. in.) = $\frac{1728}{231}$ gal. = 7.48^+ gal., or nearly $7\frac{1}{2}$ gal.

2. A **bushel** is equivalent to **2150.42 cu. in.**, but for all practical purposes it is considered sufficiently accurate to take $1\frac{1}{4}$ cu. ft. (= 2160 cu. in.) as the equivalent.

3. Learn these **accurate equivalents**:

$$\begin{aligned} 1 \text{ gallon} &= 231 \text{ cu. in.} \\ 1 \text{ bushel} &= 2150.42 \text{ cu. in.} \end{aligned}$$

4. These **approximate equivalents** are in common use:

$$\begin{aligned} 1 \text{ bushel} &= 1\frac{1}{4} \text{ cu. ft.} \\ 1 \text{ cu. ft.} &= 7\frac{1}{2} \text{ gal.} \end{aligned}$$

Written Exercises

Using accurate equivalents, change to cubic inches:

- | | | | | |
|-----------|----------|------------|------------|--------------|
| 1. 4 gal. | 4. 5 bu. | 7. 14 gal. | 10. 42 bu. | 13. 128 gal. |
| 2. 7 gal. | 5. 8 bu. | 8. 21 gal. | 11. 56 bu. | 14. 153 gal. |
| 3. 9 gal. | 6. 9 bu. | 9. 36 gal. | 12. 78 bu. | 15. 145 gal. |

Using approximate equivalents, change to cubic feet:

- | | | | | |
|------------|-------------|-------------|--------------|-------------|
| 16. 28 bu. | 20. 30 gal. | 24. 108 bu. | 28. 150 gal. | 32. 208 bu. |
| 17. 32 bu. | 21. 60 gal. | 25. 132 bu. | 29. 105 gal. | 33. 196 bu. |
| 18. 48 bu. | 22. 45 gal. | 26. 156 bu. | 30. 180 gal. | 34. 228 bu. |
| 19. 72 bu. | 23. 75 gal. | 27. 184 bu. | 31. 225 gal. | 35. 304 bu. |

Find the approximate equivalent in bushels of :

36. 20 cu. ft. 39. 65 cu. ft. 42. 105 cu. ft. 45. 175 cu. ft.
37. 35 cu. ft. 40. 80 cu. ft. 43. 120 cu. ft. 46. 240 cu. ft.
38. 50 cu. ft. 41. 95 cu. ft. 44. 215 cu. ft. 47. 325 cu. ft.

Find the approximate equivalent in gallons of :

48. 24 cu. ft. 51. 64 cu. ft. 54. 106 cu. ft. 57. 172 cu. ft.
49. 42 cu. ft. 52. 88 cu. ft. 55. 128 cu. ft. 58. 256 cu. ft.
50. 54 cu. ft. 53. 96 cu. ft. 56. 204 cu. ft. 59. 316 cu. ft.

Find the accurate equivalent in gallons of :

60. 693 cu. in. 62. 1386 cu. in. 64. 3465 cu. in.
61. 924 cu. in. 63. 2772 cu. in. 65. 8316 cu. in.

66. A box contained 7.5 cu. ft. of seed oats. About how many bushels of seed oats did it contain ?

67. A trough held $9\frac{1}{3}$ cu. ft. of water. About how many gallons of water did it hold ?

68. A gardener used 85 gal. of water to sprinkle flower beds. Express this amount, exactly, in cubic inches.

69. In a car there were 1200 cu. ft. of grain. About how many bushels of grain did the car contain ?

70. A skating rink 216 ft. by 75 ft. was flooded with 2 in. of water. About how many gallons of water were used ?

71. A filtration plant discharges 275,000 cu. ft. of water per day. Express this amount, approximately, in gallons.

72. A city uses $18\frac{3}{4}$ million gallons of water per day. Express this quantity of water, approximately, in cubic feet.

Finding the capacity of rectangular bins, boxes, etc.

Unless stated to the contrary, use approximate equivalents.

Written Exercises

Find the volume in cubic feet of a box that will hold :

1. 4 bu. 3. 6 bu. 5. 18 bu. 7. 37 bu. 9. 65 bu.
2. 5 bu. 4. 8 bu. 6. 28 bu. 8. 48 bu. 10. 72 bu.

11-20. In Ex. 1-10, find the *accurate* volume in cubic inches.

Find the capacity in bushels of a rectangular bin :

21. $8' \times 6' \times 5'$ 24. $15' \times 9' \times 8'$ 27. $10' \times 10' \times 9'6''$
22. $7' \times 5' \times 4'$ 25. $14' \times 10' \times 7'$ 28. $16' \times 15' \times 12'3''$
23. $9' \times 8' \times 5'$ 26. $16' \times 12' \times 10'$ 29. $20' \times 16' \times 10'9''$

30. A grain box is 5' by 4' and 4' deep. How many bushels of grain will it hold ?

31. How many bushels of grain are there in a bin that is 5' square and 6' deep, if it is half full ?

32. Find the value, at 93¢ a bushel, of the wheat that would fill a bin 15' square and 12' deep.

33. A car 36' by 8'4" by 8' on the inside is half full of grain. How many bushels does it contain ?

34. Find the number of tons of coal, occupying 35 cu. ft. per ton, that can be put into a bin 10' by 7' by 7'.

35. Find the capacity in bushels of a Duluth grain elevator having 2,250,000 cu. ft. of storage space.

36. How many tons of coal (34 cu. ft. per ton) will a bin 10' long, 8½' wide, and 4' deep hold ?

Finding the capacity of rectangular tanks, cisterns, etc.

Unless stated to the contrary, use approximate equivalents.

Written Exercises

Find the accurate volume of a tank that will hold :

- | | | | |
|------------|------------|-------------|-------------|
| 1. 40 gal. | 3. 64 gal. | 5. 125 gal. | 7. 268 gal. |
| 2. 35 gal. | 4. 86 gal. | 6. 172 gal. | 8. 325 gal. |

Find the volume in cubic feet of a vat that will hold :

- | | | | |
|-------------|-------------|--------------|--------------|
| 9. 35 gal. | 11. 80 gal. | 13. 120 gal. | 15. 225 gal. |
| 10. 63 gal. | 12. 90 gal. | 14. 165 gal. | 16. 260 gal. |

Find the capacity in gallons of a rectangular cistern :

- | | | |
|------------------------------|-------------------------------|-----------------------------------|
| 17. $6' \times 5' \times 5'$ | 19. $10' \times 9' \times 6'$ | 21. $12' \times 10' \times 8'6''$ |
| 18. $8' \times 7' \times 6'$ | 20. $13' \times 8' \times 9'$ | 22. $14' \times 12' \times 9'3''$ |

Find the exact capacity in gallons of a rectangular tank :

- | | | |
|-------------------------------|---|-----------------------------------|
| 23. $11' \times 7' \times 4'$ | 25. $11' \times 10' \times 7'$ | 27. $13' \times 11' \times 8'9''$ |
| 24. $11' \times 8' \times 7'$ | 26. $14' \times 11' \times 9\frac{1}{3}'$ | 28. $15' \times 14' \times 7'4''$ |

29. How many gallons of water will the tank of a locomotive tender hold, if its capacity is 890 cu. ft.?

30. A tank, when full, holds 4875 gal. of water. Find the capacity of the tank in cubic feet.

31. An aquarium is 21 in. by 12 in. Exactly how many gallons of water are needed to fill it to a depth of 11 in.?

32. A vat at a cheese factory is 11' long, 4' wide, and 3'6'' deep. How many gallons of milk will it hold?

33. How many gallons of water are there in a cistern 8' long, 6' wide, and 5' deep, if it is one third full?

EXCAVATION AND MASONRY

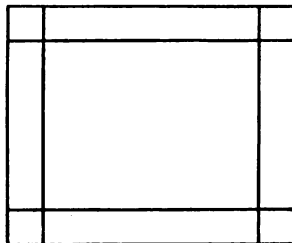
Excavation is estimated by the **cubic yard**.

Brickwork is estimated by the **thousand (M) bricks**.

Common bricks, laid in mortar, are usually reckoned at 22 per cubic foot. In this book "bricks" means "common bricks," unless otherwise stated.

Stone work is estimated by the **perch** ($24\frac{3}{4}$ cu. ft.), by the **cubic foot**, or by the **cubic yard**.

In *estimating the labor* of laying stone and brick no allowance is made for openings, and the corners are usually *counted twice* by taking the total length of wall equal to the *perimeter* of the foundation *measured on the outside*. This is to offset the extra work of building around openings and corners.



In *estimating the cost of material*, however, some allowance is made for openings and the corners are *counted only once*.

Written Exercises

Find the cost, at 45¢ per cubic yard, of the excavation for a cellar:

1. $28' \times 27' \times 4'$ 3. $39' \times 36' \times 4'$ 5. $42' \times 33' \times 6'$
2. $36' \times 33' \times 5'$ 4. $40' \times 27' \times 6'$ 6. $45' \times 42' \times 7'$

Find the cost, at \$6.50 per cu. yd., of a concrete wall:

7. $36' \times 2' \times 6'$ 9. $72' \times 3' \times 11'$ 11. $48' \times 2'6'' \times 9'$
8. $63' \times 2' \times 9'$ 10. $81' \times 3' \times 12'$ 12. $54' \times 2'6'' \times 9'$

Find, to the nearest perch, the masonry in a wall:

13. $36' \times 2' \times 5'$ 15. $81' \times 3' \times 10'$ 17. $48' \times 2'6'' \times 8'$
14. $63' \times 2' \times 8'$ 16. $72' \times 3' \times 12'$ 18. $54' \times 2'6'' \times 9'$

19. Find the cost of labor to build the brick foundation walls of a house $30' \times 28'$, the walls being 8' high and 1' thick, at \$8.25 per M.

SOLUTION. — Since in estimating the labor the corners are counted twice, the total length of wall is equal to the perimeter of the foundation measured on the *outside*, or $2 \times (30' + 28') = 116'$.

Then, the volume = $(116 \times 8 \times 1)$ cu. ft. = 928 cu. ft.

The number of bricks = $928 \times 22 = 20,416 = 20.416$ M.

Hence, the cost of labor = $20.416 \times \$8.25 = \168.43 , to nearest cent.

Find the cost of the labor, at \$7.50 per M, of laying the brick foundation walls, 8' high and 1' thick of a house:

- | | | |
|----------------------|----------------------|----------------------|
| 20. $24' \times 22'$ | 22. $42' \times 24'$ | 24. $36' \times 35'$ |
| 21. $26' \times 24'$ | 23. $40' \times 30'$ | 25. $50' \times 32'$ |

26. The walls of a house $38' \times 34'$ are 22' high and 12" thick. Find the cost of laying the bricks at \$9 per M.

27. What is the cost, at \$8.50 per M, of the bricks for the foundation mentioned in exercise 19, if 56 cu. ft. are allowed for openings?

SUGGESTION. — Since in estimating material, the corners are counted only once, and since the wall is 1' thick, the perimeter is taken as $4 \times 1'$, or 4', less than in the solution of exercise 19. Proceed as in that solution, making the allowance of 56 cu. ft. for openings.

Find the cost of the bricks, at \$8 per M, needed for the foundation walls, 8' high and 1' thick, of a house:

- | | | |
|----------------------|----------------------|----------------------|
| 28. $24' \times 24'$ | 30. $36' \times 30'$ | 32. $42' \times 32'$ |
| 29. $28' \times 25'$ | 31. $38' \times 32'$ | 33. $36' \times 36'$ |

34. Find the cost of the bricks, at \$12 per M, for a house, $32'$ by $28'$; walls, 20' high and 12" thick; openings, 240 cu. ft.

FIFTH YEAR—SECOND HALF

REVIEW

NOTATION AND NUMERATION

Written Exercises

Read the following exercises and write in words:

- | | | | | |
|--------------------|-----------|-------------------|-------------|---------------|
| 1. $\frac{2}{5}$ | 4. 65.07 | 7. $2\frac{1}{3}$ | 10. 30,620 | 13. 15.0208 |
| 2. $\frac{7}{8}$ | 5. 8.003 | 8. $5\frac{3}{4}$ | 11. 8.9007 | 14. \$640.02 |
| 3. $\frac{17}{20}$ | 6. \$6.34 | 9. $8\frac{5}{6}$ | 12. \$58.09 | 15. 2,405,698 |
16. Write with Roman numerals: 6; 14; 45; 150;
505; with Arabic numerals: IX; XV; XLI; DCCL.

Write in figures, expressing decimals in decimal form:

17. Two hundred nine; seventy-five.
18. One half; five tenths; one third.
19. Seven twelfths; five and six tenths.
20. Two hundredths; two and one fifth.
21. Twenty cents; one dollar and ten cents.
22. One thousand six; twelve and two tenths.
23. Fourteen thousand, seven hundred fourteen.
24. Nine twentieths; fifteen and one thousandth.
25. Thirty and one hundred one ten-thousandths.
26. Four dollars and eight cents; one hundredth.
27. Eighteen million, three thousand, twenty-five.

REDUCTION

Written Exercises

1. Change to thirty-sixths: $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{2}{9}$; $\frac{5}{12}$; $\frac{11}{12}$.

2. Change to sixtieths: $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{10}$; $\frac{5}{12}$; $\frac{8}{15}$.

Reduce to lowest terms:

- | | | | | | |
|--------------------|--------------------|---------------------|---------------------|---------------------|---------------------|
| 3. $\frac{20}{30}$ | 6. $\frac{18}{24}$ | 9. $\frac{18}{36}$ | 12. $\frac{18}{45}$ | 15. $\frac{25}{50}$ | 18. $\frac{32}{36}$ |
| 4. $\frac{12}{32}$ | 7. $\frac{16}{40}$ | 10. $\frac{24}{32}$ | 13. $\frac{24}{36}$ | 16. $\frac{36}{48}$ | 19. $\frac{32}{40}$ |
| 5. $\frac{15}{25}$ | 8. $\frac{25}{30}$ | 11. $\frac{24}{40}$ | 14. $\frac{12}{48}$ | 17. $\frac{20}{60}$ | 20. $\frac{48}{60}$ |

Reduce to a fraction:

Reduce to a mixed number:

- | | | | | | |
|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|
| 21. $8\frac{4}{5}$ | 24. $10\frac{3}{4}$ | 27. $6\frac{5}{12}$ | 30. $2\frac{9}{10}$ | 33. $5\frac{0}{3}$ | 36. $8\frac{2}{4}$ |
| 22. $7\frac{7}{8}$ | 25. $16\frac{2}{3}$ | 28. $8\frac{7}{15}$ | 31. $3\frac{2}{5}$ | 34. $8\frac{6}{6}$ | 37. $9\frac{9}{8}$ |
| 23. $9\frac{5}{6}$ | 26. $25\frac{3}{5}$ | 29. $9\frac{9}{16}$ | 32. $4\frac{5}{4}$ | 35. $7\frac{4}{8}$ | 38. $7\frac{8}{9}$ |

Reduce to fractions having the l. c. d.:

- | | | | |
|-------------------------------------|---------------------------------|----------------------------------|---|
| 39. $\frac{2}{3}$ and $\frac{4}{5}$ | 42. $\frac{1}{6}, \frac{2}{15}$ | 45. $\frac{3}{10}, \frac{9}{20}$ | 48. $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}$ |
| 40. $\frac{3}{4}$ and $\frac{5}{6}$ | 43. $\frac{5}{8}, \frac{7}{12}$ | 46. $\frac{5}{12}, \frac{7}{36}$ | 49. $\frac{3}{5}, \frac{2}{3}, \frac{7}{10}$ |
| 41. $\frac{5}{6}$ and $\frac{3}{8}$ | 44. $\frac{4}{9}, \frac{4}{15}$ | 47. $\frac{7}{10}, \frac{8}{15}$ | 50. $\frac{7}{8}, \frac{5}{6}, \frac{11}{12}$ |

Express as a decimal in hundredths:

- | | | | | |
|-------------------|-------------------|---------------------|---------------------|---------------------|
| 51. $\frac{1}{2}$ | 54. $\frac{3}{8}$ | 57. $\frac{9}{20}$ | 60. $\frac{11}{15}$ | 63. $\frac{17}{24}$ |
| 52. $\frac{3}{4}$ | 55. $\frac{1}{6}$ | 58. $\frac{5}{12}$ | 61. $\frac{13}{16}$ | 64. $\frac{25}{32}$ |
| 53. $\frac{1}{3}$ | 56. $\frac{5}{8}$ | 59. $\frac{19}{25}$ | 62. $\frac{29}{30}$ | 65. $\frac{31}{60}$ |

Reduce to a common fraction or a mixed number:

- | | | | | |
|---------|----------------------|----------------------|-----------------------|-----------------------|
| 66. .40 | 69. 10.5 | 72. $.87\frac{1}{2}$ | 75. $5.12\frac{1}{2}$ | 78. .0625 |
| 67. .60 | 70. .125 | 73. $.83\frac{1}{3}$ | 76. 16.05 | 79. $9.16\frac{2}{3}$ |
| 68. .75 | 71. $.33\frac{1}{3}$ | 74. 3.25 | 77. $2.66\frac{2}{3}$ | 80. 8.025 |

ADDITION

Oral Exercises

To each of the following, add 4; 7; 6; 9; 8:

1. 31 2. 3.6 3. 45 4. .52 5. 67 6. 89 7. 9.8

To each of the following, add \$.09; \$.25; \$.36:

8. \$.52 9. \$.75 10. \$.49 11. \$.93 12. \$2.12 13. \$5.34

Find the sum of:

- | | | | | |
|--------------------------------|--------------------------------|--------------------------------|--------------------------------|---------------------------------|
| 14. $\frac{3}{4}, \frac{1}{2}$ | 17. $\frac{5}{8}, \frac{1}{2}$ | 20. $\frac{2}{3}, \frac{1}{2}$ | 23. $\frac{3}{4}, \frac{1}{3}$ | 26. $\frac{1}{2}, \frac{3}{10}$ |
| 15. $\frac{3}{4}, \frac{1}{8}$ | 18. $\frac{3}{8}, \frac{1}{4}$ | 21. $\frac{1}{4}, \frac{1}{6}$ | 24. $\frac{4}{5}, \frac{1}{2}$ | 27. $\frac{2}{5}, \frac{3}{10}$ |
| 16. $\frac{2}{3}, \frac{1}{6}$ | 19. $\frac{5}{6}, \frac{1}{2}$ | 22. $\frac{1}{2}, \frac{2}{5}$ | 25. $\frac{5}{6}, \frac{3}{4}$ | 28. $\frac{1}{4}, \frac{1}{12}$ |
| 29. $3\frac{1}{2}$ | 30. $.5\frac{1}{3}$ | 31. $6\frac{1}{2}$ | 32. $.7\frac{7}{8}$ | 33. $9\frac{5}{6}$ |
| <u>2\frac{1}{4}</u> | <u>.3\frac{1}{6}</u> | <u>5\frac{3}{8}</u> | <u>.4\frac{3}{4}</u> | <u>6\frac{2}{3}</u> |
| 35. $6\frac{1}{2}$ | 36. $.8\frac{1}{3}$ | 37. $8\frac{2}{3}$ | 38. $.9\frac{1}{4}$ | 39. $5\frac{5}{9}$ |
| <u>4\frac{1}{3}</u> | <u>.3\frac{1}{4}</u> | <u>5\frac{4}{9}</u> | <u>.6\frac{1}{6}</u> | <u>3\frac{1}{2}</u> |
| 40. $7\frac{5}{6}$ | | | | <u>2\frac{3}{4}</u> |

Add and test, timing yourself:

- | | | | | | | | |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 41. 3 | 42. .7 | 43. 1 | 44. 9 | 45. 5 | 46. .8 | 47. 7 | 48. .3 |
| 4 | .6 | 3 | 8 | 2 | .9 | 6 | .9 |
| 6 | .8 | 4 | 2 | 7 | .7 | 5 | .7 |
| 9 | .5 | 6 | 1 | 3 | .9 | 4 | .8 |
| 7 | .2 | 7 | 7 | 4 | .8 | 8 | .8 |
| 2 | .1 | 8 | 3 | 6 | .9 | 3 | .9 |
| 5 | .3 | 9 | 6 | 2 | .8 | 9 | .4 |
| 4 | .9 | 4 | 5 | 8 | .7 | 1 | .7 |
| 8 | .5 | 5 | 4 | 9 | .8 | 5 | .9 |
| <u> </u> | <u> </u> | <u> </u> | <u> </u> | <u> </u> | <u> </u> | <u> </u> | <u> </u> |

Written Exercises

Add :

1. 86 <u>79</u>	2. $\frac{2}{5}$ $\frac{3}{8}$	3. \$5.89 <u>3.72</u>	4. $67\frac{4}{5}$ <u>$23\frac{2}{3}$</u>	5. 47.4 <u>25.85</u>	6. 56,425 <u>37,894</u>
7. 95 <u>68</u>	8. $\frac{4}{5}$ $\frac{3}{4}$	9. \$8.98 <u>5.79</u>	10. $74\frac{5}{6}$ <u>$48\frac{3}{8}$</u>	11. 50.39 <u>28.86</u>	12. 920.56 <u>738.78</u>
13. 67 45 <u>98</u>	14. $\frac{2}{3}$ $\frac{4}{9}$ $\frac{5}{6}$	15. \$7.62 3.74 <u>5.87</u>	16. $25\frac{2}{3}$ $72\frac{3}{4}$ <u>$69\frac{5}{6}$</u>	17. 37.65 5.38 <u>79.8</u>	18. 75,246 67,895 <u>80,879</u>
19. 87 43 <u>68</u>	20. $\frac{4}{5}$ $\frac{1}{2}$ $\frac{3}{4}$	21. \$6.54 5.08 <u>7.45</u>	22. $31\frac{1}{3}$ $89\frac{5}{6}$ <u>$78\frac{5}{8}$</u>	23. 70.81 47.7 <u>9.58</u>	24. 76.479 51.867 <u>99.680</u>
25. 28 36 54 80 <u>25</u>	26. 3.4 6.6 4 5.3 <u>7.8</u>	27. \$5.20 8.14 3.65 5.08 <u>9.50</u>	28. 465 94 728 76 <u>542</u>	29. 85.73 3.24 74.8 39.5 <u>80.75</u>	30. 16,372 84,293 51,740 28,605 <u>37,426</u>

Add and test, timing yourself :

31. 74 45 93 48 61 26 30	32. 5.7 2.3 7 4.6 2.8 6 9.2	33. \$8.06 .93 4.67 5.25 3.98 .75 6.44	34. 189 204 672 428 741 96 825	35. 9.385 .269 8.04 5.376 2.085 .728 6.18	36. 68,423 5,387 21,491 529 6,485 78,140 82,909
--	---	--	--	---	---

SUBTRACTION

Oral Exercises

From each of the following, subtract 5; 3; 7; 9; 8:

1. 25 2. 34 3. 56 4. 44 5. 67 6. 83 7. 9.5

From each of the following, subtract \$.04; \$.12; \$.06:

8. \$.24 9. \$.63 10. \$1.17 11. \$3.24 12. \$4.52

13-39. Subtract in exercises 14-40, page 289.

Written Exercises

Subtract:

- | | | | | | |
|---|---|--|---|---|--|
| 1. $\begin{array}{r} 578 \\ 347 \\ \hline \end{array}$ | 2. $\begin{array}{r} \frac{4}{5} \\ \frac{2}{3} \\ \hline \end{array}$ | 3. $\begin{array}{r} 1.89 \\ .92 \\ \hline \end{array}$ | 4. $\begin{array}{r} 57\frac{1}{2} \\ 36\frac{2}{9} \\ \hline \end{array}$ | 5. $\begin{array}{r} \$52.08 \\ 38.24 \\ \hline \end{array}$ | 6. $\begin{array}{r} 270.26 \\ 158.12 \\ \hline \end{array}$ |
| 7. $\begin{array}{r} 627 \\ 276 \\ \hline \end{array}$ | 8. $\begin{array}{r} \frac{7}{8} \\ \frac{5}{6} \\ \hline \end{array}$ | 9. $\begin{array}{r} 5.7 \\ 1.08 \\ \hline \end{array}$ | 10. $\begin{array}{r} 38\frac{2}{3} \\ 26\frac{7}{8} \\ \hline \end{array}$ | 11. $\begin{array}{r} \$63.71 \\ 45.56 \\ \hline \end{array}$ | 12. $\begin{array}{r} 34.21 \\ 17.318 \\ \hline \end{array}$ |
| 13. $\begin{array}{r} 405 \\ 189 \\ \hline \end{array}$ | 14. $\begin{array}{r} \frac{3}{4} \\ \frac{2}{5} \\ \hline \end{array}$ | 15. $\begin{array}{r} 6.24 \\ 3.8 \\ \hline \end{array}$ | 16. $\begin{array}{r} 47\frac{4}{9} \\ 36\frac{5}{6} \\ \hline \end{array}$ | 17. $\begin{array}{r} \$42.34 \\ 14.89 \\ \hline \end{array}$ | 18. $\begin{array}{r} 7 \\ 4.5145 \\ \hline \end{array}$ |

19-30. Subtract in exercises 1-12, page 290.

Find correct answers, timing yourself:

- | | | |
|----------------|-----------------------------------|----------------------|
| 31. 729 - 54 | 37. $\frac{7}{10} - \frac{1}{2}$ | 43. 512.7 - 4.36 |
| 32. 605 - 45 | 38. $\frac{5}{12} - \frac{1}{6}$ | 44. 62.35 - 3.76 |
| 33. 523 - 66 | 39. $\frac{17}{20} - \frac{3}{8}$ | 45. 5.016 - 4.18 |
| 34. 827 - .78 | 40. $\frac{7}{15} - \frac{1}{6}$ | 46. \$27.45 - \$8.32 |
| 35. 916 - .98 | 41. $\frac{9}{10} - \frac{1}{4}$ | 47. \$32.07 - \$6.25 |
| 36. 7.12 - 6.9 | 42. $\frac{7}{12} - \frac{3}{8}$ | 48. \$43.52 - \$5.53 |

MULTIPLICATION

Oral Exercises

Find:

- | | | | | |
|------------------|---------------------------|-----------------------|--------------------------------------|-----------------------------|
| 1. 2×14 | 5. $5 \times \frac{1}{2}$ | 9. $3 \times \$.16$ | 13. $\frac{3}{4} \times .8$ | 17. $3\frac{1}{2} \times 4$ |
| 2. 3×15 | 6. $6 \times \frac{3}{4}$ | 10. $2 \times \$.24$ | 14. $\frac{4}{5} \times \frac{3}{4}$ | 18. $4\frac{2}{3} \times 6$ |
| 3. 4×25 | 7. $8 \times \frac{5}{6}$ | 11. $4 \times \$.25$ | 15. $\frac{5}{6} \times .3$ | 19. $5\frac{3}{4} \times 8$ |
| 4. 5×16 | 8. $4 \times \frac{2}{3}$ | 12. $5 \times \$.50$ | 16. $\frac{3}{8} \times \frac{2}{9}$ | 20. $6\frac{2}{3} \times 9$ |

Multiply by 2; by 3; by 4; by 6:

- | | | | | | |
|--------|-------------------|-------------|--------------------|---------|-------------|
| 21. 18 | 23. $\frac{4}{5}$ | 25. $\$.21$ | 27. $5\frac{3}{4}$ | 29. 2.8 | 31. $\$.14$ |
| 22. 32 | 24. $\frac{5}{6}$ | 26. $\$.50$ | 28. $6\frac{1}{8}$ | 30. .15 | 32. $\$.40$ |

Written Exercises

Multiply by 9; by 25; by 4.8; by $\frac{3}{4}$; by $7\frac{1}{3}$:

- | | | | | | |
|---------|------------|--------------------|--------------------|----------|-----------|
| 1. 13.5 | 5. $\$.48$ | 9. $\frac{3}{4}$ | 13. $5\frac{1}{3}$ | 17. 3240 | 21. 32.76 |
| 2. 3.24 | 6. $\$.60$ | 10. $\frac{5}{6}$ | 14. $7\frac{1}{5}$ | 18. 5064 | 22. 664.8 |
| 3. 60.9 | 7. $\$.72$ | 11. $\frac{3}{10}$ | 15. $8\frac{5}{6}$ | 19. 7296 | 23. 9.852 |
| 4. .891 | 8. $\$.96$ | 12. $1\frac{1}{2}$ | 16. $9\frac{3}{8}$ | 20. 9804 | 24. 69.36 |

Find answers, timing yourself on the third column:

- | | | |
|----------------------|-------------------------|-------------------------|
| 25. 24×3205 | 33. $35 \times \$43.52$ | 41. 48×165.09 |
| 26. 37×5364 | 34. $27 \times \$56.08$ | 42. $67 \times 24,637$ |
| 27. 56×4968 | 35. $46 \times \$39.76$ | 43. 59×36.076 |
| 28. 49×6107 | 36. $63 \times \$61.95$ | 44. 62×5.7345 |
| 29. 72×8438 | 37. $58 \times \$78.69$ | 45. 8.5×42.658 |
| 30. 65×7769 | 38. $74 \times \$67.84$ | 46. $.77 \times 157.46$ |
| 31. 87×9076 | 39. $97 \times \$86.45$ | 47. $.96 \times 78,516$ |
| 32. 96×8409 | 40. $86 \times \$93.67$ | 48. 6.8×8.3495 |

DIVISION

Oral Exercises

Divide as indicated :

- | | | | | |
|-----------------|-------------------------|--------------------------|--------------------|------------------------------------|
| 1. $3.4 \div 2$ | 5. $\frac{1}{3} \div 4$ | 9. $6 \div \frac{3}{8}$ | 13. $\$.46 \div 2$ | 17. $\frac{2}{3} \div \frac{5}{6}$ |
| 2. $.45 \div 3$ | 6. $\frac{8}{9} \div 2$ | 10. $5 \div \frac{4}{5}$ | 14. $\$.75 \div 5$ | 18. $\frac{3}{4} \div \frac{3}{8}$ |
| 3. $5.6 \div 4$ | 7. $\frac{5}{6} \div 3$ | 11. $8 \div \frac{4}{9}$ | 15. $\$.90 \div 6$ | 19. $.6 \div .3$ |
| 4. $.78 \div 6$ | 8. $\frac{3}{4} \div 6$ | 12. $9 \div \frac{2}{3}$ | 16. $\$.91 \div 7$ | 20. $.8 \div .5$ |

Divide by 2; by 3; by 4; by 6:

- | | | | | | |
|--------|-------------------|-------------|--------------------|---------|-------------|
| 21. 36 | 23. $\frac{3}{4}$ | 25. $\$.60$ | 27. $2\frac{2}{3}$ | 29. 7.2 | 31. $\$.84$ |
| 22. 48 | 24. $\frac{7}{8}$ | 26. $\$.72$ | 28. $3\frac{3}{4}$ | 30. .24 | 32. $\$.96$ |

Written Exercises

Divide by 8; by 12; by $\frac{2}{3}$; by $2\frac{2}{5}$:

- | | | | | | |
|--------|-------------------|---------------------|---------------|----------------------|----------|
| 1. 632 | 5. $\frac{2}{3}$ | 9. $25\frac{1}{2}$ | 13. $\$19.44$ | 17. $113\frac{1}{4}$ | 21. 3376 |
| 2. 684 | 6. $\frac{8}{9}$ | 10. $22\frac{2}{5}$ | 14. $\$29.52$ | 18. $330\frac{2}{3}$ | 22. 4395 |
| 3. 780 | 7. $\frac{9}{10}$ | 11. $38\frac{2}{3}$ | 15. $\$38.88$ | 19. $180\frac{3}{4}$ | 23. 5400 |
| 4. 912 | 8. $\frac{8}{15}$ | 12. $76\frac{4}{5}$ | 16. $\$42.72$ | 20. $371\frac{1}{5}$ | 24. 6138 |

Find answers, timing yourself on the third column:

- | | | |
|----------------------|------------------------|-----------------------|
| 25. $5094 \div 18$ | 33. $\$91.25 \div 25$ | 41. $15,290 \div 44$ |
| 26. $7605 \div 27$ | 34. $\$67.94 \div 79$ | 42. $25,578 \div 63$ |
| 27. $7776 \div 32$ | 35. $\$39.33 \div 207$ | 43. $47,367 \div 57$ |
| 28. $82.32 \div 49$ | 36. $\$71.37 \div 117$ | 44. $43,914 \div 78$ |
| 29. $8.568 \div 36$ | 37. $\$99.18 \div 114$ | 45. $62.475 \div 75$ |
| 30. $93.84 \div 4.8$ | 38. $\$79.92 \div 148$ | 46. $65.281 \div .97$ |
| 31. $9.912 \div .56$ | 39. $\$93.06 \div 198$ | 47. $7900.2 \div 8.4$ |
| 32. $89.78 \div 6.7$ | 40. $\$86.43 \div 129$ | 48. $9.8394 \div .69$ |

Exercises in Making Change

When the grocer has announced the cost and the buyer has handed him a coin or a bill, the grocer, naming the cost first, counts out the change, giving as few pieces of money as possible.

1. Emily bought 3 cans of corn @ 10ϕ ; and 1 lb. of coffee, 34ϕ . The grocer announced the cost, "64 cents."

Emily handed him \$1, and he handed back a cent, a dime, and a 25-cent piece, saying, "64 cents, 65, 75, 1 dollar."



Announce the cost of each purchase, and count out the change as the grocer should:

Amount paid, \$.50

2. 4 lb. rice @ 8ϕ .
3. 5 lb. beans @ 7ϕ .
4. $\frac{1}{2}$ lb. lard @ 18ϕ .
5. $1\frac{1}{2}$ lb. figs @ 20ϕ .
6. 3 lb. dates @ 12ϕ .
7. 2 lb. honey @ 15ϕ .
8. $1\frac{1}{2}$ doz. eggs @ 30ϕ .
9. 4 lb. raisins @ 12ϕ .
10. $\frac{1}{2}$ lb. nutmegs @ 50ϕ .

Amount paid, \$2

11. $2\frac{1}{2}$ lb. prunes @ 14ϕ .
12. 2 lb. almonds @ 22ϕ .
13. 3 doz. lemons @ 25ϕ .
14. $3\frac{1}{2}$ lb. cheese @ 20ϕ .
15. 9 lb. crackers @ 10ϕ .
16. 5 lb. allspice @ 25ϕ .
17. 3 lb. coconut @ 19ϕ .
18. 4 lb. codfish @ $12\frac{1}{2}\phi$.
19. 4 doz. oranges @ 40ϕ .

Written Exercises

1. What is the sum of \$7.59, \$.75, and \$324.50?
2. Find the total weight of 175 cheeses of 40 lb. each.
3. Subtract one dollar and ten cents from ten dollars.
4. Find the cost at $2\frac{1}{2}\phi$ each of 600 tin cans.
5. A jobber bought 225 boxes of lemons at \$2.85 a box. Find the cost of his purchase.
6. If Ruth has read $\frac{9}{16}$ of the 336 pages in her book, how many pages has she read?
7. A National League team lost 55 games of baseball, or $\frac{5}{9}$ as many as it won. How many games did it win?
8. How much will it cost to put a cement floor in a basement 20 ft. by 30 ft. at 14ϕ per square foot?
9. A farmer raised 180 bu. of oats and sold 120 bu. What part of his crop did he sell?
10. From the sum of twenty-four and five eighths and sixteen and eleven twelfths subtract the difference.
11. A man sold 50 bu. of potatoes for \$37 $\frac{1}{2}$. At this rate find the value of his crop of 450 bu.
12. A man's expenses are $\frac{4}{5}$ of his earnings. If his expenses are \$816 a year, how much does he earn?
13. If a hog gains $11\frac{3}{4}$ lb. for each bushel of corn he eats, how many bushels will increase his weight $70\frac{1}{2}$ lb.?
14. A strawberry farm yielded 140,000 qt. of strawberries. How many bushel crates would the crop fill?
15. How many tons of coal will a bin 12 ft. by 9 ft. by 5 ft. hold, if 1 ton occupies 36 cu. ft. of space?

Orchard Problems



1. In an apple orchard there are 15 rows of 26 trees each. How many trees are there in the orchard?
2. How many acres are there in an orchard 64 rd. long and 26 rd. wide?
3. Find the cost of 175 apple barrels at 30¢ each.
4. Find the value of 248 barrels of apples at \$1.90 a barrel.
5. How long will it take a man to pick 300 barrels of apples, if he picks 12 barrels per day?
6. The yield from an apple orchard was 384 barrels. If a barrel holds $2\frac{3}{4}$ bu., find the yield in bushels.
7. In an orchard there are 288 trees. If $\frac{1}{4}$ of the trees are cherry trees, $\frac{2}{3}$ apple trees, and the rest pear trees, find the number of trees of each kind in the orchard.
8. A farmer sold 156 barrels of apples at \$2.25 each. How much had he left from the sale after paying 15¢ per barrel for picking, 14¢ per barrel for sorting and barreling, and 30¢ each for the barrels?

DENOMINATE NUMBERS

1. A number that is used with reference to some particular kind of object or unit is a **concrete number**.

6 pears, \$12, 40 miles are concrete numbers.

2. To ascertain the quantity of anything, or to *measure* it, is to find how many times it contains some established unit called the **unit of measure**.

Thus, to measure the water in a tank is to find how many times the whole quantity of water in the tank contains some *unit of measure*, as 1 gallon or 1 barrel.

3. A concrete number in which the unit of measure is established by law or custom is a **denominate number**.

4. A denominate number is **simple**, if it is composed of units of one denomination; **compound**, if it is composed of units of two or more related denominations.

10 feet is a simple denominate number; 10 feet 7 inches is a compound denominate number.

5. The facts that are *summarized* in the following tables are to be **memorized**.

LIQUID MEASURES

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

The gill is now little used.

In determining the capacity of cisterns, reservoirs, etc., $31\frac{1}{2}$ gallons are considered a *barrel* (bbl.), and 2 barrels, or 63 gallons, a *hogshead* (hhd.).

Oral Exercises

1. How many pints are there in 3 qt. ? in 16 gi. ?
2. How many quarts are there in 5 gal. ? in 12 pt. ?
3. How many gallons are there in 8 qt. ? in 24 qt. ?

Change to pints:

Change to quarts:

- | | | | |
|-----------|-----------|-----------|-------------|
| 4. 24 gi. | 6. 12 qt. | 8. 18 pt. | 10. 12 gal. |
| 5. 32 gi. | 7. 16 qt. | 9. 26 pt. | 11. 15 gal. |

The process of changing the form of a number without changing its value is **reduction**.

Reduction to a *lower* unit is **reduction descending**; to a *higher* unit, **reduction ascending**.

The student has performed reduction of denominate numbers, mostly from one unit to the next higher or lower unit. He will now take up the work a little more formally and carry the reduction somewhat farther; but he will not be asked to reduce through many successive units either here or in practice.

Written Exercises

1. A dealer has 31 gal. 2 qt. of olive oil that he wishes to put into pint bottles. How many bottles does he need ?

31, number of gal.

$$\begin{array}{r} 4 \\ \hline 124 \end{array}$$

$$\begin{array}{r} + 2 \\ \hline \end{array}$$

126, number of qt.

$$\begin{array}{r} 2 \\ \hline \end{array}$$

252, number of pt.

Since there are 4 qt. in 1 gal., in 31 gal. there are 31 times 4 qt., or 124 qt.; and in 31 gal. 2 qt. there are 124 qt. + 2 qt., or 126 qt.

Since there are 2 pt. in 1 qt., in 31 gal. 2 qt., or in 126 qt., there are 126 times 2 pt., or 252 pt.

Hence, 252 pint bottles will be needed to hold the oil.

Reduce to the next lower unit :

- | | | | |
|-----------|------------|------------|-------------|
| 2. 48 qt. | 4. 56 gal. | 6. 202 pt. | 8. 295 qt. |
| 3. 35 pt. | 5. 84 gal. | 7. 185 qt. | 9. 260 gal. |

Change to pints :

- | | | | |
|------------|-------------|-------------|--------------|
| 10. 72 qt. | 12. 64 gal. | 14. 408 qt. | 16. 142 gal. |
| 11. 96 qt. | 13. 75 gal. | 15. 561 qt. | 17. 235 gal. |

Reduce to the lowest of the given units :

- | | | |
|------------------|-------------------|-------------------------|
| 18. 24 qt. 1 pt. | 20. 48 gal. 1 qt. | 22. 24 gal. 2 qt. 1 pt. |
| 19. 18 pt. 3 gi. | 21. 67 gal. 3 qt. | 23. 38 gal. 3 qt. 1 pt. |

24. A milkman sold 479 pt. of milk. Express his sales in higher units, that is, in gallons, quarts, and pints.

2 479	Since 2 pt.=1 qt., 479 pt.=239 qt. and
4 239, + 1 pt.	1 pt. over.
59, + 3 qt.	Since 4 qt.=1 gal., 239 qt.=59 gal. and
59 gal. 3 qt. 1 pt.	3 qt. over.
	Hence, the sales were 59 gal. 3 qt. 1 pt.

Reduce to the next higher unit :

- | | | | |
|-------------|-------------|-------------|-------------|
| 25. 284 pt. | 27. 376 pt. | 29. 428 gi. | 31. 688 qt. |
| 26. 368 gi. | 28. 464 qt. | 30. 596 pt. | 32. 764 qt. |

Change to quarts :

Change to gallons :

- | | | | |
|-------------|-------------|-------------|-------------|
| 33. 488 pt. | 35. 396 pt. | 37. 372 qt. | 39. 860 qt. |
| 34. 576 gi. | 36. 832 gi. | 38. 496 pt. | 40. 952 pt. |

Express in higher units :

- | | | | |
|-------------|-------------|-------------|-------------|
| 41. 125 qt. | 43. 257 pt. | 45. 589 qt. | 47. 633 qt. |
| 42. 339 qt. | 44. 370 pt. | 46. 474 pt. | 48. 787 pt. |

DRY MEASURES

2 pints (pt.) = 1 quart (qt.)

8 quarts = 1 peck (pk.)

4 pecks = 1 bushel (bu.)

A quart or a pint dry measure contains a little more, respectively, than a quart or a pint liquid measure.

Oral Exercises

1. How many pints are there in 6 qt. ? in 8 qt. ?
2. How many quarts are there in 5 pk. ? in 16 pt. ?
3. How many pecks are there in 3 bu. ? in 48 qt. ?
4. How many quarts are there in $\frac{1}{2}$ pk. ? in $\frac{3}{4}$ pk. ?
5. How many bushels are there in 24 pk. ? in 36 pk. ?
in 44 pk. ?

Change to quarts :

Change to pecks :

- | | | | |
|-----------|------------|------------|------------|
| 6. 8 pk. | 9. 12 pk. | 12. 7 bu. | 15. 12 bu. |
| 7. 20 pt. | 10. 20 pk. | 13. 56 qt. | 16. 72 qt. |
| 8. 24 pt. | 11. 32 pt. | 14. 10 bu. | 17. 96 qt. |

Written Exercises

Reduce to the next lower unit :

- | | | | |
|-----------|-----------|------------|-------------|
| 1. 48 qt. | 4. 65 bu. | 7. 106 pk. | 10. 625 qt. |
| 2. 26 bu. | 5. 53 pk. | 8. 158 bu. | 11. 550 pk. |
| 3. 42 pk. | 6. 98 qt. | 9. 385 qt. | 12. 498 bu. |

Reduce to the next higher unit :

- | | | | |
|-------------|-------------|-------------|-------------|
| 13. 132 pk. | 15. 352 qt. | 17. 356 pt. | 19. 688 pk. |
| 14. 156 pt. | 16. 464 pk. | 18. 504 qt. | 20. 952 qt. |

21. Express $\frac{5}{8}$ bu. in lower units.

SOLUTION

$$\frac{5}{8} \text{ bu.} = \frac{5}{8} \text{ of } 4 \text{ pk.} = 2\frac{1}{2} \text{ pk.}$$

$$\frac{1}{2} \text{ pk.} = \frac{1}{2} \text{ of } 8 \text{ qt.} = 4 \text{ qt.}$$

$$\text{Then, } \frac{5}{8} \text{ bu.} = 2 \text{ pk. } 4 \text{ qt.}$$

22. What part of a bushel is 1 pk. 4 qt.?

SOLUTION

$$1 \text{ pk. } 4 \text{ qt.} = 12 \text{ qt.}$$

$$1 \text{ bu.} = 4 \text{ pk.} = 32 \text{ qt.}$$

$$\text{Then, } 1 \text{ pk. } 4 \text{ qt.} = \frac{12}{32} \text{ bu.} = \frac{3}{8} \text{ bu.}$$

Express in lower units :

23. $\frac{3}{8}$ bu.

24. $\frac{3}{16}$ pk.

25. $\frac{7}{8}$ bu.

26. $\frac{7}{16}$ pk.

What part of the next higher unit is

27. 2 qt. 1 pt.?

28. 3 pk. 4 qt.?

29. 3 qt. 1 pt.?

Reduce :

30. 3 pk. to pints.

34. 18 qt. 1 pt. to pints.

31. 5 bu. to quarts.

35. 15 bu. 3 pk. to pecks.

32. $4\frac{3}{4}$ pk. to pints.

36. 35 pk. 6 qt. to quarts.

33. $6\frac{5}{8}$ bu. to quarts.

37. 24 bu. 2 qt. to quarts.

Reduce to pecks :

Reduce to quarts :

38. 145 bu.

41. 304 qt.

44. 138 pk.

47. 288 bu.

39. 264 qt.

42. 418 bu.

45. 256 pt.

48. 824 pt.

40. 178 bu.

43. 512 qt.

46. 184 pk.

49. 376 bu.

Express in higher units :

50. 46 pt.

53. 74 pk.

56. 100 qt.

59. 326 pk.

51. 54 pk.

54. 88 qt.

57. 150 pk.

60. 562 pt.

52. 72 qt.

55. 63 pk.

58. 230 pt.

61. 437 qt.

62. If a horse eats 10 qt. of oats per day, how many days will 5 bu. of oats last him ?

WEIGHT MEASURES

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

These measures are used for groceries and heavy articles, as hay, grain, etc.

The ton of 2000 lb. is known as the *short ton*. A ton of 2240 lb., called the *long ton* (L. T.), is used in weighing iron and steel (except in bars), iron ore, and other products of mines, and for dutiable articles at U. S. customhouses.

Coal is sold (wholesale) by the long ton, and (retail) by the short ton.

A *cubic foot* of water weighs $62\frac{1}{2}$ lb.; a *gallon* of water weighs, approximately, $8\frac{1}{8}$ lb. These are *useful* equivalents.

Oral Exercises

1. How many ounces are there in 2 lb.? in 3 lb.?
2. How many pounds are there in 3 T.? in 5 T.? in 4 T.?
3. How many pounds are there in 4 cwt.? in $6\frac{1}{2}$ cwt.?
4. How many hundredweight are there in 300 lb.? in 550 lb.? in 425 lb.?
5. How many pounds are there in $\frac{1}{2}$ T.? in $\frac{1}{4}$ T.? in $\frac{3}{4}$ T.?
6. Express as tons: 4000 lb.; 2500 lb.; 5000 lb.

Written Exercises

Change to pounds:

Change to ounces:

- | | | | |
|------------------------|-------------------------|------------------------|-------------------------|
| 1. 7 T. | 5. 96 oz. | 9. 15 lb. | 13. $9\frac{3}{4}$ lb. |
| 2. $9\frac{1}{2}$ T. | 6. $16\frac{1}{4}$ T. | 10. 37 lb. | 14. $7\frac{1}{4}$ cwt. |
| 3. 64 oz. | 7. 128 oz. | 11. 6 cwt. | 15. 125 lb. |
| 4. $8\frac{3}{4}$ cwt. | 8. $22\frac{1}{2}$ cwt. | 12. $8\frac{1}{2}$ lb. | 16. 18 cwt. |

Express in tons :		Express in lower units :	
17. 360 cwt.	19. 46.5 cwt.	21. $\frac{1}{8}$ T.	23. $\frac{7}{8}$ cwt.
18. 1500 lb.	20. 7000 lb.	22. $\frac{5}{8}$ T.	24. $\frac{3}{16}$ cwt.

Express in lb. and oz. :		Express in tons and lb. :	
25. $7\frac{3}{4}$ lb.	28. 300 oz.	31. $4\frac{1}{2}$ T.	34. 3100 lb.
26. 50 oz.	29. $16\frac{3}{8}$ lb.	32. $6\frac{1}{4}$ T.	35. 4600 lb.
27. $9\frac{1}{8}$ lb.	30. 450 oz.	33. $8\frac{3}{5}$ T.	36. 7850 lb.

37. Find the cost of 3000 lb. of hay at \$20 per ton.
38. The average weight of each of 10 ears of corn was 20 oz. Find their total weight in pounds.
39. A coal dealer wishes to put $2\frac{4}{5}$ tons of coal into bags that hold 100 lb. each. How many bags does he need?
40. Find, in tons and pounds, the approximate weight of the water in a tank that contains 450 gal.
41. Cork is .24 as heavy as water. How much does a cubic foot of cork weigh?
42. Milk is about 1.03 times as heavy as water. What is the approximate weight of 600 gal. of milk?
43. During a fire the fire engines used 1200 lb. of cannel coal costing \$18 per ton. Find the cost of the coal.
44. A cubic foot of one kind of marble weighs 163.75 lb. How many times as heavy as water is it?
45. If 420 tons of drinking water are used on a vessel during a voyage, about how many gallons are used?
46. A steel firm pays 12¢ per bushel of 14 lb. for charcoal. At this rate find the cost per long ton.

In states that regulate the weight of a bushel the following are standard weights for a few important products:

Wheat	60 lb.	
Corn (shelled)	56 lb.	(Except in Cal.)
Potatoes	60 lb.	(Except in Md., Pa., Va.)
Oats	32 lb.	(Except in Ida., Md., N. J., Ore., Va.)
Barley	48 lb.	(Except in Ala., Cal., Ga., Ky., La., Ore., Pa.)

Remember also that a barrel of flour contains 196 pounds.

Written Exercises

Find the weight of:

1. 6 bu. oats.
2. 5 bu. corn.
3. 8 bu. oats.
4. $7\frac{3}{4}$ bu. corn.
5. 42 bbl. flour.
6. 56 bu. barley.
7. $84\frac{1}{2}$ bu. wheat.
8. 72.5 bu. barley.
9. 96 bu. potatoes.

Find the weight and the cost of:

10. 20 bbl. salt, each 280 lb., at \$1.15 per barrel.
11. 120 bbl. pork, each 200 lb., at \$24 per barrel.
12. 144 bu. buckwheat, each 48 lb., at 75¢ per bushel.
13. 96 bu. clover seed, each 60 lb., at \$12.75 per bushel.
14. 25 bu. timothy seed, each 45 lb., at \$3.25 per bushel.

Find the gain on each of the following:

ARTICLE	BUYING PRICE	SELLING PRICE
15. 12 25-lb. boxes apricots	$9\frac{1}{2}$ ¢ per lb.	12¢ per lb.
16. 6 boxes dates, each 30 lb.	\$2.50 per box	10¢ per lb.
17. 8 tubs butter, each 60 lb.	$28\frac{1}{2}$ ¢ per lb.	32¢ per lb.
18. 25 kegs nails, each 100 lb.	\$2.25 per keg	4¢ per lb.
19. 18 tons meal, 100-lb. bags	\$1.20 per bag	\$1.30 per bag

REVIEW OF LENGTH, AREA, AND VOLUME MEASURES

Written Exercises

Refer to pages 263, 266, and 277, respectively, for the tables.

Reduce to the lower of the given units :

- | | | |
|-----------------|------------------------|------------------------|
| 1. 18 yd. 1 ft. | 4. 2 mi. 160 rd. | 7. 4 cu. yd. 6 cu. ft. |
| 2. 16 ft. 6 in. | 5. 3 A. 5 sq. rd. | 8. 8 sq. rd. 5 sq. yd. |
| 3. 20 rd. 4 yd. | 6. 5 sq. ft. 9 sq. in. | 9. 7 cu. ft. 8 cu. in. |

Express in higher units :

- | | | | |
|------------|-------------|-----------------|------------------|
| 10. 78 in. | 13. 650 rd. | 16. 500 sq. in. | 19. 4000 cu. in. |
| 11. 88 yd. | 14. 190 in. | 17. 650 cu. ft. | 20. 4961 sq. yd. |
| 12. 67 ft. | 15. 331 ft. | 18. 900 sq. rd. | 21. 6539 sq. ft. |

Reduce :

- | | |
|-----------------------|--|
| 22. 4 rd. to feet. | 26. 605 sq. yd. to acres. |
| 23. .5 yd. to inches. | 27. $\frac{2}{3}$ cu. yd. to cubic inches. |
| 24. 198 ft. to rods. | 28. $1\frac{1}{3}$ sq. rd. to square feet. |
| 25. 880 yd. to miles. | 29. 648 sq. in. to square yards. |
30. Find the cost of constructing $4\frac{1}{2}$ mi. of macadam road 14 ft. wide, at 50¢ per square yard.
31. Ice is .92 as heavy as water. Find the exact weight of a block of ice 2 ft. 6 in. by 2 ft. by 12 in.
32. Find the exact weight of the water in a tank 8' square and 7'6" deep when it is full.
33. Find the weight of a piece of granite 4 ft. square and 30 in. thick, if granite is 2.72 times as heavy as water.
34. Steel is about 7.84 times as heavy as water. Find the weight of a steel plate 90 in. by 56 in. and $\frac{3}{8}$ in. thick.

TIME MEASURES

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 year (yr.)

10 years = 1 *decade*; 100 years = 1 *century*. Each hundredth year, counting from the birth of Christ (B.C.), is called a *centennial year*.

The earth revolves around the sun in 365 days 5 hours 48 minutes 46 seconds. This is the *solar year*, and is nearly $365\frac{1}{4}$ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years whose number is divisible by 400 and other years whose number is divisible by 4 are lengthened 1 day, Feb. 29. These years are **leap years**, and each contains **366 days**.

There are **12 months (mo.)** in a **year**, as follows :

- | | |
|-------------------------------------|-----------------------------|
| 1. January (Jan.) 31 da. | 7. July (July) 31 da. |
| 2. February (Feb.) 28 da. or 29 da. | 8. August (Aug.) 31 da. |
| 3. March (Mar.) 31 da. | 9. September (Sept.) 30 da. |
| 4. April (Apr.) 30 da. | 10. October (Oct.) 31 da. |
| 5. May (May) 31 da. | 11. November (Nov.) 30 da. |
| 6. June (June) 30 da. | 12. December (Dec.) 31 da. |

Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

The day is divided into two parts; from midnight to noon (M.)

is called *forenoon* (A.M.) and from noon to midnight, *afternoon* (P.M.). Thus, "ten minutes after nine o'clock in the morning of the fourth of July in the year 1914" is written, "9.10 A.M., July 4, 1914."

Oral Exercises

Unless otherwise specified, "year" means "common year."

1. How many seconds are there in 2 min.? in 3 min.?
2. How many minutes are there in 4 hr.? in 6 hr.?
3. How many days are there in 4 wk.? in 48 hr.?
4. How many hours are there in 2 da.? in 300 min.?
5. How many weeks are there in 14 da.? in 35 da.?
6. What part of a year is 2 mo.? 3 mo.? 6 mo.?

Tell which of the following are leap years:

7. 1892 8. 1909 9. 1900 10. 2000 11. 1912

Written Exercises

Reduce to the next lower unit:

- | | | | |
|------------|------------|-------------|-------------|
| 1. 91 da. | 4. 168 hr. | 7. 420 min. | 10. 312 hr. |
| 2. 72 hr. | 5. 182 da. | 8. 540 min. | 11. 336 da. |
| 3. 105 da. | 6. 216 hr. | 9. 840 min. | 12. 768 hr. |

13-24. In exercises 1-12 reduce to the next higher unit.

Change to minutes:

Change to days:

- | | | | |
|------------|--------------|------------|-------------|
| 25. 18 hr. | 28. 720 sec. | 31. 96 hr. | 34. 104 wk. |
| 26. 22 hr. | 29. 840 sec. | 32. 18 wk. | 35. 288 hr. |
| 27. 29 hr. | 30. 960 sec. | 33. 14 yr. | 36. 116 yr. |

Reduce to the lowest unit:

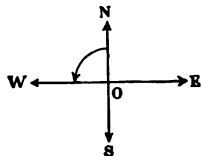
Reduce to higher units:

- | | | |
|----------------------------|--------------|-------------|
| 37. 12 da. 9 hr. | 41. 475 sec. | 45. 174 da. |
| 38. 4 yr. 128 da. | 42. 520 min. | 46. 268 hr. |
| 39. 6 wk. 5 da. 6 hr. | 43. 850 sec. | 47. 548 hr. |
| 40. 15 hr. 20 min. 15 sec. | 44. 789 min. | 48. 485 da. |

49. How many days are there in $\frac{4}{5}$ of a year?
50. Mrs. Dee roasted a turkey 2 hr. 35 min. It was ready to serve at 2 P.M. At what time was it put in the oven?
51. If a watch ticks 286 times in a minute, how many times does it tick in an hour?
52. A workman can mold 2 flowerpots in 3 min. At this rate, how many can he mold per hour?
53. If my watch gains 3 sec. a day, how many minutes will it gain during April?
54. A mason works from 8 A.M. to 12 M. and from 1 P.M. to 5 P.M. each day at 60¢ an hour. Find his daily wages.
55. A machine puts out 450 sanded covers for match boxes per minute. How many are produced in 8 hr.?
56. My watch runs 36 hr. after being wound. How many times must I wind it during November?
57. Allowing 20 min. for each pound in roasting pork, how many hours will it take to roast $8\frac{3}{4}$ lb.?
58. A box-making machine produces 25 boxes in 3 min. How many boxes does it produce in a 10-hour day?
59. Two watchmen relieved each other every 6 hr. How many times did each one watch during January?
60. The point of the minute hand of a large clock travels $11\frac{1}{2}$ in. in 30 sec. How many feet does it travel per hour?
61. A track-laying machine lays 7 rails on each side of the track in 5 min. At that rate, how many rails will it lay in 8 hr.?

CIRCULAR MEASURES

1. What part of a complete turn do you make in turning from north to west? from west to south? from south to east? from north to south?

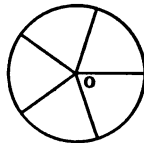
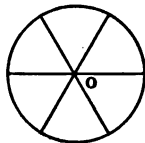
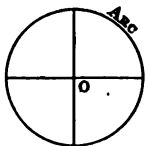
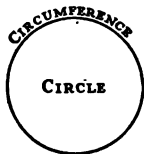


The size of an angle is usually expressed in **degrees** ($^{\circ}$).

A complete turn measures 360° .

Each angle at O is an angle of 90° , or a **right angle**.

2. The boundary line of a circle is its **circumference**; any part of the circumference is an **arc**.



3. In each of the three figures on the right, what part of 360° is each angle? of the circumference is each arc?

4. In these figures O, the *center* of each circle, is also the *vertex* of each angle. Such angles are **central angles**.

5. Since an arc is the same part of a circumference that its central angle is of 360° , we measure *arcs* as well as *angles* in *degrees*, calling a circumference 360° .

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^{\circ}$)
360 angle degrees	= 4 right angles
360 arc degrees	= 1 circumference (cir.)

Written Exercises

Change to minutes :

- | | | | | |
|---------------|------------|-----------------|--------------|-----------------|
| 1. 15° | 5. $780''$ | 9. 116° | 13. $1320''$ | 17. 450° |
| 2. 18° | 6. $840''$ | 10. 235° | 14. $2040''$ | 18. 675° |
| 3. 24° | 7. $900''$ | 11. 340° | 15. $4260''$ | 19. 738° |
| 4. 36° | 8. $960''$ | 12. 375° | 16. $5580''$ | 20. 840° |

Reduce to the lowest of the given units :

- | | | | |
|--------------------|--------------------|----------------------|-------------------------|
| 21. $22^\circ 30'$ | 24. $58' 45''$ | 27. $5^\circ 8' 6''$ | 30. $12^\circ 10' 15''$ |
| 22. $45' 50''$ | 25. $78^\circ 30'$ | 28. $8^\circ 5' 9''$ | 31. $25^\circ 35' 30''$ |
| 23. $37^\circ 30'$ | 26. $55' 39''$ | 29. $9^\circ 7' 5''$ | 32. $45^\circ 16' 45''$ |

Express in lower units :

Express in higher units :

- | | | | | | |
|-------------------------|-------------------------|------------------------|-------------|-------------|--------------|
| 33. $\frac{1}{2}^\circ$ | 37. $\frac{5}{8}'$ | 41. $\frac{1}{4}$ cir. | 45. $500''$ | 49. $800'$ | 53. $5060''$ |
| 34. $\frac{3}{4}'$ | 38. $\frac{4}{5}'$ | 42. $\frac{3}{5}$ cir. | 46. $650'$ | 50. $750''$ | 54. $4580'$ |
| 35. $\frac{2}{3}'$ | 39. $\frac{3}{8}^\circ$ | 43. $\frac{5}{6}$ cir. | 47. $864''$ | 51. $485'$ | 55. $6270''$ |
| 36. $\frac{1}{5}^\circ$ | 40. $\frac{7}{8}^\circ$ | 44. $\frac{5}{8}$ cir. | 48. $775'$ | 52. $976''$ | 56. $7400'$ |

57. What part of a circumference is 30° ? 120° ? 270° ?

58. How many degrees are there in $\frac{2}{3}$ of a circumference?
in .625 cir.? in $\frac{5}{6}$ cir.? in .875 cir.?

59. Through how many degrees does the hour hand of a clock pass in 6 hr.? in 12 hr.? in 1 day?

60. How many degrees are there in the angle made by the hour and minute hands of a clock at 1 o'clock?

61. The length of a minute of the circumference of the earth at the equator is a nautical mile. Find the circumference of the earth at the equator in nautical miles.

VALUE MEASURES

The unit of **English money** is the **pound**, or **sovereign**.

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)

The sign £ precedes the number of pounds.

The unit of **French money** is the **franc**.

100 centimes (c.)	= 1 franc (fr.)
-------------------	-----------------

Centimes is pronounced *sān'tēms'*.

The unit of **German money** is the **mark**.

100 pfennigs (pf.)	= 1 mark (M.)
--------------------	---------------

Written Exercises

Express in marks :

Express in francs :

- | | | | |
|-------------|-------------|------------|------------|
| 1. 1600 pf. | 3. 2450 pf. | 5. 2700 c. | 7. 4110 c. |
| 2. 3000 pf. | 4. 5675 pf. | 6. 3500 c. | 8. 6325 c. |

Reduce :

- | | |
|-------------------------|----------------------------|
| 9. 4800 c. to francs. | 12. 475 s. to pence. |
| 10. £ 25 to shillings. | 13. 675.5 M. to pfennigs. |
| 11. 1256 far. to pence. | 14. 58.75 fr. to centimes. |

Reduce to the next lower unit :

- | | | | | |
|-----------|-----------|-----------|-----------|-----------|
| 15. 156d. | 17. 340s. | 19. 452d. | 21. 590s. | 23. 844d. |
| 16. 280s. | 18. 396d. | 20. 665s. | 22. 711d. | 24. 935s. |

25-34. In exercises 15-24 reduce to higher units.

These official **equivalents** may be learned.

1 pound	= \$4.8665
1 franc	= \$.193
1 mark	= \$.238

In estimates we think of the pound as \$ 5, the franc as 20 ¢, and the mark as 25 ¢.

Written Exercises

Find, to the nearest cent, the value of :

- | | | |
|-------------|---------------|---------------|
| 1. £ 420 | 5. 1800 fr. | 9. 1215 M. |
| 2. £ 24.75 | 6. 2340 fr. | 10. 3025 M. |
| 3. £ 56.125 | 7. 144.75 fr. | 11. 85.75 M. |
| 4. £ 520.75 | 8. 6258.5 fr. | 12. 408.25 M. |

13. How much less than a quarter of a dollar is a franc worth ? a mark ?

14. Find, to the nearest cent, the value of an English shilling ; an English penny.

15. A traveler in Paris exchanged \$ 38.60 for francs. How many francs did he receive ?

16. A German mason earned 51 marks per week. Find his weekly wages in United States money.

17. If 92 pigeons were sold for 3772 fr., find the average value of each pigeon in United States money.

18. The average amount of ivory obtained from an elephant is worth £ 60. Find its value in U. S. money.

19. A coal miner's pay is 43*d.* per ton of coal. Find, to the nearest cent, the value of his pay.

COUNTING TABLE

20 things = 1 score
12 things = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
12 gross = 1 great gross (G. gr.)

1 gross = 144; 1 great gross = 1728.

Written Exercises

Find the gain from each purchase and sale:

ARTICLES	QUANTITY AND BUYING PRICE	SELLING PRICE
1. Pens	1 great gross @ \$9.60	90¢ per gross
2. Cuffs	1 doz. pairs @ \$2.70	25¢ per pair
3. Collars	20 doz. @ \$1.12	15¢ each
4. Hinges	1 doz. pairs @ \$1.75	25¢ per pair
5. Screws	100 gross @ 25¢	5¢ per doz.
6. Pencils	4 gross @ \$2.25	2 for 5¢
7. Brackets	6 doz. pairs @ \$1.50	18¢ per pair
8. Door knobs	5 doz. pairs @ 96¢	10¢ per pair
9. Coat hooks	18 gross @ \$1.75	22¢ per doz.
10. Indian clubs	30 pairs @ 28¢	35¢ per pair
11. Rubber bands	25 gross @ 11¢	15¢ per gross
12. Eggs	1 case (30 doz.), \$6.60	25¢ per doz.
13. Soap	1 box (240 cakes), \$7.50	6 for 25¢
14. Corn	1 case (24 cans), \$1.70	3 for 25¢
15. Beans	1 case (24 cans), \$2.50	2 for 25¢
16. Peaches	1 case (24 cans), \$4.50	1 for 25¢
17. Oranges	1 crate (96 oranges), \$3	50¢ per doz.
18. Oranges	1 crate (126 oranges), \$2.80	40¢ per doz.

STATIONERS' MEASURES

Folded sheets of paper are often sold by these measures:

24 sheets = 1 quire (qr.)	2 reams = 1 bundle
20 quires = 1 ream (R.)	5 bundles = 1 bale

Paper in tablets is sold by the 100, 200, 250, 500, and 1000 sheets, and by the pound or 100 pounds.

Written Exercises

1. If a girl uses 1 doz. sheets of paper and 1 doz. envelopes per month, how many quires of paper and how many dozen envelopes will she use in a year?
2. A boy buys a $2\frac{1}{2}$ -quire box of writing paper. How many letters of 1 sheet each can he write on the paper?
3. Mr. Stevens bought a 5-quire box of writing paper and a " $\frac{1}{8}$ -thousand" box of envelopes. How many more envelopes than sheets of paper did he buy?
4. Find the cost of 5 lb. of paper at 80¢ per pound and 500 envelopes at \$ 3.40 per thousand.
5. How many quires of paper are required to furnish 48 pupils 2 sheets apiece during an examination?
6. Find the cost of 4 qr. of paper at \$ 1.20 per ream.
7. If legal cap that weighs 14 lb. to the ream costs 45¢ per $\frac{1}{4}$ ream, find the weight and cost of a quire.
8. Find the cost of 100 invitations, each requiring a sheet of paper and an envelope, if the paper costs 18¢ per quire and the envelopes \$ 3 a thousand.

Miscellaneous Exercises

Reduce to a fractional part of the next higher unit :

1. 48 min. 3. 210° 5. 96 sq. rd. 7. 2 pk. 4 qt.
2. 750 lb. 4. 3s. 4d. 6. 1 ft. 6 in. 8. 1296 cu. in.

Reduce to a decimal part of the next higher unit :

9. 800 lb. 11. 280 rd. 13. 108 sq. in. 15. 1 pk. 4 qt.
10. 219 da. 12. 1 pt. 1 gi. 14. 2 ft. 3 in. 16. 216 cu. in.

Reduce :

17. 14 yr. 5 mo. to months. 22. 8 bu. to quarts.
18. 36 pk. 7 qt. to quarts. 23. 95 yd. to inches.
19. 32 ft. 10 in. to inches. 24. 3.375 bu. to quarts.
20. 58 lb. 12 oz. to ounces. 25. $2^{\circ}24'18''$ to seconds.
21. 45 gal. 2 qt. to quarts. 26. 9 cu. yd. to cubic inches.

27. Reduce $\frac{7}{8}$ gal. to quarts and pints.

28. Reduce 240 qt. to bushels and pecks.

29. Reduce $263\frac{1}{3}$ ft. to yards, feet, and inches.

30. Reduce $654\frac{2}{3}$ min. to hours, minutes, and seconds.

31. How many bushels of peanuts will be needed to plant 7 acres of ground, if 5 pk. are used per acre ?

32. If 1 lb. of shelled corn yields 4 oz. of starch, how many pounds of starch will a bushel of corn yield ?

33. At 10d. per hour, how many pounds does a British bricklayer earn in 6 days, working 9 hr. per day ?

34. Find the weight in tons of the rails required for a railroad 30 mi. long, if each rail weighs 90 lb. to the yard.

Maple Sugar Problems

1. Find the production of maple sugar from 625 trees, if the average yield of each tree is 3 lb. of sugar.
2. Find the cost of 1200 sap spouts at \$ 2 per 100 and 800 sap buckets at 20¢ each.
3. How many gallons of maple sirup can be made from 6840 gal. of sap, if 38 gal. of sap make 1 gal. of sirup?
4. How many pounds of maple sugar will 12 bbl. of sap make, if 4 gal. of sap make 1 lb. of sugar?
5. Ten acres of land were planted to sugar maples with one seedling to every 36 sq. ft. How many were used?
6. If 1 gal. of maple sirup is equivalent to $9\frac{1}{2}$ lb. of sugar, for how much per pound should sugar be sold when sirup is worth \$ 1.14 per gallon?
7. At a sugar party 115 dishes of sugar were sold at 10¢ each. Find the profit, if $3\frac{1}{2}$ gal. of sirup, at \$ 1.10 per gallon, were used.

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

Written Exercises

1. Add 12 ft. 6 in. and 14 ft. 9 in.

ft.	in.	
12	6	
14	9	
27	3	

The sum of the inches is 15 in., or 1 ft. 3 in.; consequently, 3 is written in the column for inches, and 1 is reserved to be united with the numbers in the column for feet. The sum of the feet is 27 ft.

Hence, the entire sum is 27 ft. 3 in.

Add:

ft.	in.
2. 9	4
8	10

yd.	ft.
3. 5	21½
4	21¼

gal.	qt.
4. 55	31½
16	21½

mi.	rd.
5. 22	100
10	300

lb.	oz.
6. 8	4
	13

hr.	min.
7. 8	40
6	50

wk.	da.
8. 17	4
12	3

mo.	da.
9. 4	12
3	14

hr.	min.	sec.
10. 1	20	45
	39	15

11. Subtract 5 lb. 7 oz. from 21 lb. 2 oz.

(1)	(2)		
lb.	oz.	lb.	oz.
21	2	20	18
5	7	5	7
15	11	15	11

Since 7 oz. cannot be subtracted from 2 oz., we may express 21 lb. 2 oz. as 20 lb. 18 oz. Then we subtract the ounces and pounds separately. The remainder is 15 lb. 11 oz.

In practice, we subtract without writing the numbers a second time as shown in (1).

We think of 1 lb., or 16 oz., as subtracted from 21 lb. and added to 2 oz. Then 18 oz. - 7 oz. = 11 oz., and 20 lb. - 5 lb. = 15 lb.

- 12-20. Subtract in exercises 2-10.

21. How long was it from Apr. 10, 1899 to Feb. 2, 1913?

yr.	mo.	da.	The later date is written as the minuend and the earlier date as the subtrahend, with the number of the month instead of its name.
1913	2	2	
1899	4	10	Subtract as in denominate numbers, considering 30 days as a month and 12 months as a year. The remainder is the difference in time as accurately as it can be expressed in years, months, and days.
13	9	22	

time as accurately as it can be expressed in years, months, and days.

Find the time from to-day to the following dates:

22. Apr. 3, 1916 25. Nov. 15, 1923 28. Dec. 31, 1927
 23. Jan. 1, 1920 26. Sept. 21, 1925 29. Oct. 28, 1918
 24. Feb. 8, 1915 27. Apr. 18, 1930 30. Aug. 11, 1922

31. Subtract 12 gal. 3 qt. 1 pt. from 24 gal. 2 qt.
 32. Subtract 22 bu. 3 pk. 1 qt. from 32 bu. $1\frac{1}{2}$ pk.
 33. Add 25 ft. 6 in.; 32 ft. 4 in.; 28 ft. 8 in.; 56 ft.
 34. Add 12 T. 8 cwt.; 9 T. 48 lb.; 12 cwt. 65 lb.

35-36. Find by this table the total time required to make 100 medium-grade coats by hand; by machinery.

OPERATIONS	TIME FOR EACH OPERATION, PER 100 COATS			
	By Hand		By Machinery	
Shrinking cloth	11 hr.	40 min.	1 hr.	48 min.
Cutting cloth	33	20	4	32.5
Sewing seams	1000		66	40
Cutting buttonholes	3	20		17.5
Working buttonholes	275		6	17.8
All others	1978	23	1296	20.5

37-43. Find the difference in time for hand and machine work for each operation; for all.

44. Add £ 865 4s. 8d.; £ 82 9s. 10½d.; £ 48 18s.

45. Add £ 642 3s. 2d.; £ 66 0s. 6½d.; £ 34 15s.

46-51. Subtract each amount of money in exercise 45 from the amount above it in exercise 44; add these amounts.

52. Some boys gathered 10 bu. of walnuts and sold all of them but 1 bu. 3 pk. Find the quantity sold.

53. How long was a man away from home, if he left at 9.30 A.M. Thursday and returned at 5.45 P.M. the next Wednesday?

54. A man owned 84 A. 82.5 sq. rd. of land and bought 16 A. 147.5 sq. rd. How much land did he then own?

55. The leakage from a 63-gallon hogshead of rock-candy sirup was 2 qt. 1 pt. How much sirup remained?

56. How much higher is Lake Superior, 602 ft. 11 in. above sea level, than Lake Huron, 576 ft. 9 in. above?

57. In a snowshoe race, the winner's time was 5 hr. 15 min. 49 sec. and that of the one following the winner was 5 hr. 18 min. 38 sec. Find the difference in their time.

58. Find the length of a floating dock, whose two sections are 277 ft. 9 in. and 207 ft. 8 in. long.

Find the age of each of the following men at the time when he became President of the United States:

NAME	BORN	BECAME PRESIDENT
59. George Washington	Feb. 22, 1732	Apr. 30, 1789
60. Abraham Lincoln	Feb. 12, 1809	Mar. 4, 1861
61. Ulysses S. Grant	Apr. 27, 1822	Mar. 4, 1869
62. Woodrow Wilson	Dec. 28, 1856	Mar. 4, 1913

MULTIPLICATION OF DENOMINATE NUMBERS

Written Exercises

1. Multiply 16 bu. 3 pk. by 7.

bu.	pk.	
16	3	7 times 3 pk. = 21 pk. = 5 bu. 1 pk.
	7	7 times 16 bu. = 112 bu.; 112 bu. + 5 bu. = 117 bu.
117	1	The product is 117 bu. 1 pk.

Multiply :

- | | |
|-------------------------|---------------------------------|
| 2. £ 8 16s. by 5 | 20. 11° 0' 16" by 15 |
| 3. 9 qt. 1 pt. by 7 | 21. 20 rd. 3 ft. by 22 |
| 4. 7 ft. 8 in. by 6 | 22. 75 T. 50 lb. by 25 |
| 5. 6 bu. 3 pk. by 4 | 23. 4 yr. 108 da. by 20 |
| 6. 7 gal. 3 qt. by 5 | 24. 18 cwt. 48 lb. by 40 |
| 7. 2 hr. 9 min. by 8 | 25. 9 qr. 16 sheets by 13 |
| 8. 3 mi. 80 rd. by 7 | 26. 5 yd. 2 ft. 8 in. by 30 |
| 9. 16° 35' 20" by 4 | 27. 4 bu. 1 pk. 2 qt. by 12 |
| 10. 24 lb. 9 oz. by 16 | 28. 40 A. 120 sq. rd. by 25 |
| 11. 4 yr. 11 mo. by 11 | 29. 32 pk. 7 qt. 1 pt. by 22 |
| 12. 11 yd. 10 in. by 15 | 30. 8 yr. 4 mo. 15 da. by 16 |
| 13. 10 wk. 6 da. by 13 | 31. 5 T. 10 cwt. 12 lb. by 21 |
| 14. 18 R. 15 qr. by 24 | 32. 25 sq. yd. 8 sq. ft. by 14 |
| 15. 25 lb. 10 oz. by 18 | 33. 12 gal. 2 qt. 1 pt. by 24 |
| 16. £ 20 15s. 8d. by 8 | 34. 8 sq. ft. 72 sq. in. by 12 |
| 17. 10 gr. 8 doz. by 20 | 35. 125 cu. yd. 12 cu. ft. by 9 |
| 18. 13 gal. 1 pt. by 14 | 36. 5 da. 6 hr. 45 min. by 18 |
| 19. 40 rd. 11 yd. by 12 | 37. 20 cu. ft. 44 cu. in. by 24 |

38. If a horse travels on an average a mile in 10 min. 15 sec., how long will it take him to go 6 mi. ?
39. How many rods long is a fence inclosing a square field, each side of which is 25 rd. 11 ft. ?
40. If a skillful operator stamps the name on 24 gr. 4 doz. pens per hour, how many will he stamp in 9 hr. ?
41. Find the cost of 12 tons of coal at £1 6s. per ton.
42. The average yield from each of 16 acres of corn was 48 bu. 3 pk. 6 qt. Find the total yield.
43. The point of the hour hand of a large clock travels 6 ft. $9\frac{1}{2}$ in. per hour. Find the distance traveled per day.
44. An electric fountain used 2083 gal. $1\frac{1}{2}$ qt. of water per hour. How much did it use in 24 hr. ?
45. If a passenger locomotive uses 86 lb. 9 oz. of coal per mile, how much coal will be used on a trip of 95 mi. ?
46. If the circumference of a wagon wheel is 3 yd. 2 ft. 6 in., how far will it go in making 225 revolutions ?
47. Find the weight of a shipment of candles consisting of 124 cases, each containing 21 lb. 14 oz. of candles.
48. A large windmill pumps 8333 gal. $1\frac{1}{4}$ qt. of water per hour. How much water does it pump per day ?
49. The Empire State Express went a distance of 55 mi. in 1 hr. 1 min. 52 sec. At that rate, how long would it take to go from New York to Buffalo, 440 mi. ?
50. The toll on loaded vessels passing through the Suez Canal is 8 fr. 50 c. per ton of the vessel's capacity. Find the toll on a loaded vessel whose capacity is 3964 tons.

DIVISION OF DENOMINATE NUMBERS

Written Exercises

1. Divide 33 hr. 21 min. 45 sec. by 15.

hr.	min.	sec.
15)33	21	45
2	13	27

$$33 \text{ hr.} + 15 = 2 \text{ hr. and } 3 \text{ hr. over.}$$

$$3 \text{ hr. } 21 \text{ min.} = 201 \text{ min.}$$

$$201 \text{ min.} + 15 = 13 \text{ min. and } 6 \text{ min. over.}$$

$$6 \text{ min. } 45 \text{ sec.} = 405 \text{ sec.; } 405 \text{ sec.} + 15 = 27 \text{ sec.}$$

Divide:

- | | |
|---|-----------------------------------|
| 2. 53 ft. 6 in. by 6 | 9. $202^{\circ} 2' 44''$ by 11 |
| 3. 81 lb. 9 oz. by 5 | 10. £ 85 10s. 9d. by 9 |
| 4. 55 bu. 2 pk. by 6 | 11. 143 gr. 9 doz. by 15 |
| 5. 40 gal. 1 qt. by 7 | 12. 102 mi. 120 rd. by 14 |
| 6. 92 yd. 27 in. by 9 | 13. 25 cu. ft. 40 cu. in. by 20 |
| 7. 196 R. 4 qr. by 18 | 14. 280 sq. ft. 120 sq. in. by 30 |
| 8. 8 T. 250 lb. by 25 | 15. 48 hr. 45 min. 12 sec. by 12 |
| 16. How many times does 144 ft. 6 in. contain 8 ft. 6 in. ? | |

SOLUTION. — Reducing to a *common denomination*, in this case to feet, we have 144 ft. 6 in. = 144.5 ft. and 8 ft. 6 in. = 8.5 ft.

Dividing 144.5 ft. by 8.5 ft., we have for the quotient 17.

Hence, 144 ft. 6 in. contains 8 ft. 6 in. 17 times.

17. Divide 31 hr. 8 min. 18 sec. by 2 hr. 13 min. 27 sec.

SUGGESTION. — Reduce both dividend and divisor to seconds.

18. Divide 239 ft. 7 in. by 9 ft. 7 in.
19. Divide 48 gal. 3 qt. by 3 gal. 1 qt.
20. Divide 158 T 1200 lb. by 12 T. 400 lb.
21. Divide 50 bu. 3 pk. 4 qt. by 4 bu. 2 pk. 4 qt.

22. My lamp holds $1\frac{1}{4}$ qt. of oil. How many times can I fill it from a 5-gallon can of oil?

23. If a horse eats 12 qt. of oats per day, how long will $31\frac{1}{2}$ bu. of oats last him?

24. A house was built on a lot 86 ft. 3 in. wide. How wide was the house, if it was $\frac{1}{3}$ as wide as the lot?

25. If a train travels 143 mi. in 3 hr. 15 min., find its rate per hour.

26. A man picked 94 bu. 2 pk. of cranberries in 12 days. What quantity did he pick per day, on the average?

27. The circumference of a wheel is 8 ft. 3 in. How many revolutions will it make in going a mile?

28. What is the average weight of the members of a football team, if the 11 men weigh together 1 T. 35 lb.?

29. The distance around a square farm is 3 mi. 240 rd. Find the length of each side in rods.

30. A 12-inch gun on the *South Carolina* fired 16 shots in 4 min. 51 sec. Find the average time used for each shot.

31. In a trial run of 176 mi. a torpedo boat consumed 220 gal. of gasoline. How much did it use per mile?

32. An ice boat sailed 20 mi. in 39 min. 50 sec. At this rate, how long would it take the boat to sail 5 mi.?

33. A prize bundle of asparagus weighed 21 lb. 9 oz. If there were 120 heads of asparagus in the bundle, find the average weight of each head in ounces.

34. A window shade factory turns out 60,000 yd. of shade cloth per day. How many shades $7\frac{1}{2}$ ft. long can be made from one day's output of cloth?

Miscellaneous Exercises

Add :

1.	hr. 24	min. 50	5.	180° 44°	45'	9.	T. 5	cwt. 8	lb. 40
	16						2	3	60
2.	ml. 4	rd. 80	6.	ml. 12	ft. 1000	10.	hr. 8	min. 45	sec. 18
	3	120		8	4500		6	30	56
3.	ft. 15	ln. 9	7.	yr. 5	mo. 2	11.	T. 3	cwt. 4	lb. 75
	6	10		3	8			9	45
4.	cu. yd. 148	cu. ft. 10	8.	bu. 35	pk. 0	qt. 2	12.	8° 14'	48"
	85	24		20	1	4		5° 0'	24"

13-24. Subtract in exercises 1-12.

Multiply :

25. 9 gal. 3 qt. by 6
 26. 8 ft. 11 in. by 8
 27. 24 lb. 12 oz. by 15
 28. 10 hr. 15 min. by 25
 29. £15 12s. 10d. by 18
 30. 12 bu. 2 pk. 5 qt. by 12

Divide :

31. 154° 39' by 9
 32. 25 wk. 1 da. by 11
 33. 40 yr. 6 mo. by 18
 34. 80 mi. 56 rd. by 12
 35. 184 R. 10 qr. by 18
 36. 36 A. 120 sq. rd. by 20

37. Add 48 gal. 3 qt.; 24 gal. 2 qt. 1 pt.; 50 gal. 1 qt.

38. Add 36 gal. 1 qt.; 18 gal. 3 qt. 1 pt.; 40 gal. 2 qt.

39-44. Subtract each denominate number in exercise 38 from the one above it in exercise 37; add these numbers.

45. How much less than a right angle is $61^{\circ} 25' 48''$?
46. How much flaxseed was required to sow 12 acres, if 2 bu. 2 pk. were sown per acre?
47. Find the number of barrels required to contain a crop of 814 bu. of onions, if each barrel holds 11 pk.
48. Mr. Wolfe works each day from 8 A.M. to 12 M. and from 1 P.M. to 4.30 P.M. How long does he work per day?
49. An athlete ran 18 times around a track $293\frac{1}{3}$ yd. in circumference. How many miles did he run?
50. Find the average time of passage through the Suez Canal of a vessel that went through it 3 times in 15 hr. 32 min.; 19 hr. 58 min.; and 17 hr. 15 min.
51. How many bags that hold 2 bu. each will be required to contain 3 T. 384 lb. of shelled corn?
52. A farmer sold four loads of hay containing 1 T. 250 lb.; 1975 lb.; 1 T. 75 lb.; and 1 T. 480 lb. How much did he receive for the hay at \$20 per ton?
53. How much change should I receive out of a sovereign in paying for 15 lb. of apples @ 3d. and 20 lb. @ 5d.?
54. A farmer sowed 2 bu. 2 pk. of wheat and harvested 45 bu. Find the average yield per bushel of seed sown.
55. If 1 lb. of India tea makes 7 gal. 2 qt. of liquid tea of a certain flavor, how much liquid tea will 4 oz. make?
56. A stove dealer received by freight, in one shipment, several lots of stoves, weighing: 1 T. 14 cwt. 64 lb.; 3 T. 49 lb.; 2 T. 9 cwt.; 5 T. 18 cwt. 87 lb. Find the freight charges at \$.25 per hundredweight.

BUSINESS PROBLEMS

BILLS AND STATEMENTS OF ACCOUNT

Review pages 144 and 145 in regard to bills.

Written Exercises

Make out, foot or balance, and receipt bills, supplying dates, names, and addresses when lacking:

- | | |
|-----------------------------------|--|
| 1. $\frac{1}{4}$ lb. tea @ \$.60 | 6. 6 lb. bacon @ 25¢ |
| 8 lb. rice @ \$.09 | 8 lb. rib roast @ 28¢ |
| 4 doz. eggs @ \$.25 | 9 lb. chicken @ 30¢ |
| 2. 12 hoes @ 59¢ | 7. 2 doz. lemons @ \$.25 |
| 18 rakes @ 50¢ | 3 doz. oranges @ \$.35 |
| 24 shovels @ 75¢ | 4 doz. bananas @ \$.20 |
| 3. $1\frac{1}{2}$ qt. cream @ 32¢ | 8. $2\frac{1}{2}$ lb. cheese @ \$.25 |
| 50 lb. sugar @ $6\frac{1}{2}$ ¢ | 12 lb. crackers @ \$.09 |
| $5\frac{1}{4}$ lb. butter @ 32¢ | $1\frac{1}{2}$ bu. tomatoes @ \$.60 |
| 4. 6 trunks @ \$9.75 | 9. 24 baseballs @ \$.87 $\frac{1}{2}$ |
| 9 suit cases @ \$6.50 | 4 catchers' mitts @ \$4.75 |
| 6 hand bags @ \$3.25 | 5 fielders' gloves @ \$.95 |
| 9 card cases @ \$.98 | 3 catchers' masks @ \$2.25 |
| 5. 8 yd. silk @ \$1.25 | 10. 3 shotguns @ \$14.75 |
| 6 yd. lace @ \$1.19 | 12 shell bags @ \$.60 |
| 9 yd. linen @ \$.75 | 8 pairs leggings @ \$.59 |
| 12 yd. muslin @ \$.18 | 3 shotgun covers @ \$ 1.50 |

11. 85 yd. carpet @ \$1.25; 24 yd. linoleum @ \$1.35.
12. 9 yd. linen sheeting @ \$1.25; 6 yd. table damask @ \$1.40.
13. J. Osgood is debtor to Dr. E. Cook for : 25 day visits @ \$2; 7 night visits @ \$3.
14. George Case is debtor to the Municipal Gas Co. for: 6300 cu. ft. of gas for lighting @ 95¢ per 1000 cu. ft.; 11,200 cu. ft. of gas for fuel @ 80¢ per 1000 cu. ft.
15. Mar. 15, Pearl Roberts bought of the Haller Publishing Co.: 1 encyclopedia, \$48; 3 books @ \$1.50.
16. 12 neckties @ \$.65; 9 pairs gloves @ \$1.35; 6 shirts @ \$1.10; 9 caps @ \$.75; 6 bath robes @ \$3.75.
17. 6 doz. gas mantles @ \$1.10; 4 doz. chimneys @ \$1.30; 6 gas plates @ \$2.20; 9 gas stoves @ \$7.50.
18. 12 pairs opera glasses @ \$9.25; 6 pairs field glasses @ \$12.50; 9 reading glasses @ \$.75.
19. 14 yd. silk @ \$1.75; 2 pairs gloves @ \$1.50; 9 yd. flannel @ 37½¢; 4 doz. handkerchiefs @ \$2.75; 3 tablecloths @ \$3.75; 1 rug, \$32.50; 4 hassocks @ 75¢.
20. 2 brass beds @ \$18.50; 2 steel folding cots @ \$3.89; 2 mattresses @ \$15.
21. 4 sets knives and forks @ \$22.50; 4 sets fruit knives @ \$4.85; 6 carving sets @ \$7.50; 24 butcher knives @ \$.50; 60 pocket knives @ \$.37½.
- 22-26. Make out a bill for 5 purchases at a shoe store; at a fruit store; at a meat market; at a grocery store; at a dry goods store.

Dealers often send to each customer at certain intervals, a statement showing the charges against him, **debits**; and the payments he has made, **credits**.

The customer is the **debtor** and the dealer the **creditor**.

Dr. stands for *debits* or *debtor* and Cr. for *credits* or *creditor*.

The following is a **statement of account** :

Acct. stands for *account* and Mdse. for *merchandise*.

Syracuse, N.Y., <u>April 30, 1914.</u>							
Mr. <u>A. V. Lord,</u>							
<u>74 Madison Ave.</u>							
In account with Mix Furniture Co.							
422 Washington Ave.							
		Dr.					
Mar.	31	To acct. rendered	25	36			
Apr.	15	• Mdse.	37	80	63	16	
		Cr.					
	10	By cash	15	—			
	23		20	75	35	75	
		Balance			27	41	

The difference between the sum of the amounts owed and the sum of the amounts paid is the **balance**.

In the above statement, the first debit *item* shows the balance due Mar. 31 when the last statement was sent.

What is the sum of the debits? the credits? What is the balance April 30?

NOTE. — An *itemized statement*, showing a detailed list of all the debit and credit items, is often sent. Such a statement is equivalent to a bill. If not paid in a certain time, it is followed by another similar to the above, simply stating the balance of the account rendered,

Written Exercises

Make out and balance statements of these accounts, supplying names, dates, and addresses when lacking:

1. Henry Blue in account with Douglass Brothers.
Debits. — Aug. 31, Acct. rendered, \$61.25.
Credits. — Sept. 12, labor, \$22; Sept. 29, cash, \$30.25.
2. G. L. Lang in account with E. W. Burrows.
Debits. — Nov. 1, Mdse., \$52; Nov. 28, Mdse., \$33.75.
Credits. — Dec. 4, cash, \$45.65; Dec. 15, labor, \$25.
3. George Moore in account with Vars & Son.
Debits. — May 6, Mdse., \$37.38; May 26, Mdse., \$62.50.
Credits. — June 1, cash, \$48.75; July 1, cash, \$23.
4. D. B. Slater in account with Coy Furniture Co.
Debits. — Feb. 28, Acct. rendered, \$177.50.
Credits. — Mar. 3, cash, \$60.75; April 6, labor, \$55.
5. Lewis N. Sweet in account with Drake & Clinton.
Debits. — Aug. 1, Mdse., \$108; Aug. 25, Mdse., \$237.
Credits. — Sept. 9, cash, \$200; Sept. 25, cash, \$65.35.
6. *Debits.* — Dec. 2, Mdse., \$37.68; Jan. 1, Mdse., \$83.
Credits. — Jan. 1, cash, \$10; Jan. 25, labor, \$15.50;
Feb. 20, cash, \$20.35; Mar. 10, cash, \$25.75.
7. *Debits.* — Jan. 1, Mdse., \$231.65; Jan. 25, Mdse., \$72.30; Feb. 6, Mdse., \$58.87; Mar. 20, Mdse., \$37.75.
Credits. — Mar. 1, cash, \$200; May 1, cash, \$135.50.
8. *Debits.* — May 13, Mdse., \$82.64; May 25, Mdse., \$44; June 6, Mdse., \$97.50; June 25, Mdse., \$88.75.
Credits. — June 20, cash, \$150; July 1, cash, \$105.50.

RECEIPTS

When bills are not rendered for services, purchases, rent, etc., the following form of receipt is generally used.

No. 32	Buffalo, N. Y. June 23, 1914
Received from	C. M. McKay
Thirty-four and ¹⁵ / ₁₀₀	Dollars
For	professional services
\$ 34 ¹⁵ / ₁₀₀	H. F. Norton

A receipt should contain the :

1. Date when the money is paid.
2. Name of the person who pays the money.
3. Name of the person who receives the money.
4. Amount of money paid, in figures and words.

It is well to indicate the nature of the indebtedness as in the form.

Written Exercises

Supply necessary data and make out a receipt for each :

1. W. Cox paid M. Dix \$25.50, on May 1, 1913, for goods purchased.
2. Henry Collins paid Frank S. Mason \$20, for professional advice, on October 18, 1912.
3. \$8.40, gas.
4. \$45.75, rent.
5. \$20.25, board.
6. \$12.35, on Acct.
7. \$68.85, fuel purchased.
8. \$50.25, goods purchased.
9. \$64.75, services rendered.
10. \$50.50, professional advice.

SIMPLE ACCOUNTS

A person may desire to keep a record of money he expends for various purposes. This is an **expense account**.

An expense account may be footed each day, week, or month. The footing will show the total expenses for that time.

When an expense account is joined with a record of the money received, that is, with the person's *receipts*, it constitutes the simplest form of **cash account**.

Such forms of account are useful records of personal transactions or of a small business. They are largely used by farmers in keeping account of receipts and expenses for the whole farm, any particular crop, or the dairy, the poultry, etc. The difference, or *balance*, between the footings of the receipts and expenses shows whether these things are paying or not. The following illustrates such an account:

Receipts

1914				
April	1	20 bushels potatoes @ 60¢	\$12.00	
"	3	18 dozen eggs @ 25¢	4.50	
"	7	2 tons hay @ \$16.00	32.00	
"	15	1 cow sold to R. Johnson	47.50	
"	23	30 dozen eggs @ 22¢	6.60	
"	28	3 bushels seed potatoes @ \$1.00	3.00	\$105.60

Expenses

1914				
April	4	1 ton cottonseed meal for dairy	\$35.00	
"	8	Strap for work harness	.35	
"	18	Garden seeds	8.00	
"	"	Express on seeds	.35	
"	21	2 milk pails	2.00	
"	25	Repairing plow	1.20	\$47.40

Tell the sum of the receipts; of the expenses. Find the balance.

Written Exercises

Supply dates, when lacking, make out, and find the balance of each of these simple accounts:

1. Philip True's cash account for week ending Jan. 10.

Receipts.—Jan. 3, wages preceding week, 54 hr. @ 35¢.

Expenses.—Jan. 4, coal, \$7.10; shirt, \$1; Jan. 5, groceries, \$4.28; Jan. 7, rubbers, \$1.05.

2. School fair, treasurer's report.

Receipts.—112 admissions @ 10¢; candy booth, \$3.78; refreshment booth, \$6.75; 75 carnations @ 5¢.

Expenses.—Rent of hall, \$3.50; advertising, \$1.30; ice cream, \$3; 80 carnations @ 2¢.

3. James Wright's dairy account from June 1 to 15.

Receipts.—June 9, 2 cows @ \$44; 1 yearling, \$19; June 15, milk and cream, \$114.25.

Expenses.—June 2, 1 ton mixed feed, \$28; 2 milk cans @ \$3.50; June 10, 1 cow, \$57.50; $\frac{1}{2}$ ton bran @ \$31.

4. Ruth Mason's cash account for one week.

Receipts.—Allowance, 50¢; embroidering a sofa pillow, 50¢; keeping house for mother one day, 25¢.

Expenses.—4 car fares @ 5¢; notebook, 10¢.

5. A farmer's account from April 2 to 9.

Receipts.—April 2, 22 bu. oats @ 42¢; 1 calf, 165 lb. @ 6¢; April 5, 6 pigs @ \$2.75; April 8, 12 gal. maple sirup @ \$1.10.

Expenses.—April 3, shoeing horses, \$2.40; 2 bbl. salt @ \$2.25; April 5, 2 plowshares @ \$1.75; April 9, repairing tools, \$15.50.

6. Your own cash account for one week.

7. Frank Wells' poultry account for April.

Receipts.—12 doz. eggs @ 24¢; 4 settings of eggs @ \$1.75; 48 chicks @ 10¢; fowls, 28 lb. @ 18¢.

Expenses.—75 lb. cracked corn at \$1.80 per 100 lb.; 50 lb. chick feed at \$1.90 per 100 lb.; chicken netting, 50¢.

8. Monthly report of the treasurer of a debating society.

Receipts.—Cash on hand, \$4.57; initiation fees, 11 @ 25¢; monthly dues, 35 members @ 15¢; back dues, 9 members @ 15¢; speaking contest, 125 tickets @ 10¢.

Expenses.—Advertising speaking contest, \$1.15; stationery, 25¢; parliamentary rules, 11 copies @ 35¢; rent of hall, \$6.

9. Fifth Grade Baseball Club, manager's account.

Receipts.—Contributions from 45 pupils @ 5¢; parents' subscriptions, \$25.75.

Expenses.—11 baseball suits @ \$1.90; 3 baseballs @ 25¢; 3 bats @ 25¢; catcher's mitt, 75¢; catcher's mask, \$1; catcher's body protector, 50¢; first baseman's mitt, 50¢; contribution toward championship banner, 75¢.

10. Earl Swift's cash account for May.

Receipts.—Allowance, 4 wk. @ 25¢; May 7, spading up flower bed, 4 hr. @ 7½¢; May 12, delivering meat, 8 hr. @ 7½¢; May 15, running errands, 23¢; May 19, raking lawn, 6 hr. @ 7½¢; May 26, planting garden, 45¢.

Expenses.—May 1, tablet, 5¢; pencil, 2¢; May 3, ¼ lb. candy @ 40¢; May 6, baseball, 10¢; May 10, Boy Scout hatchet, 50¢; May 23, baseball glove, 25¢.

REVIEW

Written Exercises

1. Write three decimal fractions; two mixed decimals.

How many figures are needed to express tenths? hundredths? thousandths?

How many units of one order make one unit of the next higher order? Where is the decimal point placed?

Read, then write in words:

2. 7.9 4. .072 6. .0058 8. 153.29 10. 25.6793
3. .06 5. 1.43 7. 4.267 9. 24.708 11. 59.0138

Reduce to the next lower unit:

12. 54 pt. 14. 84 hr. 16. 642 doz. 18. 243 sq. ft.
13. 37 pk. 15. 960 A. 17. 820 min. 19. 920 sq. rd.

20-27. In exercises 12-19 reduce to the next higher unit.

Write in figures as decimals or mixed decimals:

28. $\frac{7}{100}$ 29. $5\frac{9}{10}$ 30. $\frac{16}{1000}$ 31. $18\frac{45}{100}$ 32. $9\frac{536}{10000}$

33. 3 tenths; 15 hundredths; 107 thousandths.

34. 9 hundredths; 5 and 1 tenth; 3 thousandths.

35. 27 thousandths; 306 and 25 ten-thousandths.

Reduce to a decimal not beyond four places:

36. $\frac{2}{5}$ 37. $\frac{3}{4}$ 38. $\frac{7}{8}$ 39. $\frac{15}{16}$ 40. $\frac{11}{12}$ 41. $\frac{14}{15}$

42. Reduce to a common fraction: .25; .375; $.33\frac{1}{3}$.

Subtract each of the following from 8.072; from 6:

43. .45 44. .0267 45. 2.073 46. 3.764 47. 5.6328

48. Reduce 12 wk. 5 da. to minutes.
 49. Reduce $\text{£} \frac{7}{8}$ to shillings and pence.
 50. Reduce 63 pt. to pints, quarts, and pecks.
 51. Reduce $22^{\circ} 30'$ to a fractional part of a right angle.

Multiply, timing yourself:

52. 7.75 by 6.2 56. 2 ft. 7 in. by 8 60. 16.25 by 21
 53. .832 by .78 57. 7 lb. 4 oz. by 7 61. 217.4 by 18
 54. 60.9 by $9\frac{2}{3}$ 58. 3 pk. 5 qt. by 6 62. 3438 by .59
 55. .956 by $8\frac{3}{4}$ 59. 6 da. 9 hr. by 9 63. .0384 by 4.5
 64. Divide 194.4 by 10; by 100; by 1000; by .24.

How is a decimal divided by 10, 100, or 1000?

How is a decimal divided by a decimal?

Divide, timing yourself:

- | To nearest tenth | To nearest hundredth | To nearest thousandth |
|--|----------------------|-----------------------|
| 65. 473 by 25 | 68. \$ 4.16 by 19 | 71. 20.461 by 47.2 |
| 66. 26.5 by 18 | 69. 89.73 by 4.2 | 72. \$ 60.07 by 551 |
| 67. 5.61 by 3.6 | 70. 7.329 by .38 | 73. 7.1382 by 6.89 |
| 74. Add 21 bu. 3 pk. 5 qt.; 15 bu. 2 pk.; 34 bu. 7 qt. | | |
| 75. Add 17 bu. 2 pk. 6 qt.; 9 bu. 3 pk.; 16 bu. 4 qt. | | |
| 76-81. Subtract each denominate number in exercise 75 from the one above it in exercise 74; add these numbers. | | |

How many years have elapsed since each event?

82. Atlantic cable, 1858. 86. First life preserver, 1805.
 83. Morse telegraph, 1844. 87. First railroad in U. S., 1826.
 84. First cotton gin, 1794. 88. First steamboat in U. S., 1786.
 85. Marconi telegraph, 1896. 89. First newspaper in U. S., 1704.

Divide :

90. 12 lb. 8 oz. by 8 93. 8 sq. ft. 59 sq. in. by 7
 91. 15 da. 6 hr. by 6 94. 42 yd. 2 ft. by 5 yd. 1 ft.
 92. 21 bu. 6 pk. by 5 95. 17 gal. 1 qt. by 5 gal. 3 qt.

Find the cost of 360 articles @ :

96. 40 ¢ 99. 75 ¢ 102. $62\frac{1}{2}$ ¢ 105. \$1.50 108. $\$1.12\frac{1}{2}$
 97. 60 ¢ 100. 80 ¢ 103. $66\frac{2}{3}$ ¢ 106. \$1.30 109. $\$1.33\frac{1}{3}$
 98. 90 ¢ 101. $8\frac{1}{3}$ ¢ 104. $83\frac{1}{3}$ ¢ 107. \$1.80 110. $\$1.16\frac{2}{3}$

Find the number of articles that can be bought for :

111. \$76 @ 25 ¢ 114. \$234 @ $37\frac{1}{2}$ ¢ 117. \$351 @ $\$1.08\frac{1}{3}$
 112. \$91 @ 70 ¢ 115. \$448 @ $87\frac{1}{2}$ ¢ 118. \$455 @ $\$1.62\frac{1}{2}$
 113. \$96 @ 80 ¢ 116. \$524 @ $66\frac{2}{3}$ ¢ 119. \$693 @ $\$1.83\frac{1}{3}$

Fill the blanks to the nearest hundredth :

FIGURE	BASE	ALTITUDE	AREA
120. Rectangle	5 ft. 4 in.	3 ft. 6 in.	—— sq. ft.
121. Triangle	16 ft. 8 in.	5 ft. 2 in.	—— sq. ft.
122. Square	22 ft. 9 in.	22 ft. 9 in.	—— sq. ft.
123. Parallelogram	25 ft. 7 in.	20 ft. 4 in.	—— sq. ft.

Add and test, timing yourself :

124. 6.72	125. 5832	126. 38.78	127. 3287	128. 29.0746
4.8	149	5.41	4653	.5275
.98	6586	23.6	9024	4.8368
1.46	2045	8.3	3580	43.2054
8.07	871	94.27	916	37.0725
.85	5357	25.69	5471	9.5167

Problems in House Furnishing

1. One spring Mr. Reynolds paid \$36.75 for papering and painting some of the rooms in his house, and \$14.88 for staining the floors. Find the total cost.

Find the cost of furnishings for :

2. KITCHEN		4. DINING ROOM	
Coal range	\$ 45.00	Rug	\$ 24.50
Gas range	17.25	Table	24.00
Kitchen cabinet	18.50	6 dining chairs @ \$ 4.75	—
Table	2.50	Buffet	22.50
High stool	.85	China cabinet	21.45
Chair	1.25	Pictures	8.65
Stepladder chair	2.00	2 pairs curtains @ \$ 2.85	—
3. LIVING ROOM		5. BEDROOM	
Rug	\$ 35.00	Brass bed	\$ 25.00
Library table	22.60	Mattress	12.50
Bookcase	21.50	Pillows	7.25
Desk and chair	18.25	Lace bed set	5.75
2 rockers @ \$ 6.75	—	Dresser	24.85
Morris rocker	11.95	Chair	5.40
Morris chair	14.35	Rocker	5.98
Couch	24.95	Small table	4.75
Portières	25.00	3 rugs @ \$ 3.25	—
Curtains	30.00	2 pairs curtains @ \$ 1.98	—
Pictures	45.00	Pictures	8.25

6. Veranda: 3 porch chairs @ \$1.50; couch hammock, \$8.75; porch shades, \$9; rug, \$12.75.

7. Laundry: tubs, \$12.75; wash machine, \$9; clothes wringer, \$6.25; wash boiler, \$3.75; washboard, \$.85; laundry stove, \$12.25; pail, \$.40; clothes basket, \$.75; clothespins, \$.50; ironing board, \$1.75; flatirons, \$2.25.

Problems for Girls



1. How many handkerchiefs, 12 in. square before being hemmed, can be cut from 1 yd. of linen 36 in. wide?
2. If 60 in. of tape is required in making a book bag, how many yards of tape are required for 12 such bags?
3. Ethel bought 2 skeins of embroidery floss at 5¢ each and 3 packages of needles at 8¢ each. Find the cost.
4. Ruth made a workbag by sewing together 4 strips of ribbon of equal length. If she used 1 yd. of ribbon, how many inches long was each strip?
5. How many pieces of ruffling 10 in. wide can be cut crosswise from $2\frac{1}{2}$ yd. of silk?
6. How many bean-bag covers 6 in. by 12 in. can be cut from 1 yd. of denim 36 in. wide?
7. At $12\frac{1}{2}$ ¢ a yard, find the cost of the lace required to put around a handkerchief 9 in. square, if $\frac{1}{3}$ of the distance around the handkerchief is allowed for fullness.

8. Find the cost of the braid for a hat, if 10 yd. are required and the braid costs \$1.25 for 6 yd.

9. A milliner paid \$1.50 a dozen for hat frames. Find her gain on 1 doz. frames, if she sold them at 25¢ each.

10. Helen made 12 fern baskets, paying 36¢ for the reed and 10¢ apiece for the ferns to put into the baskets. What was the cost of each filled basket?

11. Alice made 5 laundry bags, using $1\frac{1}{4}$ yd. of towel-ing and 3 yd. of ribbon for each bag. At 40¢ a yard for the toweling, and 10¢ a yard for the ribbon, find the cost.

12. I bought a hat frame, 50¢; a piece of braid, 50¢; 2 bunches of flowers @ 75¢; and a lining, 10¢. How much did the material for the hat cost?

13. How many yards of ruffling are required for the bottom of an apron that measures 62 in., if $\frac{1}{2}$ of the width of the apron is allowed for fullness?

14. A milliner tucked some 18-inch silk for a hat facing. If she put in 12 tucks each $\frac{3}{8}$ in. deep, how wide was the silk after it was tucked?

15. Make out, foot, and receipt the bill for Mrs. Day's hat, the items of which were: hat, \$2.50; 3 yd. ribbon @ 75¢; $\frac{1}{4}$ yd. velvet @ \$1.25; work, 75¢.

16. A ruffle is to be 10 in. wide. How wide must it be cut, if it has a 1-inch hem and 3 $\frac{1}{4}$ -inch tucks?

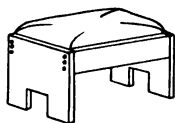
SUGGESTION. — Allow $\frac{1}{4}$ in. for the first turning, in making hems.

17. A doll's one-piece skirt was 10 in. long when finished. If there was a $1\frac{1}{2}$ -inch hem with 5 $\frac{1}{8}$ -inch tucks above it, how long was the piece of which the skirt was made?

Problems for Boys

1. At 5¢ per linear foot, find the cost of enough ash lumber to make a pair of skis each $7\frac{1}{2}'$ long.

2. Find the total cost of the material for this stool:



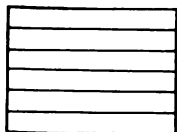
2 end pieces, $12'' \times 9''$, at 12¢ per sq. ft.

2 rails, each $18''$ long, at 5¢ per foot.

$\frac{1}{4}$ sq. yd. imitation leather @ \$1.20.

Screws, stain, and varnish, 25¢.

3. I made an ironing board $4' 4''$ long and $15''$ wide. How much did it cost at 15¢ per square foot?



4. This drawing board is $32''$ long by $27''$ wide and is made of boards $4\frac{1}{2}''$ wide.

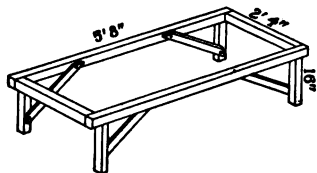
How long a board will just make it?

Find its cost at 4¢ per linear foot.

5. How long a board $3''$ wide will just make such a drawing board?

6. How many boards $4''$ wide and $12'$ long must be bought to make a box cover $4'$ square?

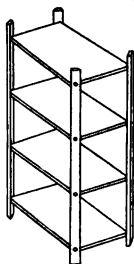
7. From the lengths of the strips given on the drawing, find the number of linear feet of oak strips needed for the frame and legs of this Boy Scout cot.



8. Allowing $4''$ in length and $6''$ in width, over the dimensions given, find, to the nearest square

yard, the amount of canvas needed for the top of the cot. Find the cost of the canvas at 36¢ per square yard.

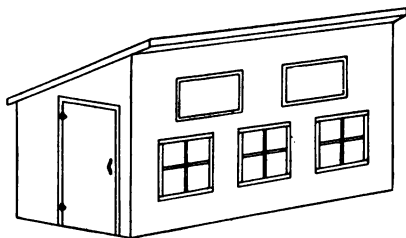
9. Find the cost of the material for this magazine rack at these prices: shelves, each $15'' \times 9''$, at 12ϕ per square foot; legs, each $29''$ long, at $1\frac{1}{2}\phi$ per linear foot; and screws, 20ϕ .



10. The distance between the shelves is $7''$ and the shelves are $\frac{3}{4}''$ thick. What is the distance between the top and bottom shelves? How far is it from the top of the top shelf to the bottom of the bottom shelf?

The sides and roof of this poultry house are boarded and then covered with tarred felt.

11. Make out a bill of materials, extend items, and find the total cost:



5 pieces, $2'' \times 4'' \times 12'$, for plate and sills @ 20ϕ .

1 piece, $2'' \times 4'' \times 14'$, for plate, 25ϕ .

16 pieces, $1'' \times 12'' \times 12'$, for sides @ 32ϕ .

7 pieces, $1'' \times 12'' \times 14'$, for roof boards @ 37ϕ .

3 windows at 60ϕ each.

1 roll three-ply tarred felt for roof, $\$1.05$.

1 roll one-ply tarred felt for sides, 85ϕ .

1 yd. muslin, 9ϕ .

Nails, hinges, and door fastener, 35ϕ .

12. Allowing $4\frac{1}{2}$ sq. ft. of floor space per fowl, how many fowls may be kept in a poultry house, $12'$ long and $6'$ wide?

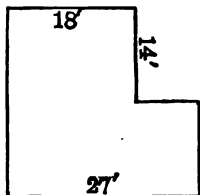
Problems in Measurements

1. Find the cost of painting the front of an apartment house 28 ft. wide and 54 ft. high at 20¢ per square yard.

2. How many yards of unfigured carpet 27 in. wide are required to cover a floor 15 ft. by 14 ft.?

3. Find the expense of paving $\frac{5}{8}$ mi. of street 45 ft. wide with asphalt, at a cost of \$ 1.96 per square yard.

4. How many bricks are required to build a wall 6 ft. high, 36 ft. long, and 16 in. thick?



5. This is a diagram of a cellar. Find the cost of paving the cellar floor with concrete at \$ 1.15 per square yard.

6. The cellar was dug 6 ft. deep. Find the cost of excavation at 45¢ per cubic yard.

7. Find the approximate capacity in barrels of a rectangular cistern 16 ft. by 8 ft. 9 in. by 7 ft. 6 in.

8. A grain elevator had a bin $7\frac{1}{8}$ ft. square and 80 ft. deep. About how many bushels would it hold?

9. Ice, cut into blocks of a certain size and well packed, occupies 40 cu. ft. per ton. How many tons of ice can thus be packed into a space 24 ft. by 20 ft. by 18 ft.?

10. At 35¢ per square yard, find the cost of lathing and plastering the walls and ceiling of a room 18' by 18' and 9'3" high, deducting 8 sq. yd. for openings.

11. How many strips of sod 9 ft. long and 1 ft. wide are required to sod a lawn 45 ft. by 30 ft. that has a path 3 ft. wide running through it lengthwise?

TABLES

DENOMINATE NUMBER TABLES

Length Measures

12 inches	= 1 foot
3 feet	= 1 yard
5 $\frac{1}{4}$ yards	} = 1 rod
16 $\frac{1}{4}$ feet	
320 rods	= 1 mile (statute)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A nautical mile (knot) = 6080.27 ft., or approximately 1.15 mi.

A furlong = $\frac{1}{8}$ mi.; a fathom, used in measuring the depth of water, is 6 ft.; a hand, used in measuring the height of horses, is 4 in.

Area Measures

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30 $\frac{1}{4}$ square yards	= 1 square rod
160 square rods	= 1 acre
640 acres	= 1 square mile

1 acre = 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 ft. on a side.

A tract of land 1 mile square is often called a section.

100 sq. ft. of roofing, flooring, or slating is called a square.

Volume Measures

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, or 128 cu. ft. of wood, is a cord; a cubic yard of earth is a load; 24 $\frac{1}{2}$ cu. ft. of stone is a perch.

Liquid Measures

4 gills	= 1 pint
2 pints	= 1 quart
4 quarts	= 1 gallon

1 gal. = 231 cu. in.; 1 cu. ft. = 7 $\frac{1}{2}$ gal., approximately. A gallon of water weighs about 8 $\frac{1}{4}$ lb.; a cubic foot of water weighs about 62 $\frac{1}{2}$ lb.

In measuring the capacity of cisterns, etc., 31 $\frac{1}{2}$ gal. = 1 barrel.

63 gal. = 1 hogshead.

Dry Measures

2 pints	= 1 quart
8 quarts	= 1 peck
4 pecks	= 1 bushel

1 bu. = 2150.42 cu. in., or approximately 1 $\frac{1}{4}$ cu. ft.

Our bushel is the Winchester bushel. In form it is a cylinder 18 $\frac{1}{4}$ in. in diameter and 8 in. deep. This has been displaced in England by the imperial bushel of 2218.192 cu. in.

Weight Measures

16 ounces = 1 pound
 100 pounds = 1 hundredweight
 2000 pounds = 1 ton
 1 long or gross ton = 2240 pounds.
 1 lb. = 7000 grains (gr.); 1 oz. =
 437½ gr.

Standard Weight of a Bushel

In New York and most other states:
 Oats, 32 lb. Corn (shelled), 56 lb.
 Wheat, 60 lb. Buckwheat, 48 lb.
 Barley, 48 lb. Potatoes, 60 lb.
 A barrel of flour contains 196 lb.

Time Measures

60 seconds = 1 minute	Thirty days have September,
60 minutes = 1 hour	April, June, and November.
24 hours = 1 day	All the rest have thirty-one,
7 days = 1 week	Save February, which alone
365 days = 1 year	Has twenty-eight, and one day more
366 days = 1 leap year	We add to it one year in four.
10 years = 1 decade; 100 years = 1 century.	

United States Money

10 cents = 1 dime
 10 dimes = 1 dollar
 10 dollars = 1 eagle

English Money

4 farthings = 1 penny
 12 pence = 1 shilling
 20 shillings = 1 pound

Foreign Money Equivalents

English: 1 pound (£) = \$4.8665
French: 1 franc (fr.) = \$.193
German: 1 mark (M.) = \$.238

Circular Measures

60 seconds (") = 1 minute (')
 60 minutes = 1 degree (°)
 360 degrees = 4 right angles or 1 circumference
 90° of angle = 1 right angle; 90° of arc = 1 quadrant.

Counting Table

20 = 1 score
 12 = 1 dozen
 12 dozen = 1 gross
 12 gross = 1 great gross

Stationers' Measures

24 sheets = 1 quire
 20 quires = 1 ream
 2 reams = 1 bundle
 5 bundles = 1 bale

MULTIPLICATION TABLES

1 × 1 = 1	1 × 2 = 2	1 × 3 = 3	1 × 4 = 4
2 × 1 = 2	2 × 2 = 4	2 × 3 = 6	2 × 4 = 8
3 × 1 = 3	3 × 2 = 6	3 × 3 = 9	3 × 4 = 12
4 × 1 = 4	4 × 2 = 8	4 × 3 = 12	4 × 4 = 16
5 × 1 = 5	5 × 2 = 10	5 × 3 = 15	5 × 4 = 20
6 × 1 = 6	6 × 2 = 12	6 × 3 = 18	6 × 4 = 24
7 × 1 = 7	7 × 2 = 14	7 × 3 = 21	7 × 4 = 28
8 × 1 = 8	8 × 2 = 16	8 × 3 = 24	8 × 4 = 32
9 × 1 = 9	9 × 2 = 18	9 × 3 = 27	9 × 4 = 36
10 × 1 = 10	10 × 2 = 20	10 × 3 = 30	10 × 4 = 40
11 × 1 = 11	11 × 2 = 22	11 × 3 = 33	11 × 4 = 44
12 × 1 = 12	12 × 2 = 24	12 × 3 = 36	12 × 4 = 48
1 × 5 = 5	1 × 6 = 6	1 × 7 = 7	1 × 8 = 8
2 × 5 = 10	2 × 6 = 12	2 × 7 = 14	2 × 8 = 16
3 × 5 = 15	3 × 6 = 18	3 × 7 = 21	3 × 8 = 24
4 × 5 = 20	4 × 6 = 24	4 × 7 = 28	4 × 8 = 32
5 × 5 = 25	5 × 6 = 30	5 × 7 = 35	5 × 8 = 40
6 × 5 = 30	6 × 6 = 36	6 × 7 = 42	6 × 8 = 48
7 × 5 = 35	7 × 6 = 42	7 × 7 = 49	7 × 8 = 56
8 × 5 = 40	8 × 6 = 48	8 × 7 = 56	8 × 8 = 64
9 × 5 = 45	9 × 6 = 54	9 × 7 = 63	9 × 8 = 72
10 × 5 = 50	10 × 6 = 60	10 × 7 = 70	10 × 8 = 80
11 × 5 = 55	11 × 6 = 66	11 × 7 = 77	11 × 8 = 88
12 × 5 = 60	12 × 6 = 72	12 × 7 = 84	12 × 8 = 96
1 × 9 = 9	1 × 10 = 10	1 × 11 = 11	1 × 12 = 12
2 × 9 = 18	2 × 10 = 20	2 × 11 = 22	2 × 12 = 24
3 × 9 = 27	3 × 10 = 30	3 × 11 = 33	3 × 12 = 36
4 × 9 = 36	4 × 10 = 40	4 × 11 = 44	4 × 12 = 48
5 × 9 = 45	5 × 10 = 50	5 × 11 = 55	5 × 12 = 60
6 × 9 = 54	6 × 10 = 60	6 × 11 = 66	6 × 12 = 72
7 × 9 = 63	7 × 10 = 70	7 × 11 = 77	7 × 12 = 84
8 × 9 = 72	8 × 10 = 80	8 × 11 = 88	8 × 12 = 96
9 × 9 = 81	9 × 10 = 90	9 × 11 = 99	9 × 12 = 108
10 × 9 = 90	10 × 10 = 100	10 × 11 = 110	10 × 12 = 120
11 × 9 = 99	11 × 10 = 110	11 × 11 = 121	11 × 12 = 132
12 × 9 = 108	12 × 10 = 120	12 × 11 = 132	12 × 12 = 144

DIVISION TABLES

1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9
10 ÷ 1 = 10	20 ÷ 2 = 10	30 ÷ 3 = 10	40 ÷ 4 = 10
11 ÷ 1 = 11	22 ÷ 2 = 11	33 ÷ 3 = 11	44 ÷ 4 = 11
12 ÷ 1 = 12	24 ÷ 2 = 12	36 ÷ 3 = 12	48 ÷ 4 = 12
5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
50 ÷ 5 = 10	60 ÷ 6 = 10	70 ÷ 7 = 10	80 ÷ 8 = 10
55 ÷ 5 = 11	66 ÷ 6 = 11	77 ÷ 7 = 11	88 ÷ 8 = 11
60 ÷ 5 = 12	72 ÷ 6 = 12	84 ÷ 7 = 12	96 ÷ 8 = 12
9 ÷ 9 = 1	10 ÷ 10 = 1	11 ÷ 11 = 1	12 ÷ 12 = 1
18 ÷ 9 = 2	20 ÷ 10 = 2	22 ÷ 11 = 2	24 ÷ 12 = 2
27 ÷ 9 = 3	30 ÷ 10 = 3	33 ÷ 11 = 3	36 ÷ 12 = 3
36 ÷ 9 = 4	40 ÷ 10 = 4	44 ÷ 11 = 4	48 ÷ 12 = 4
45 ÷ 9 = 5	50 ÷ 10 = 5	55 ÷ 11 = 5	60 ÷ 12 = 5
54 ÷ 9 = 6	60 ÷ 10 = 6	66 ÷ 11 = 6	72 ÷ 12 = 6
63 ÷ 9 = 7	70 ÷ 10 = 7	77 ÷ 11 = 7	84 ÷ 12 = 7
72 ÷ 9 = 8	80 ÷ 10 = 8	88 ÷ 11 = 8	96 ÷ 12 = 8
81 ÷ 9 = 9	90 ÷ 10 = 9	99 ÷ 11 = 9	108 ÷ 12 = 9
90 ÷ 9 = 10	100 ÷ 10 = 10	110 ÷ 11 = 10	120 ÷ 12 = 10
99 ÷ 9 = 11	110 ÷ 10 = 11	121 ÷ 11 = 11	132 ÷ 12 = 11
108 ÷ 9 = 12	120 ÷ 10 = 12	132 ÷ 11 = 12	144 ÷ 12 = 12

GLOSSARY

Abstract number. A number that is used without reference to any particular kind of object or unit.

Account. A record of business transactions.

Addends. Numbers added.

Addition. The process of uniting two or more numbers into one number.

Aliquot part. A part of a number that exactly divides the number.

Altitude. The height of a figure measured on a line perpendicular to the base.

Analysis. Any course of reasoning by which a problem is solved.

Angle. The difference in the direction of two lines that meet.

Arabic numerals. The ten figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used to write numbers.

Arc. Any part of a circumference.

Area. The number of square units that a surface contains.

Austrian division in decimals. The method of placing the decimal point in the quotient directly *over* the decimal point in the dividend in long division (*under* in short division) after changing the divisor to an integer (when it is not one) and making a corresponding change in the dividend.

Austrian subtraction. Subtraction by addition, that is, finding the number that added to the subtrahend gives the minuend.

Balance of an account. The difference between the sum of the debits and the sum of the credits.

Base. The side of a figure upon which it is assumed to stand.

Bill. A detailed written statement of indebtedness for goods sold or services rendered.

Cancellation. The process of shortening work in division by rejecting equal factors from both dividend and divisor.

Cash account. A record of money paid out and money received.

Central angle. An angle whose vertex is at the center of a circle.

Circle. A figure bounded by a curved line every point of which is equally distant from a point within called the *center*.

Circumference. The boundary line of a circle.

Common denominator. A denominator common to two or more fractions.

Common divisor. A number that is an exact divisor of two or more numbers.

Common factor. Same as *common divisor*.

Common fraction. Same as *fraction*.

Common multiple of several numbers. A number that is divisible by each of those numbers.

Compound denominate number. A denominate number composed of units of two or more related denominations.

Concrete number. A number that is used with reference to some particular kind of object or unit.

Creditor. A person to whom a debt is owed.

Credits. Amounts paid.

Cube. A rectangular solid whose faces are equal squares.

Debits. Amounts owed.

Debtor. A person who owes a debt.

Decimal. Same as *decimal fraction*.

Decimal fraction. A fraction that expresses tenths, hundredths, thousandths, etc.

Decimal point. The period written at the left of tenths.

Denominate number. A concrete number in which the unit of measure is established by law or custom.

Denominator. The number in a common fraction that shows into how many equal parts the unit is divided. It is written below the line.

Difference. The result found by subtracting.

Digits. The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Dissimilar decimals. Decimals that have a different number of decimal places.

Dividend. The number divided by another number.

Division. The process of finding how many times one number contains another, or the process of separating a number into equal parts.

Divisor. The number by which another number is divided.

Even number. A number that is exactly divisible by 2.

Exact divisor. A number that divides another number without a remainder.

Expense account. A record of money expended for various purposes.

Factor. An exact divisor of a number.

Factoring. The process of separating a number into its factors.

Fraction. One or more of the equal parts of a unit.

Fundamental operations. The operations of addition, subtraction, multiplication, and division.

Greatest common divisor. The *largest* number that is an exact divisor of two or more numbers.

Horizontal line. A line that is parallel to the horizon.

Improper fraction. A fraction whose numerator is equal to or greater than its denominator.

Integer. A number that stands for one or more whole units.

Least common denominator. The *smallest* common denominator to which two or more fractions can be reduced.

Least common multiple of several numbers. The *smallest* number that is divisible by each of those numbers.

Long division. Division in which all the steps are written.

Lowest terms. A fraction is expressed in its *lowest terms* when its terms have no common divisor except 1.

Minuend. The number from which another number is subtracted.

Mixed decimal. A number expressed by an integer and a decimal.

Mixed number. A number expressed by an integer and a fraction.

Multiple of a number. A number that is exactly divisible by that number.

Multiplicand. The number multiplied by another number.

Multiplication. The process of taking one number as many times as there are units in another. A short way of adding equal numbers.

Multiplier. The number by which another number is multiplied.

Notation. Any method of representing numbers by characters.

Numeration. Any method of reading numbers represented by characters.

Numerator. The number in a common fraction that shows how many parts form the fraction. It is written above the line.

Odd number. A number that is not exactly divisible by 2.

Orders of units. The units, tens, and hundreds of each period in a number.

Parallel lines. Lines that cannot meet however far they are extended.

Parallelogram. A four-sided figure whose opposite sides are parallel.

Perimeter. The distance around a figure or object.

Periods of figures. Groups of three (or less) figures.

Perpendicular. A line that forms a right angle with another line.

Prime factor. A factor that is a prime number.

Prime number. A number that has no factors except itself and 1.

Product. The result found by multiplying.

Proper fraction. A fraction whose numerator is less than its denominator.

Quotient. The result found by dividing.

Receipt. A statement acknowledging the taking or receiving of anything.

Rectangle. A figure that is bounded by four straight lines and has four right angles.

Rectangular solid. A solid having six rectangular faces.

Reduction. The process of changing the form of a number without changing its value.

Reduction ascending. Reduction to a higher unit.

Reduction descending. Reduction to a lower unit.

Reduction of fractions. The process of changing the form of a fraction without changing its value.

Remainder. In subtraction, same as *difference*. In division, the part of the dividend that is left when the division is inexact.

Right angle. When a straight line meets another straight line forming two equal angles, each angle is a *right angle*.

Roman numerals. The seven letters I, V, X, L, C, D, M, used to write numbers.

Short division. Division in which the operations of multiplication and subtraction are done mentally.

Sign of addition is +, read "*and*" or "*plus*."

Sign of division is \div , read "*divided by*."

Sign of equality is =, read "*are*" or "*equal*" (sometimes "*is*" or "*equals*").

Sign of multiplication is \times , read "*times*" or "*multiplied by*," according as it precedes or follows the multiplicand.

Sign of subtraction is -, read "*less*" or "*minus*."

Similar decimals. Decimals that have the same number of decimal places.

Simple denominate number. A denominate number composed of units of one denomination.

Square. A rectangle whose sides are equal.

Statement of account. A written list of debits and credits of an account.

Subtraction. The process of finding how much greater one number is than another, or the process of finding what number added to one of two numbers will give the other.

Subtrahend. The number subtracted from another number.

Sum. The result found by adding.

Terms of a fraction. The numerator and the denominator.

Test. Any method of verifying a result.

Triangle. A figure bounded by three straight lines.

Unit. Any single thing.

Unit of measure. The standard by which the quantity of anything is ascertained.

Value of a fraction. The quotient of the numerator divided by the denominator.

Vertex of an angle. The point in which the sides meet.

Vertical line. A line that is perpendicular to a horizontal line.

Volume. The number of cubic units that any solid contains.

Whole number. Same as *integer*.

INDEX

- Abstract number, 847.
 Account, 144, 847.
 Accounts, 144-146, 826-838.
 Addenda, 18, 847.
 Addition, 18-18, 68-66, 116-117, 129-181, 154-160, 169-170, 176-177, 178-185, 280-282, 289-290, 817-819, 847.
 defined, 847.
 miscellaneous, 289-290.
 of decimals, 280-282.
 of denominate numbers, 817-819.
 of fractions, 154-160, 169-170, 176-177.
 of integers, 18-18, 68-66, 116-117.
 of mixed numbers, 178-185.
 of U. S. money, 129-181.
 sign of, 18, 850.
 Allquot part, 847.
 Allquot parts, 256-262.
 Altitude, 267, 847.
 Analysis, 206-210, 847.
 Angle, defined, 267, 847.
 measures of, 809-810, 844.
 Arabic notation, 7-12, 111-118.
 Arabic numerals, 111, 847.
 Arc, defined, 809, 847.
 measures of, 809-810, 844.
 Area, 61-62, 108, 267-276, 847.
 Area measures, 61-62, 102, 266, 805, 848.
 Austrian division, 247, 847.
 Austrian subtraction, 22, 28, 25, 27, 847.

 Balance of account, 828, 881, 847.
 Barrel, 297, 848.
 Base, 267, 847.
 Bill, 144, 847.
 Bills and accounts, 144-146, 826-838.
 Boy Scout problems, 142.
 Bundle of laths, 272.
 Business problems, 826-838.

 Calculating, 274.
 Camp Fire Girl problems, 211.
 Cancellation, 147-151, 847.
 Capacity, measures of, 99-100, 152-153, 281-284, 297-801, 848.
 Carpeting, 275-276.
 Cash account, 881, 847.
 Center of circle, 809, 847.
 Central angle, 809, 847.
 Chicken problems, 95.
 Circle, 54, 809, 847.
 Circular measures, 809-810, 844.
 Circumference, 809, 844, 847.
 Coins, 126.
 Common denominator, 168, 847.
 Common divisor, 171, 847.

 Common factor, 171, 848.
 Common fractions, 54-56, 154-170, 174-205, 848.
 addition of, 154-160, 169-170, 176-177.
 defined, 162, 224, 848.
 division of, 196-202.
 multiplication of, 186-195.
 notation and numeration of, 162.
 reduction of, 154-160, 164-168, 174-175, 849.
 subtraction of, 154-160, 169-170, 176-177.
 Common multiple, 172, 848.
 Comparison, analysis by, 207-208.
 Compound denominate number, 297, 848.
 Concrete number, 297, 848.
 Counting exercises, 7-11, 15, 104, 116.
 Counting table, 818, 844.
 Creditor, 828, 848.
 Credits, 828, 848.
 Cube, 212, 278, 848.

 Dates, time between, 818, 819.
 Debits, 828, 848.
 Debtor, 828, 848.
 Decimal fractions, 228-255, 848.
 addition of, 230-232.
 defined, 224, 848.
 division of, 242-251.
 multiplication of, 235-241.
 notation and numeration of, 228-226.
 reduction of, 227-229, 244.
 subtraction of, 238-234.
 Declinal point, 128, 228, 848.
 Degree, 809, 844.
 Denominate numbers, 57, 99-102, 126, 152-153, 268-264, 266, 277, 280, 281-282, 297-325, 848.
 addition of, 817-819.
 defined, 297, 848.
 division of, 822-828.
 multiplication of, 820-821.
 reduction of, 57, 99-102, 126, 152-153, 268-264, 266, 277, 280, 281-282, 298-816.
 subtraction of, 817-819.
 Denominator, 162, 848.
 Difference, 19, 848.
 Digits, 96, 848.
 Dissimilar decimals, 227, 848.
 Dividend, 44, 47, 848.
 Divisibility, tests of, 96-97.
 Division, 44-52, 54-56, 75-92, 128-125, 188-140, 196-202, 242-251, 298, 822-828, 848.
 Austrian method of, 247, 847.
 defined, 848.

 miscellaneous, 298.
 of decimals, 242-251.
 of denom. numbers, 822-828.
 of fractions, 196-202.
 of integers, 44-56, 75-92, 123-125.
 of mixed numbers, 197, 199, 201, 202.
 of U. S. money, 188-140.
 sign of, 44, 850.
 tables of, 44, 80-81, 846.
 Divisor, 44, 47, 848.
 Divisors and factors, 96-98, 219, 848.
 Divisors and multiples, 171-178.
 Drawing to scale, 59-60, 62, 108, 265.
 Dry measures, 152-153, 281-288, 300-301, 348.

 English money, 811-812, 844.
 Equality, sign of, 18, 850.
 Equivalents, capacity, 281-284, 848.
 money, 812, 844.
 Even number, 96, 848.
 Exact divisor, 96, 848.
 Excavation and masonry, 285-286.
 Expense account, 881, 848.

 Factor, 97, 848.
 Factoring, 97-98, 109, 219, 848.
 Fathom, 268, 848.
 Footing, 144, 881.
 Foreign money, 811-812, 844.
 Fractional relations, 160-161, 203-205.
 Fractions, common, 54-56, 154-170, 174-205, 848.
 decimal, 228-255, 848.
 French money, 811-812, 844.
 Fundamental operations, defined, 848.
 Furlong, 268, 848.

 German money, 811-812, 844.
 Greatest common divisor, 171-172, 848.

 Hand, 268, 848.
 Hoghead, 297, 848.
 Horizontal, 61, 848.
 House furnishing problems, 887.

 Imperial bushel, 848.
 Improper fraction, 167, 848.
 Integer, 166, 849.

 Knot, 268, 848.

 Lathing and plastering, 272-273.
 Laths, 272.

- Least common denominator, 168, 175, 849.
 Least common multiple, 172-173, 849.
 Length measures, 57-58, 100-101, 266-264, 805, 843.
 Liquid measures, 99-100, 281-282, 234, 297-299, 843.
 Long division, 84-90, 92, 125, 140, 243-244, 248-251, 298, 849.
 Long ton, $\frac{9}{2}$, 844.
 Lowest terms, 165, 174, 849.
 Making change, 127, 294.
 Maple sugar problems, 816.
 Masonry, 285-286.
 Measurements, 58-60, 62, 108, 215, 265, 267-276, 279-279, 288-289, 842.
 Measures, tables of, 57, 62, 99, 100, 102, 126, 152, 214, 266, 266, 277, 280, 281, 297, 800, 802, 804, 806, 809, 811, 812, 818, 814, 843-844.
 Minuend, 19, 22, 849.
 Miscellaneous exercises, 28, 43, 53, 93-94, 141, 194-195, 202, 241, 251, 271, 315, 324-325.
 Mixed decimal, 224, 849.
 Mixed number, 166, 849.
 Multiple, 120, 172, 849.
 Multiplicand, 29, 32, 87, 849.
 Multiplication, 29-42, 70-74, 120-122, 185-187, 186-195, 285-241, 292, 320-321, 849.
 defined, 32, 849.
 miscellaneous, 292.
 of decimals, 225-241.
 of denom. numbers, 820-821.
 of fractions, 186-195.
 of integers, 29-42, 70-74, 120-122.
 of mixed numbers, 188, 191, 192, 193, 194-195.
 of U. S. money, 185-187.
 sign of, 29, 38, 850.
 tables of, 29, 80-81, 845.
 Multiplier, 29, 32, 87, 849.
 Nautical mile, 268, 843.
 Notation, defined, 849.
 Notation and numeration, 7-12, 111-115, 128, 154-160, 162, 223-226, 287.
 miscellaneous, 287.
 of decimals, 223-226.
 of fractions, 154-160, 162.
 of integers, 7-12, 111-113.
 of U. S. money, 128.
 Roman, 114-115.
 Numeration, defined, 849.
 Numerator, 162, 849.
 Odd number, 96, 849.
 Orchard problems, 296.
 Orders of units, 111, 849.
 tables of, 111, 225.
 Painting and calclmning, 274.
 Parallel lines, 269, 849.
 Parallelogram, 269, 849.
 Partial product, 87, 41.
 Perch, 285, 843.
 Perimeter, 60, 849.
 Periods of figures, 111, 849.
 Perpendicular lines, 267, 849.
 Plastering, 272-278.
 Prime factor, 97, 849.
 Prime number, 97, 849.
 Principles of fractions, 168.
 Problem interpretation, 209.
 Problem making, 148, 190, 210.
 Problems, topical, 95, 134, 142, 211, 296, 316, 338-339, 840-841.
 Boy Scout, 142.
 Camp Fire Girl, 211.
 Chicken, 95.
 for boys, 840-841.
 for girls, 338-339.
 House furnishing, 337.
 Maple sugar, 816.
 Orchard, 296.
 School savings bank, 134.
 Product, 29, 32, 87, 849.
 Proper fraction, 167, 849.
 Quadrant, 844.
 Quotient, 44, 47, 849.
 Receipting bills, 144.
 Receipts, 830, 849.
 Rectangle, 60-62, 108, 267-268, 849.
 Rectangular solid, 212-215, 278-279, 849.
 Reduction, 57, 99-102, 126, 152-153, 154-160, 164-163, 174-175, 227-229, 244, 269-264, 266, 277, 280, 281-282, 298, 299-316, 849.
 defined, 164, 298, 849.
 miscellaneous, 288.
 of decimals, 227-229, 244.
 of denom. numbers, 57, 99-102, 126, 152-153, 269-264, 266, 277, 280, 281-282, 298-316.
 of fractions, 154-160, 164-163, 174-175, 849.
 Reduction ascending, 298, 849.
 Reduction descending, 298, 849.
 Remainder, in division, 49, 50, 52, 83, 90, 350.
 in subtraction, 19, 22, 850.
 Review, 68-79, 104-110, 111-113, 116-125, 216-222, 262-265, 287-296, 334-342.
 Right angle, 267, 809, 850.
 Roman notation, 114-115.
 Roman numerals, 114, 850.
 Scale drawing, 59-60, 62, 108, 265.
 School savings bank problems, 134.
 Sewing problems, 338-339.
 Shop problems, 840-841.
 Short division, 47-48, 50-52, 56, 76-79, 82-83, 91, 124, 139, 242, 246, 248, 298, 350.
 Sign of equality, 18, 350.
 Similar decimals, 227, 850.
 Simple accounts, 331-338.
 Simple denominate number, 297, 350.
 Speed tests, 18, 24, 26, 39, 41, 42, 66, 68, 73, 78, 90, 94, 117, 119, 180, 138, 220, 231, 258, 289, 290, 291, 292, 298, 335, 336.
 Square, 54, 60, 61, 62, 102, 108, 267, 268, 350.
 Statement of account, 328, 350.
 Stationers' measures, 814, 844.
 Subtraction, 19-27, 67-69, 118-119, 132-134, 154-160, 169-170, 176-177, 178-185, 238-234, 291, 317-319.
 Austrian method of, 22, 23, 25, 27, 847.
 defined, 850.
 miscellaneous, 291.
 of decimals, 238-244.
 of denom. numbers, 817-819.
 of fractions, 154-160, 169-170, 176-177.
 of integers, 19-27, 67-69, 118-119.
 of mixed numbers, 178-185.
 of U. S. money, 182-184.
 sign of, 19, 350.
 Subtrahend, 19, 22, 850.
 Sum, 13, 850.
 Tables, of division, 44, 80-81, 846.
 of measures, 57, 62, 99, 100, 102, 126, 152, 214, 263, 266, 277, 280, 281, 297, 800, 802, 804, 806, 809, 811, 812, 813, 814, 843-844.
 of multiplication, 29, 80-81, 845.
 Terms of a fraction, 162, 850.
 Test, 850.
 Tests of divisibility, 96-97.
 Time measures, 306-308, 844.
 Triangle, 60, 270, 850.
 Unit, 162, 850.
 Unitary analysis, 206.
 United States money, 126-142, 844.
 Unit of measure, 297, 350.
 Value, measures, 811-812, 844.
 of a fraction, 162, 850.
 Vertex of an angle, 809, 850.
 Vertical, 61, 850.
 Volume, 212-215, 278-286, 850.
 Volume measures, 212-214, 277, 805, 843.
 Weight measures, 302-304, 844.
 Whole number, 166, 850.
 Winchester bushel, 843.
 Wood measure, 280, 843.



